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AN ANALYTICAL R-CURVE EXPRESSION OF A BRITTLE MATRIX COMPOSITE CONTAINING DISCONTINUOUS FIBRES

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INTRODUCTION

The ultimate tensile strength, resistance to impact and ability to inhibit cracking in a brittle material such as cement can be increased by the addition of short fibres such as those of asbestos or glass. Toughness is an important parameter with such an inhomogeneous composite material which inevitably contains defects such as microcracks, holes and discontinuities at the fibre-matrice interface. The measured critical stress intensity factors for this type of material have been found not to be independent of the specimen geometry and/or the crack growth [1, 2]. These effects can be interpreted with the help of resistance (R) curves which reflect the fracture mechanisms and the role of the fibres. In using the R-curve concept one is not searching for a unique value of toughness as is usual in fracture mechanics but attempting to solve the problem for plane stress. Krafft et al [3] postulate that for a given material and thickness, there is a unique relationship between the crack growth and the strength. This relationship is represented by R-curves and can explain the geometric effects on the factor K_c [4]. In this study an analytical expression has been found which relates the fracture toughness to the characteristics of the materials.

MATERIAL

We have studied the behaviour of cement reinforced by asbestos fibre which is a composite material consisting of a brittle matrix and discontinuous fibres. Specimens were cut from industrially manufactured flat sheets having a thickness of 6.3 mm. The matrix was of Portland cement and the asbestos fibres were randomly distributed in two dimensions. The volume fraction (V_f) was 0.054, the ratio by weight of water to cement was 0.3 and all tests were conducted on material aged for one year.

EXPERIMENTAL TECHNIQUE

The compact tension form of specimen which was used in this study is illustrated in Figure 1. In order to guide the fracture, side grooves were machined into the sample thereby reducing the thickness by 30%. The notch root was cut with a 0.4 mm jeweller's saw. Displacement was measured by an extensometer over a 20 mm gauge length and tests were conducted in an Instron tensile machine.

Figure 2 shows a typical load-elongation curve exhibiting three distinct stages:

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- a linear region
- a non linear region representing stable, slow crack growth visible to the naked eye. During this stage absestos fibres could be seen bridging the crack.
- a discontinuous unstable crack growth region.

The crack length was determined by a compliance method. The compliance curves obtained with a notched specimen and a semifractured specimen with which fibre bridges could be seen, were identical and their slope agrees with that calculated for an isotropic material using the geometric factor (Y) and the elastic modulus (E). On unloading however the semifractured specimens showed a residual opening (δ_r) which could be related to the crack length (Figure 3). In order to draw the R curve, the following was necessary.
- graph of load against crack opening $P - \delta$
- choose an increase in crack size Δa_n
- with the help of specimen curves and knowing the residual opening δ_{rn}, the compliance C_n of the specimen was determined at all points of the curve.
- calculation of the stress intensity factor K_r at each point on the load-extension graph by determining the compliance (inset sketch of Figure 3).

By repeating these operations N times a series value was obtained for K_R and a, from which the R curve is drawn.

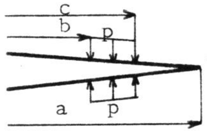
EXPERIMENTAL RESULTS AND DISCUSSION

If during a test the rupture is stable it is possible to draw all of the R curve and to distinguish three stages, see Figure 4.
1. Initially the crack resistance increases very steeply until a crack length of a_1 is reached.
2. In the second region the rate of increase of the resistance is lowered.
3. For an initial crack length of "a" the resistance attains a maximum value and thereafter, in the fourth region remains constant.

The form of the R-curve can help explain the mechanisms of crack growth and the role played by the fibres. The detection and localisation of acoustic emission (A.E.) from the specimen shows that damage is occurring in front of the crack during the first stage after the point A in Figure 4 and is attributed to microcracking of the matrix. This zone increases in size progressively before and during the stable propagation until it reaches a maximum Z_m at the point C. From then onwards in regions 3 and 4 the size of the zone remains constant. The asbestos fibres which bridge the major crack and the microcracks exert a pressure p which opposes the opening of the crack and microcracks and so increases the critical stress intensity factor, by K_2. This is analogous to the Dugdale model [5]

$$K_R = K_o + K_2 \qquad (1)$$

where K_o is the critical stress intensity factor of the matrix in the presence of fibres. Paris [6] has proposed an analytical expression which enables the coefficient K_2 to be calculated.



$$K_2 = \frac{pa^{1/2}}{2\pi^{1/2}}\left[\sin^{-1}\frac{c}{a} - \sin^{-1}\frac{b}{a} - \left(1 - \frac{c^2}{a^2}\right)^{1/2} + \left(1 - \frac{b^2}{a^2}\right)^{1/2}\right] \qquad (2)$$

If the assumption is made that the pressure exerted by the fibre on the microcracks and the major crack are identical, it is possible to consider the zone of microcracking as an extension of $z = \alpha Z (0 < \alpha < 1)$ to the crack on which there is a uniform closing pressure p. With the Paris equation it is possible to draw a theoretical curve and K_o is obtained from the load at which A.E. commences.

The experimentally obtained R-curve determined from the fracture mechanics formulae using the measured crack length gives us the values of the resistance to cracking of the composite which correspond to the maximum size of the zone of microcracking without crack propagation. This value is obtained by extrapolating the parts of the R curve in zones 1 and 3 to the point E in Figure 4. The value of $(K_R - K_o)$ (z_m) relates to the Paris equation which determines the relationship between the pressure p and the size of z_m. By successive approximations it is possible to determine a pressure p which gives the same slope to the experimental and theoretical R-curves in region 3. Having determined p and z_m it is possible to draw the analytical curve, see Figure 5 and comparison of the two curves shows an evolution of the size of the zone of microcracking during the propagation in the regions 1b and 2.

The ratio between the size Z obtained by A.E. and the length of the postulated crack extension (z) given by the R-curve determines the value of the coefficient α. This value, which is of the order of 1/3 remains constant between the points A and C and justifies the use of such a model.

The pressure p can be related to material factors by the relation

$$p = \eta_\theta V_f \bar{\sigma}_f \qquad (3)$$

η_θ being the efficiency factor of the fibres due to their orientation and $\bar{\sigma}_f$ the average fibre pull-out stress.

The value p (8.5 MPa) is 20% less than the elastic limit of the material, the difference can be explained by the triaxiality of the stresses, volume effects and the importance of the microstructure in the zone ahead of the crack.

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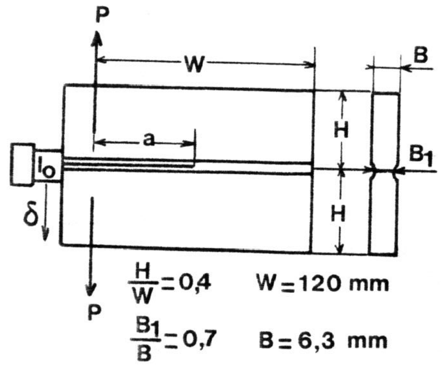

Figure 1 Schema of the Compact Tension Specimen used in this Study

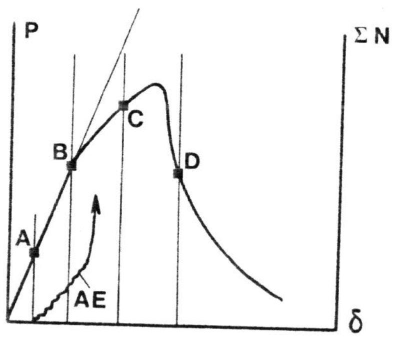

Figure 2 Curves of Load and Summation Acoustic Emission, against Displacement During a Test on a Compact Tension Specimen

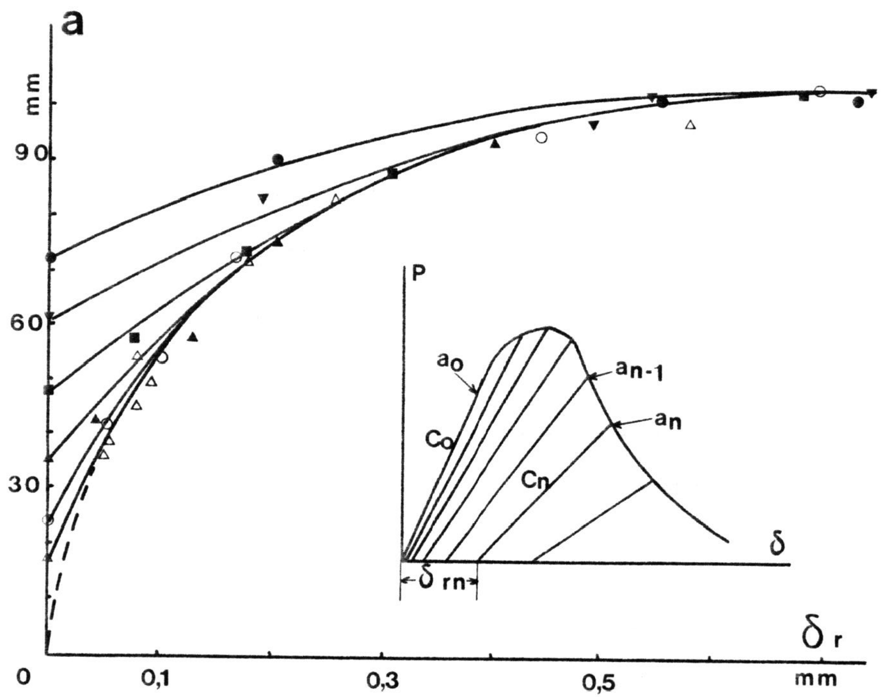

Figure 3 The Relationship Between Crack Length and a Residual Crack Opening

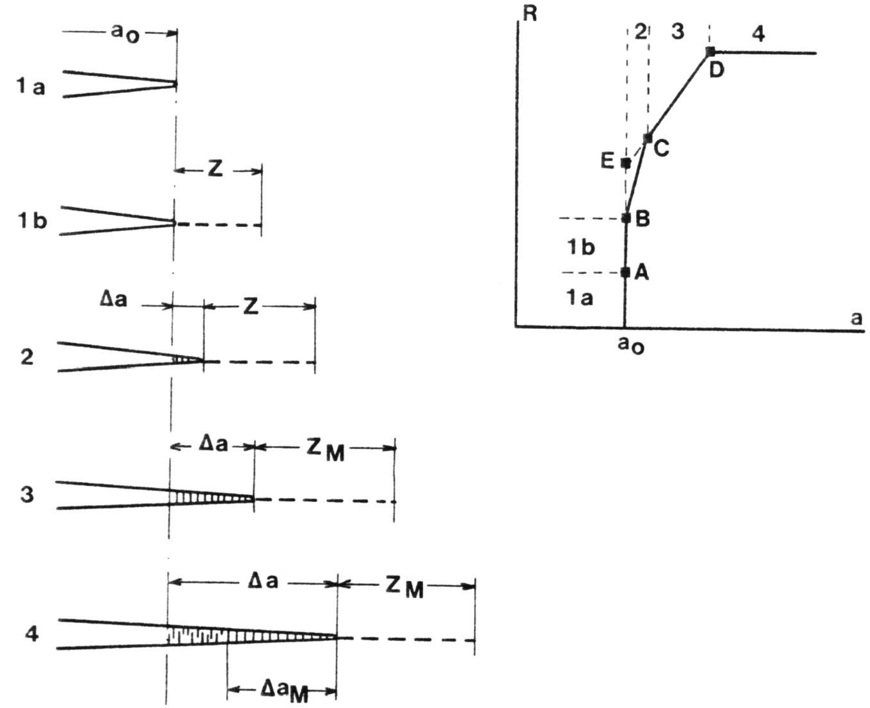

Figure 4 Different Stages of the R-Curve Related to Crack Configuration

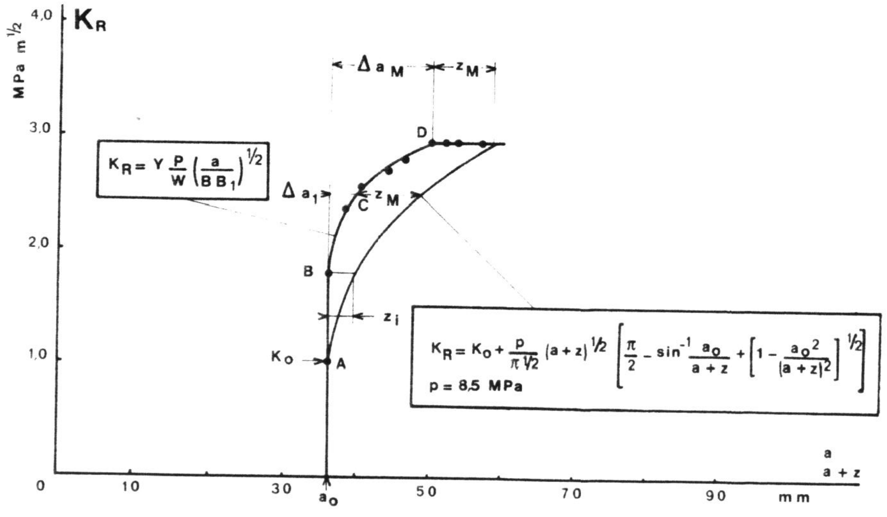

Figure 5 Comparison of Analytical and Experimental R-Curves