Fracture 1977, Volume 3, ICF4, Waterloo, Canada, June 19 - 24, 1977

A CRACK NEAR DOUBLY RIVETED STIFFENERS

Wang Ke-Jen* and D. J. Cartwright**

INTRODUCTION

Stiffening elements are frequently attached to sheets in structural engineering to improve the strength and stability of the structure. Such stiffeners can also be used to provide a means of slowing down or arresting the growth of cracks in the sheet. The stress intensity factor is less in the vicinity of an intact stiffener because the load is concentrated in the stiffener and hence the stress is lowered in the sheet. However if the stiffener breaks under the increased load, the stress intensity factor increases dramatically. The reinforcement of the sheet may be obtained by making it integral with the stiffener or by attaching a separate stiffener to the sheet by welding, bonding or riveting. Stress intensity factors have been obtained for a crack near a continuously attached stiffener [1-4] and for a crack near discretely attached stiffeners [5-8]. In all these studies it was assumed that the stiffener would either fracture completely or remain intact as the crack passed under it. In practice partial failure of the stiffener may occur, such a situation has been considered both theoretically [8-12] and experimentally [13]. Theoretical work on improving the existing models has been carried out in which the effects of rivet deflection, out-of-plane bending of the stiffener and biaxial stress in the sheet were examined [14,15]. The models of stiffeners so far developed have considered the attachment to be along a single line. Stiffeners are often attached to the sheet with a double row of rivets with the distance between the rows of order 25% of the mean stiffener spacing. In such a situation it may be unrealistic to assume that the stiffener is concentrated along a single line as would be necessary with existing solutions. The purpose of this paper is to examine the effects of a doubly riveted stiffener on the stress intensity factor of a nearby crack and to compare these with the singly riveted stiffener models for some typical stiffened panels.

METHOD OF ANALYSIS

The approach followed to determine the stress intensity factor is based on the force-displacement matching technique as used by Bloom and Sanders [6], Poe [7] and Swift [15]. The formulation is extended, using the work of Erdogan [16] and Mushkelishvili [17], to a configuration in which the stiffeners may have arbitrary spacing. In the present case, satisfying compatibility in the sheet and stiffener between the rth and the (r+1)th rivet $o \leq r \leq R$, the zeroth rivet being on the crack line, gives for the sth stiffener $1 \leq s \leq S$.

---

* Academy of Science, Peking, China
**University of Southampton, England

$$\frac{(1+\nu)f}{E_{sh}B_{sh}} \sum_{j=1}^{S}\sum_{i=1}^{R} P_{i,j}\left[v^b(z_{i,j};z_{s,r+1}) - v^b(z_{i,j};z_{s,r})\right]$$

$$+ \frac{\ell}{A_{st}E_{st}}\sum_{j=r+1}^{R} P_{s,j}$$

$$= \frac{\ell\sigma}{E_{sh}} - \frac{f(1+\nu)\sigma a}{2E_{sh}}\left[v^a(z_{s,r+1}) - v^a(z_{s,r})\right] \qquad (1)$$

where $\ell = 2c_o$ with $f = 2$ for $r = o$ (c_o = distance to the first rivet from the crackline in each stiffener) and $\ell = c_1$ the rivet pitch, with $f = 1$ for $r \geq 1$. The unknown rivet forces in the sheet are $P_{i,j}$. The function $v^a(z_{s,r})$ is the displacement at $z = z_{s,r}$ ($z = x+iy$) in the sheet, containing a crack of length 2a along the x axis with centre at $z = o$. subjected to a uniform stress σ remote from and perpendicular to the crack line. The function $v^b(z_{i,j};z_{s,r})$ is the displacement in the sheet at $z = z_{s,r}$ due to a symmetrical pair of opposing unit forces located at $z = \pm z_{i,j}$ acting away from and in a direction perpendicular to the crack line. The quantities E_{sh} and ν are the elastic modulus and Poisson's ratio of the sheet material and B_{sh} is the sheet thickness. The stiffener has elastic modulus E_{st} and transverse area A_{st}. The displacement functions v^a and v^b have been determined from the work of Erdogan [16] and Mushkelishvili [17]. For the case in which the sth stiffener is broken at the crack line equation (1) takes the form, for $r = o$.

$$\frac{1}{A_{st}E_{st}} \sum_{j=1}^{R} P_{s,j} - \frac{\sigma}{E_{sh}} = o. \qquad (2)$$

By considering each rivet interval on all stiffeners a series of S x R simultaneous equations are obtained for the unknown rivet forces $P_{i,j}$. The opening mode stress intensity factor K_I is then determined from

$$K_I = \sigma\sqrt{\pi a}\left[1 + \frac{1}{\sigma\sqrt{\pi a}} \sum_{j=1}^{S}\sum_{i=1}^{R} P_{i,j}\, g(z_{i,j})\right] \qquad (3)$$

where $g(z_{i,j})$ is the Green's function for a pair of collinear and opposing unit forces in the sheet at $\pm z_{i,j}$ acting away from and perpendicular to the line of the crack.

In the present work each doubly riveted stiffener of area A_{st} having rivet rows distance h apart is represented by two stiffeners h apart each having an area $A_{st}/2$. These two half-stiffeners either extend independently (double stiffener model) or are constrained such that each pair of rivets which are h apart move by equal amounts (coupled stiffener model). The double stiffener model represents a doubly riveted stiffener in which the in-plane shear deformation between the two rivet rows is unrestrained. The coupled stiffener model represents the other extreme in which the in-plane shear deformation between the two rivet rows is completely restrained.

RESULTS AND DISCUSSION

Equations (1), (2) and (3) were solved using a CDC 7600 computer. The number of rivets per stiffener was increased until either the sum of the rivet forces on the most highly loaded stiffener changed by less than 5% or the stress intensity factor changed by less than 0.01%. For a symmetrical array of stiffeners the stress intensity factors from equations (1) and (3) were found to be less (∿ 6%) than those given by Poe [7] particularly when the crack tip was close to the stiffener. This difference arises because the stiffener assumed by Poe is not concentrated at the sheet surface and is thus slightly more flexible than a concentrated stiffener of the same area. For a rigid stiffener ($\mu = 1$ in reference [7] where $\mu = A_{st}E_{st}/(A_{st}E_{st}+wB_{sh}E_{sh})$ and w = stiffener spacing) the stress intensity factors will not depend on the different stiffener models and for this case the results of Poe [7] and those from equations (1) and (2) were found to be within 1%. A comparison with results of Bloom and Sanders, who used a concentrated stiffener, showed agreement within 1% for both a single intact and a single broken stiffener.

Stress intensity factors for the double stiffener and coupled stiffener models, for the panel in Figure 1, are shown in Figure 2. At crack lengths less than a/w = 0.5 the two models lead to similar results because the crack is dominated by the central stiffener for which, because of symmetry, each adjacent rivet interval either side of the crack centre extends by equal amounts irrespective of any coupling. For semi-crack lengths less than the stiffener spacing the coupled stiffener gives a lower stress intensity factor. This occurs because for the coupled stiffener model the inner half of the stiffener is constrained by the outer half and therefore its extension is correspondingly reduced. This results in higher rivet forces at the rivets of the inner half-stiffener and lower rivet forces at those of the outer half-stiffener. Because of the Green's function distribution, equation (3), this change in the rivet forces causes a reduction in the stress intensity factor for a tip approaching the stiffener and an increase when the crack tip has passed the stiffener compared to that for the double stiffener. In Figure 3 the effect of rivet spacing is shown for the single stiffener $c_2/c_1 = o$ and the double stiffener $c_2/c_1 = 2$ for the panel in Figure 1. The ratio of stress intensity factor for the double stiffener K_I^D and that for the single stiffener K_I^S are shown in Figures 4 and 5. It is shown in Figure 4 that except near the stiffener the stress intensity factor for the double stiffener is less than that for the single stiffener with equal pitch rivets: Figure 5 shows that this reduction is less when the single stiffener has an equal number of rivets rather than equal pitch rivets. In Figure 6 the effect of a broken double stiffener across the line of the crack is shown for the stiffened panel in Figure 1. For semi-crack lengths longer than one quarter of the bay all results are within approximately 10% of each other for the stiffness ratio shown.

ACKNOWLEDGEMENTS

The work presented in this paper was carried out whilst one of us (WK) was on leave from the Chinese Academy of Sciences at the Department of Mechanical Engineering, The University of Southampton, England under the auspices of the British Council.

REFERENCES

1. ISIDA, M. and ITAGAKI, Y., Proc. Fourth U.S. National Congress on Applied Mechanics, 1962.
2. GREIF, R. and SANDERS, J. L., J. appl. Mech., 32, 1965, 59.
3. SANDERS, J. L., NASA Tech. Report R-13, 1959.
4. KANAZAWA, T., MACHIDA, S. and OHYAGI, M., J. Soc. Naval Arch. Japan, 121, 1967, 225 (RAE Library Translation, No. 1648, 1972.)
5. ROMOUALDI, J. P., FRASIER, J. T. and IRWIN, G. R., NRL Report 4956, Washington, 1957.
6. BLOOM, J. M. and SANDERS, J. L., J. appl. Mech., 33, 1966, 561.
7. POE, C. C., NASA Tech. Report R-358, 1971.
8. KANAZAWA, T., MACHIDA, S. and OHYAGI, M., J. Soc. Naval Arch. Japan, 122, 1967, 200 (RAE Library Translation, No. 1648, 1971).
9. MADISON, R. B., Application of fracture meachanics to bridges. Ph.D. Thesis, Lehigh University, 1969.
10. MILLER, M. and CARTWRIGHT, D. J., Int. J. Engng. Sci., 12, 1974, 353.
11. CARTWRIGHT, D. J. and MILLER, M., Int. J. Fracture, 11, 1975, 925.
12. ARIN, K., Engng. Fract. Mech., 6, 1974, 133.
13. POE, C. C., ASTM, STP 486, 1971, 79.
14. SWIFT, T., AGARDograph 176, 1974, 227.
15. SWIFT, T., Proc. Conf. on Prospects of Fracture Mechanics, 1974.
16. ERDOGAN, F., Proc. Fourth U.S. National Congress of Applied Mechanics, 1, 1962, 547.
17. MUSHKELISHVILI, N. I., Some basic problems of the mathematical theory of elasticity (translation) Noordhoff, 1953.

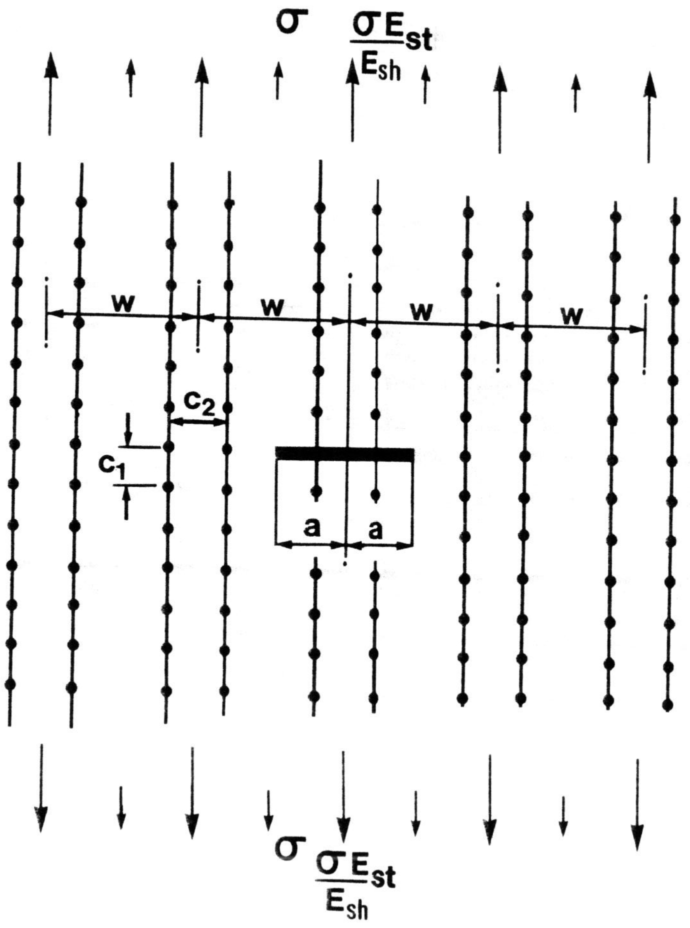

Figure 1 Panel with doubly riveted stiffeners

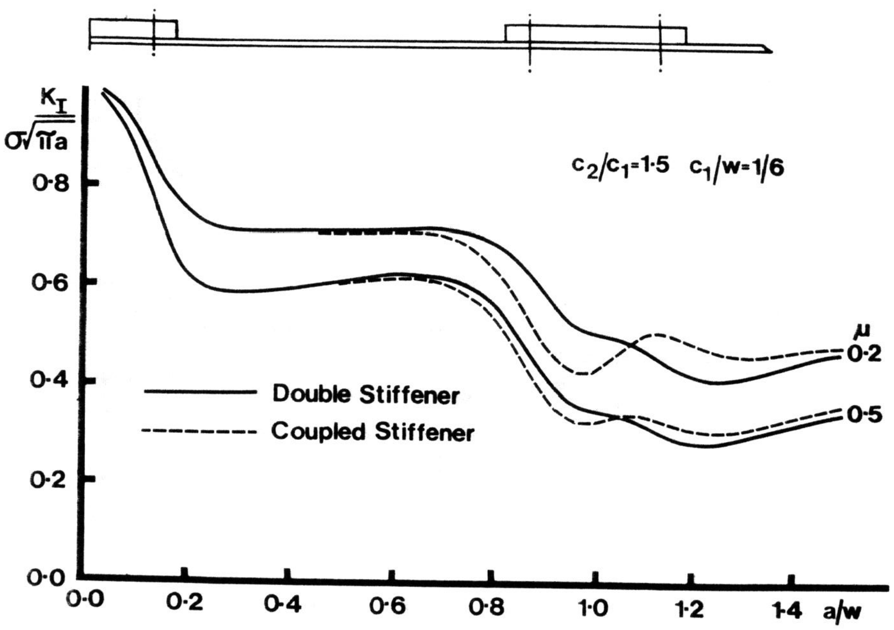

Figure 2 Comparison of stress intensity factors for the double stiffener and coupled stiffener.

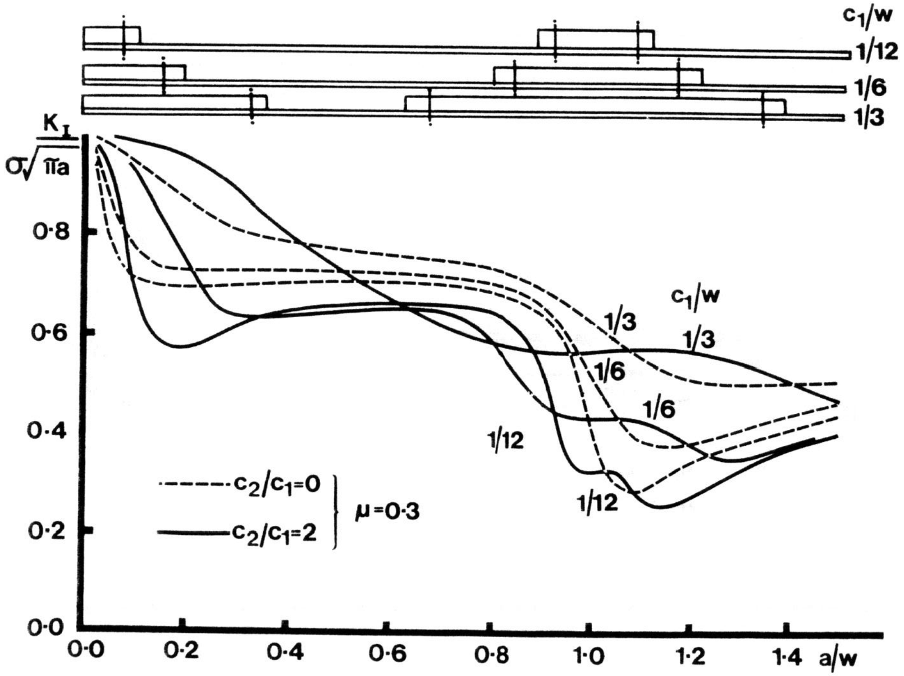

Figure 3 Comparison of stress intensity factors for the double stiffener and a single stiffener showing the effect of rivet pitch.

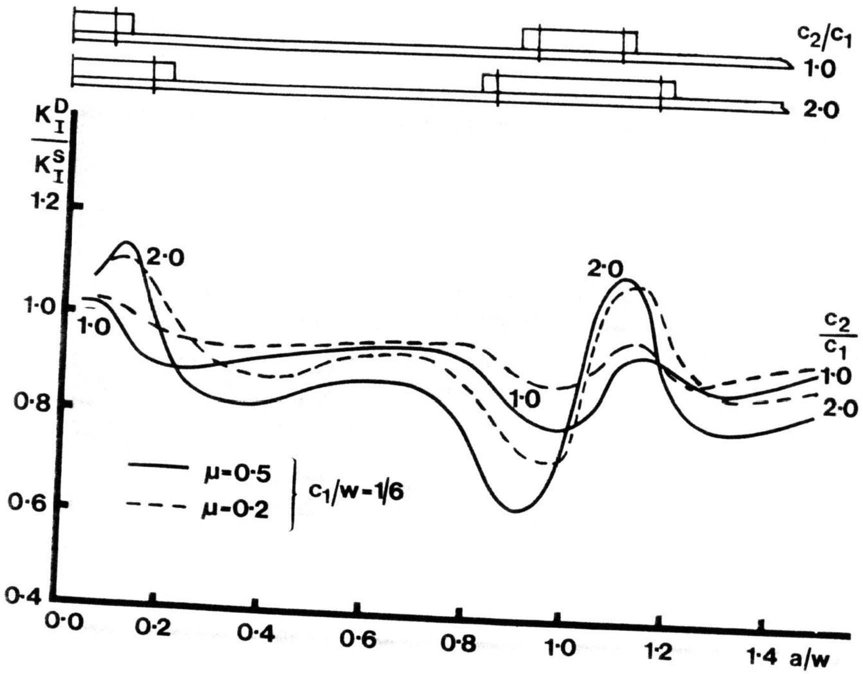

Figure 4 Ratio of stress intensity factors for the double stiffener and a single stiffener having equal pitch rivets.

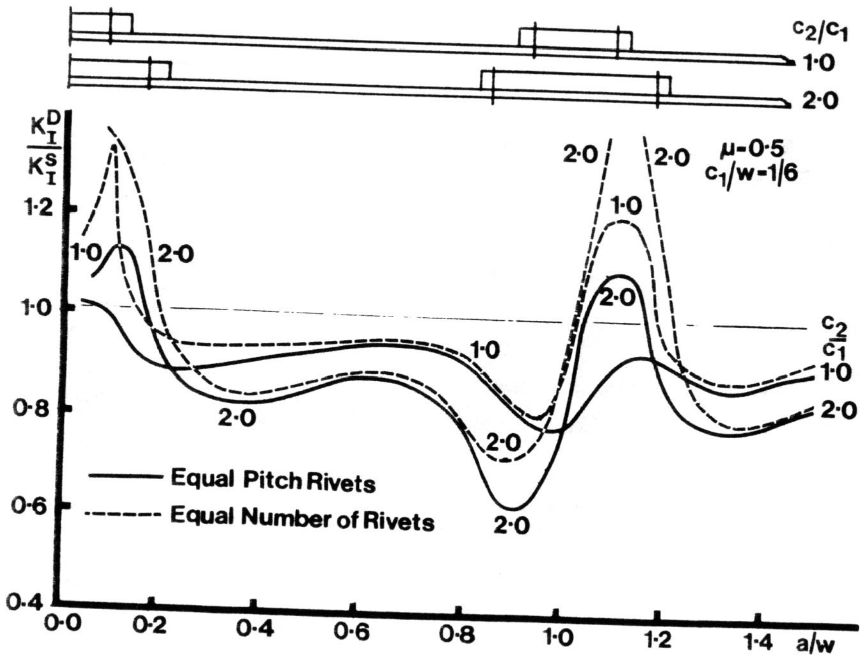

Figure 5 Ratio of stress intensity factors for the double stiffener and a single stiffener having an equal number of rivets or equal pitch rivets.

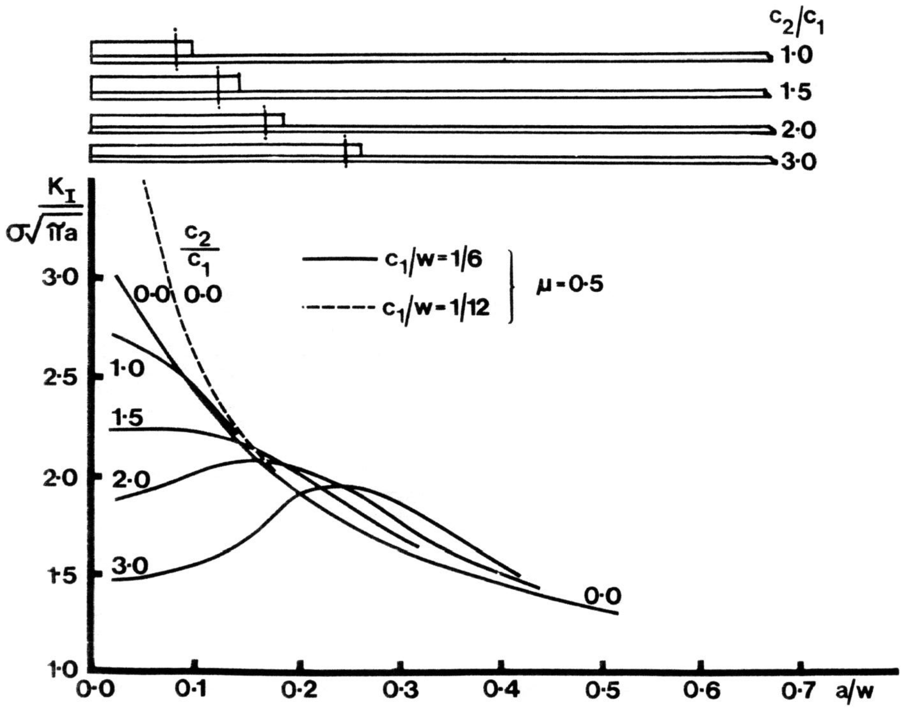

Figure 6 Effect of a central broken stiffener on the stress intensity factor for a single stiffener and the double stiffener.