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A FINITE-ELEMENT ANALYSIS OF AN IMPACT TEST

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INTRODUCTION

In 1974 Madison and Irwin [1] published results of a fracture test programme begun at Lehigh in 1966. The purpose of the programme was to determine fracture toughness (K_c) values for structural steels at temperatures and loading rates representative of service conditions. The tests employed precracked three-point-bend specimens measuring 76 mm deep, 30 mm long and 25 mm thick. The supported span was 250 mm, and the fatigue crack length, including a starter notch, was approximately 25 mm. Fracture toughness values were computed using the observed maximum load and the initial crack length adjusted to account for plastic-zone size. Loading times as brief as 0.50 ms were judged by Madison and Irwin to be "...small enough for evaluation of minimum dynamic toughness and long enough to permit static stress anslysis of the specimen." It is with this contention that the present paper takes issue.

For a simply supported beam of flexural stiffness EI, mass M and span S, the fundamental frequency of vibration ω is given (see, for example, [2] p.331) by

$$\omega = \pi^2 \sqrt{\frac{EI}{MS^3}} \qquad (1)$$

For steel of dimensions appropriate to the test specimen (1) yields a fundamental period of about 0.37 ms -- a figure much too near the least loading period to warrant neglecting inertia effects. The elementary modal analysis producing (1) neglects shear deformation and rotary inertia, beam overhang and the presence of the crack. But since these are all effects tending to increase the computed fundamental period, a static analysis seems all the more suspect. This suspicion is later confirmed by employing a finite-element model having over 300 degrees of freedom and capable of representing the neglected effects mentioned above.

MADISON-IRWIN PROCEDURE FOR DETERMINING K_c

In the Madison-Irwin exerpiments, the specimen was loaded by an instrumented striking tup mounted in a freely falling weight. The instrumentation provided an oscilloscope trace of the applied load. Two-peak load histories were reported for some of the tests, which Madison and Irwin attributed to obscuring inertia effects. They associated the first peak with inertia effects, while the second peak was judged to be the significant specimen-load record. By placing loading cushions between the specimen and the striking tup, Madison and Irwin obtained a load record with a single peak. This was accepted as evidence that inertia effects had

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been eliminated, and they supposed in their calculations that the peak recorded load was the specimen load at the onset of crack propagation. Figure 1 shows a best-estimate reproduction of a published oscilloscope trace recorded in a -40°C test of a 25 mm thick specimen. The load history depicted in Figure 1 is the type deemed acceptable by Madison and Irwin for static analysis of the problem.

Madison and Irwin used a slightly modified Gross-Srawley [3] formula in conjunction with the peak load from the specimen-load record to obtain a first-estimate value of K_c. Their formula,

$$K_I = \frac{3PS\sqrt{a}}{2BW^2}\left[1.93 - 3.12\,\frac{a}{W} + 14.7\left\{\frac{a}{W}\right\}^2 - 25.3\left\{\frac{a}{W}\right\}^3 + 25.9\left\{\frac{a}{W}\right\}^4\right], \quad (2)$$

gives K_I in MPa$\sqrt{m}$ for P in MN with beam span S, thickness B, depth W and crack length a all m. For the peak load (55.6 KN) obtained from Figure 1, (2) yields a first-estimate K_c of 43.7 MPa$\sqrt{m}$.

Such figures were subsequently revised upward by adjusting for plastic-zone size. Briefly this amounted to increasing the fatigue crack length by the plastic-zone radius

$$r_Y = \frac{1}{2\pi}\left\{\frac{K_I}{\sigma_Y}\right\}^2, \quad (3)$$

in which σ_Y is the yield stress. Equations (2) and (3) were then used repeatedly until the iteration scheme produced practically constant values for r_Y and K_c. Since the thrust of the present paper has to do with assessing inertia effects rather than plasticity effects, we shall make no plasticity adjustments to either our results or those of Madison and Irwin.

FINITE-ELEMENT ANALYSIS OF A MADISON-IRWIN TEST

Figure 2 shows a finite-element representation of a Madison-Irwin test specimen. Due to symmetry about the plane of the crack, only the left half of the specimen is modelled. The model consists of 163 nodes, 273 constant-strain triangles and 1 eight-node crack-tip singularity element. The singularity element ABCDE is required to accurately represent the locally severe stress gradients in the neighbourhood of the fatigue-crack tip at D. Consistent with symmetry requirements, nodes along the crack's prolongation DG are restrained against horizontal displacement. A vertical force equal to half the specimen load is applied at G. A vertical restraint at H simulates the specimen support. The fatigue-crack starter notch was not represented, and the two-dimensional idealization of the problem was taken to be the one corresponding to plane stress.

The singularity element used to numerically characterize the near-tip stress field has been successfully employed in many and varied static applications [4] and has performed satisfactorily in a number of problems involving transient stresses near the tip of a stationary crack [5, 6]. These problems generally represent more severe tests of the finite-element analysis than does the present problem. Details of the singularity element's formulation are given in these references. Briefly, it incorporates as generalized coordinates for stiffness and inertial characterization the

first thirteen symmetric Williams' [7] eigenfunctions and the three parameters associated with plane rigid-body motion. The Newmark - β method with β = 1/4 is the time-integration scheme used for dynamic applications.

Figure 3 shows the time dependence of the stress-intensity factor K_I for three different numerical representations of a Madison-Irwin experiment. The solid line indicates the quasi-static response of the model shown in Figure 2; i.e. $K_I(t)$ appropriate to a massless specimen subjected to the load as taken from the oscilloscope trace (Figure 1). The computed value of K_I at peak load (42 MPa$\sqrt{m}$) is in reasonable agreement with the Madison-Irwin estimate (43.7 MPa$\sqrt{m}$) obtained using (2), but based on the results of previous confirmed static applications, the 4% discrepancy is somewhat more than can be attributed to the numerical method. Notwithstanding the small difference, the quasi-static response shown in Figure 3 is used as a basis for assessing inertia effects in the two companion dynamic executions.

The locus of empty circles in Figure 3 is $K_I(t)$ for a model with inertia characteristics corresponding to steel and subjected to the time-dependent load of Figure 1. The integration time step was 10^{-2} ms and K_I was computed at each time step. This is the order of the transit time of longitudinal waves through the depth of the specimen and consequently the transmission and reflection of individual stress waves is not represented. However, the interest here is in the response over a relatively longer time during which the lower modes of vibration dominate. The shape of the response confirms the expected dominance of the first mode and the earlier estimate of its period. The considerable difference between the dynamic and quasi-static responses is exclusively the result of specimen inertia. When specimen inertia is included, the peak stress-intensity factor is elevated by more than 8% above the maximum quasi-static value. More importantly, the peaks occur at different times. So, for the particular geometry and loading rate under consideration, the dynamic result is in clear conflict with an assumption that the crack begins to propagate at the peak load registered by the oscilloscope. Such a conclusion, of course, rests on the tacit assumption that the oscilloscope trace is in fact an accurate time record of the contact force between the specimen and striking tup.

To illustrate the importance of hammer-tup mass and stiffness, the finite-element programme was executed for the model in Figure 2 with a lumped mass of 45.4 kg attached at G. The lumped mass was given an initial velocity corresponding to a free-fall drop of 0.152 m. It is not clear from a reading of the Madison-Irwin paper that these values for mass and drop height are appropriate for the oscilloscope trace in Figure 1, but the paper does imply that these are probably minimum values for the test programme. The solid circles in Figure 3 indicate computed values of K_I for this representation. These stress-intensity factors are unrealistically high as might be anticipated from the use of such a model in a time span in which non-rigid motions of the hammer are likely to be significant.

CONCLUSIONS

The writers do not claim a successful prediction of time-dependent stress-intensity factors for the impact test that has been discussed. Rather the analyses which have been presented call attention to the danger of ignoring specimen inertia or of an oversimplified model of the hammer. It is the writers' opinion that for relatively high-velocity impact, involving a hammer and specimen of similar materials, an analytical model which accounts for the elastodynamics of the specimen and at least that portion of the hammer tup between the specimen and the load transducer is required.

ACKNOWLEDGEMENTS

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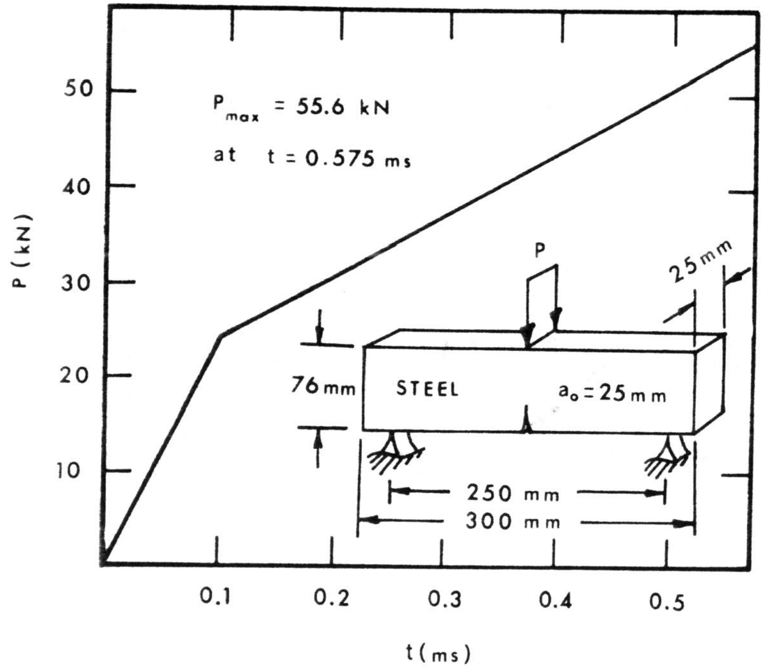

Figure 1

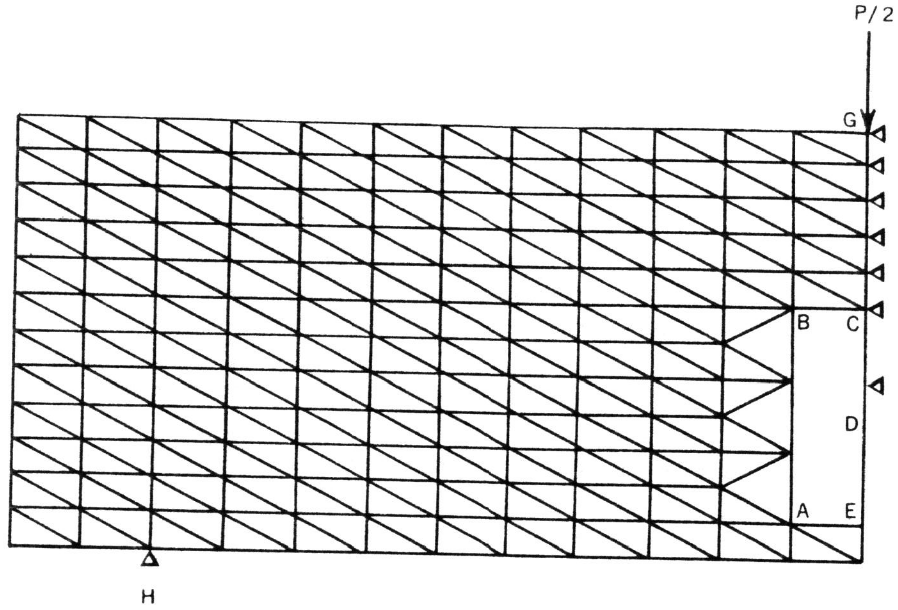

Figure 2

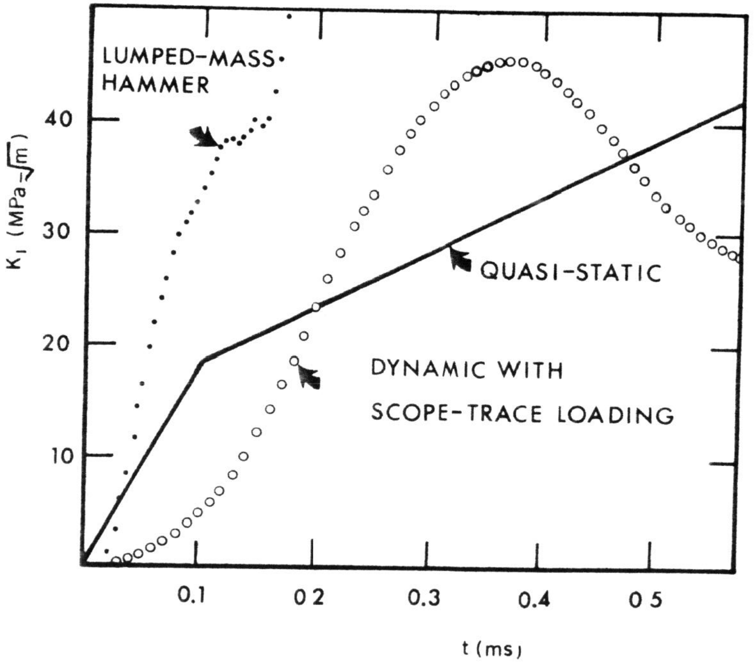

Figure 3