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A CONTRIBUTION TO THE ANALYSIS OF QUASI STATIC CRACK GROWTH IN SHEET MATERIALS

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INTRODUCTION

In general, aircraft structures have to satisfy conditions with respect to strength, stiffness and weight. These requirements have led to the application of thin walled stiffened structures made of high strength aluminium alloys. The linear elastic behaviour of these structures can be described accurately by the simple mathematical models from the linear theory of elasticity, and in combination with the finite element technique the stiffness of complex aircraft structures can be analysed accurately for general loading cases.

With respect to the analysis of strength the situation is much more complicated. Non-linear effects such as buckling and plastic deformation have a pronounced influence on the load level that can be applied to the structure. Another important aspect that frequently has to be considered is the sensitivity of structural strength to fatigue damage. As a result of increasing lifetimes and the character of the loading conditions, initiation of fatigue cracks cannot always be prevented. In this situation it is important that the rate of crack growth is limited, and also that the strength of the weakened structure is not reduced in such a way that its reliability is endangered.

In the present investigation the behaviour of cracked structures subjected to high peak loading conditions is considered. Using the Finite Element Method the phenomenon of slow stable crack growth can be analysed. In this way detailed information is obtained about the behaviour during crack growth of well known crack growth parameters such as the value of the path-independent J integral, the crack extension force G and the crack opening displacement. It is thought that for an analysis of residual strength of structures such information can be very useful.

Of course, the occurrence of a peak loading situation also affects the subsequent crack growth behaviour under fatigue loading conditions. Nowadays, it is known that the crack growth rate is strongly affected by peak loads. In the present paper some aspects of crack growth retardation and acceleration are considered.

For both studies the Finite Element Method is applied intensively. The results obtained are verified by a comparison with experimental data.

SCOPE OF THE PRESENT STUDY

Plasticity effects and the complex changes of boundary conditions associated with crack growth, crack opening and crack closure can be treated by

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the Finite Element Method. At the NLR laboratory a FEM computer program was developed specifically to deal with these situations. In the present investigation crack growth is realised by disconnecting finite elements. In view of the incremental description of plasticity effects this procedure has to be executed in small steps. In section 3 it is shown that application of the disconnecting procedure has consequences with regard to the energy dissipation in the crack tip zone. These consequences are analysed.

In the study of slow stable crack growth the decision whether elements must be disconnected or not is taken in such a way that the load versus crack length curve that was measured for the specimen under consideration is realised also in the FEM analysis of that particular specimen. Using this procedure experimental results can be analysed in detail. The method is applied in an analysis of the crack growth behaviour in two different sheet specimens weakened by central fatigue cracks of lengths respectively 61 and 123 mm. The results are presented in section 4. In the same section it is concluded that soon after initiation of crack growth the angle between the crack surfaces near the crack tip seems to be constant. This result provides a ground for the assumption that after initiation of crack growth the severity of the crack tip situation can be characterised by the crack tip opening angle CTOA. The critical value of the CTOA can also be measured accurately using an optical microscope, a fact that is an important advantage with regard to other crack growth parameters.

In the analysis of peak loading effects on fatigue crack growth, three aspects are considered. Firstly, a peak load is applied to a structure weakened by a flat central initial fatigue crack. At the maximum load level the elastic-plastic deformations and the crack opening displacements are computed. Then the loading forces are decreased. During this part of the loading trajectory the crack closure behaviour and the reversed plastic flow near the crack tip are studied. It will be shown in section 4 that both effects depend strongly on the geometry of the deformed crack surfaces in the maximum peak load situation. The results show the important influences of the peak load level and the geometry of the deformed crack on the stress distribution in the crack tip zone. Finally, subsequent fatigue crack growth through the plastic zone is analysed. In the finite element model the crack length can be increased by a simple procedure. In this way the effect of a relatively large plastic zone, located at a fixed position near the initial crack tip, on the crack extension force was computed for different actual crack lengths. From this computation it can be concluded that in terms of the distance between initial and actual crack tip, the effect of a simple overload is limited. Nevertheless, it is thought that the results obtained justify further effort to improve the methods used here and to arrive at a more complete model of fracture.

DISCRETIZATION OF CRACK GROWTH

In an application of the Finite Element Method crack growth can be realised in different ways. In this investigation a nodal force relaxation technique was applied. Adopting this method the forces connecting two finite elements are relaxed in a stepwise manner. For reasons of simplicity, in this analysis linear displacement finite elements (trim 3) were used. During relaxation of a nodal force the crack length cannot be defined in an unambiguous manner, but after complete relaxation the crack length has increased by the length of one element side.

It is noted that, in an analysis of energy dissipation near the crack tip, the work rate associated with the relaxation of nodal forces has to be considered. To estimate the significance of this complication, during relaxation a linear relation is assumed between the connecting force p and the displacement u at a node. Denoting the work rate by $\Delta R/\Delta a$, it follows that

$$\Delta R/\Delta a = \frac{1}{2}\sum_i p_i^a\, u_i^{a+\Delta a}/\Delta a \qquad (1)$$

where Δa is identified with the discrete increment of crack growth (= the element size). Further, p_i^a and $u_i^{a+\Delta a}$ denote respectively the connecting forces before and the displacements after completion of the relaxation procedure. In the present 2D-analysis the crack line is the axis of symmetry. Thus, in p_i^a two components of equal magnitude are significant. Therefore, in equation (1) the subscript i will be dropped. In the case of purely elastic material behaviour it is clear that $\Delta R/\Delta a$ equals -G, where G denotes the crack extension force. In the case of ideal plastic material behaviour the value of $\Delta R/\Delta a$ is estimated. It will be assumed that

$$p^a = -\beta\,\sigma_y\Delta a\ ,\quad u^{a+\Delta a} = \alpha\,\Delta a/2 \qquad (2)$$

In these formulas β is a constraint fractor and σ_y is the uniaxial yield limit. The scalar α stands for the crack tip opening angle CTOA. In section 4 the value of α is identified as a material constant. Then after substitution of the formulas from (2) into equation (1), for one crack tip, it follows that

$$\Delta R/\Delta a = -\,\alpha\,\beta\,\sigma_y\Delta a/2 \qquad (3)$$

From this consideration it appears that the work rate $\Delta R/\Delta a$ varies in proportion to the element size selected at the crack tip, and approaches zero if Δa tends to zero. As shown in Figure 1 for the real material behaviour a similar relation was found from the result of the FEM computations. This, in fact, invalidates a computation of the energy dissipation in the crack tip zone. However, the apparent error can be made arbitrarily small by proper choice of the element size. In the present investigation the element size was chosen in such a way that value of $\Delta R/\Delta a$ is small when compared with the plastic energy dissipation.

ANALYSES AND RESULTS

In the present analysis of slow stable crack growth the behaviour of two different sheet specimens weakened by central fatigue cracks of lengths respectively 61 and 123 mm is studied. The specimens were cut from 2024-T3 aluminium. The sheet thickness was 2 mm and the width of the specimens 600 mm.

From the FEB solutions obtained, the value of the J-integral was computed along two different contours through the purely elastic part of the specimen. Path independence was verified. The values of J are presented in Figure 2. The initial crack lengths are indicated. It is seen that the value of J depends on the amount of crack growth $a-a_o$. Further, the significant terms in the balance between rates of deformation energy quantities and the work rate associated with the external loading forces were computed. The balance was verified. From the results it follows that the elastic deformation energy is not strongly affected by plasticity effects: at the onset of unstable crack growth a difference of about 5 per cent was found between the FEM result and the value computed using an analytical formula that is based on purely elastic material behaviour. However, the rate of plastic energy dissipation and thus also the work rate associated with the external loading forces was much larger than predicted by certain analytical formulas. Moreover, the rate of energy dissipation seems to depend on $(a-a_o)/a_o$, instead of a-a as frequently suggested in the R-curve approach. In Figure 3 the values of $\Delta W/\Delta a$ are plotted in relation to $(a-a_o)/a_o$. The present results raise some questions regarding the validity of the R-curve approach in its current formulation.

At different load levels the crack opening displacements were computed. These displacements determine the actual shape of the crack. The results are given in Figure 4. It is concluded that soon after initiation of crack growth the angle between the crack surfaces near the tip is constant. This angle will be called the Crack Tip Opening Angle CTOA. It was also observed that this angle is nearly independent of the element size applied in the crack tip region. The computed values of the CTOA are given in Figure 5. The crack opening displacements were also measured using an optical microscope. In Figure 6 the measuring result is indicated. It is seen that the CTOA value obtained from the tangent modulus of this curve is in agreement with the computational result. These results provide grounds for the assumption that after initiation of crack growth the severity of the crack tip situation can be characterised by the CTOA. In this circumstance it seems logical to assume that the crack grows when the value of the CTOA exceeds some critical value. Thus, a deformation type of crack growth criterion is obtained. It is to be noted that the critical value of the CTOA can be obtained by direct measurement. Hence, adopting the CTOA as crack growth parameter, some important sources of inaccuracy associated with the determination of values for the usual crack growth parameters can be eliminated.

From the displacement pattern shown in Figure 4 an important conclusion can be drawn: for load levels below the level for initiation of stable crack growth the crack tip is blunted, while for higher peak load levels a sharp crack tip is observed. To analyse the effect of the geometry of the crack on the subsequent crack closure behaviour the loading forces are reduced. From the FEM solutions obtained at different load levels during this unloading procedure, it follows that the sharp crack tip immediately starts closing. Subtracting the "elliptic" elastic solution from the linear pattern from Figure 4 this behaviour can be verified. Further, it is concluded that the compressive stresses acting at the closed crack surfaces considerably delay reversed plastic flow and, thus preserve the plastic strain field produced as result of the peak load.

The FEM solutions obtained for a crack with blunted tips show a completely different behaviour. It was found that reversed plastic flow is immediately initiated, while crack closure is first observed at about zero load level midway between the crack tips. From there on crack closure proceeds towards the tips. However, all FEM results obtained for a blunted crack tip show that the tip itself remains open, even after application of compressive loading forces of the same order of magnitude as the original tensile peak load. From these observations it is concluded that stable crack growth has a tremendous effect on the deformation fields in the crack tip zone. Although, in general, the load level for initiation of stable crack growth is not sharply defined, it is thought that insight into the extreme situations discussed here will help to explain the effect of peak loads on the subsequent fatigue crack growth behaviour.

Analysing subsequent fatigue crack growth, it is recognised that the crack grows through the relatively large plastic zone associated with the peak load. Then, after a certain amount of fatigue crack growth, the plastic zone is situated behind the crack tip and the distance between plastic zone and crack tip is increasing. As a result of this increasing distance the effect of the peak load on the actual crack tip situation will diminish. Simulating fatigue crack growth by the nodal force relaxation model applied at a relatively low load level this effect can be analysed by the FEM. The method was applied and after completion of each nodal force relaxation the elastic deformation energy in the structure was computed. From this array values of the crack extension force G were obtained for different distances between plastic zone and actual crack tip. From the results it follows that after an amount of crack growth of the same order of magnitude as the plastic zone size the effect of the peak load on G has damped out. So, in terms of crack length the effect of a single peak load on G is limited.

CONCLUSIONS

1) The value of the J integral seems to be a useful parameter to predict initiation of stable crack growth in centrally cracked sheets.
2) During stable crack growth the crack tip opening angle CTOA is nearly constant and independent of the element size applied to discretize the crack tip zone.
3) The R curve concept needs revision.
4) After initiation of stable crack growth the CTOA seems to be an attractive parameter to describe stable crack growth.
5) The effect of peak loads on subsequent fatigue crack growth can be analysed by the Finite Element Method.

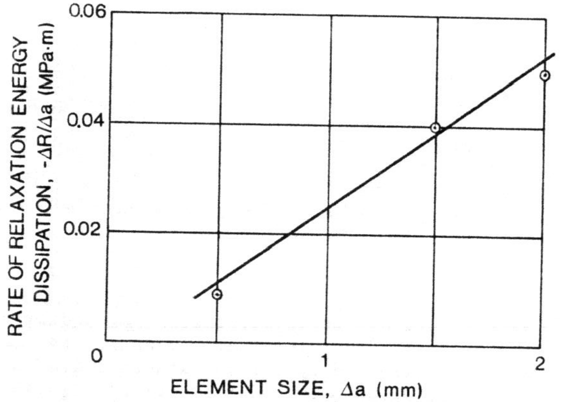

Figure 1 The relation between the work rate associated with the relaxation of nodal forces and the element size applied in the crack tip region

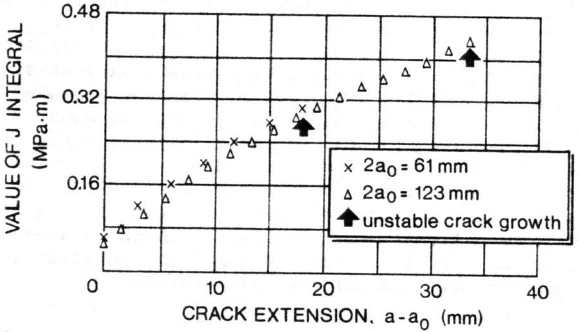

Figure 2 The relation between the value of J and the amount of crack growth

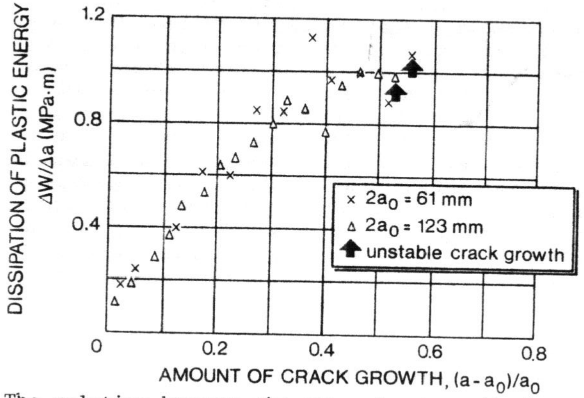

Figure 3 The relation between the rate of energy dissipation in the plastic zone and the amount of crack growth

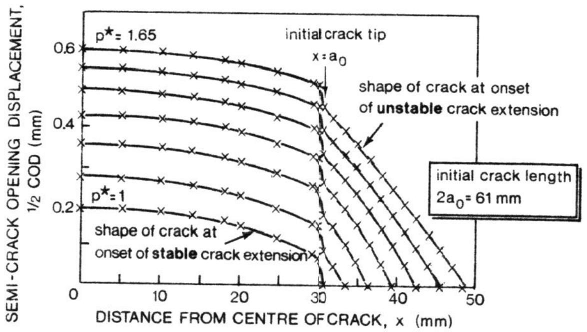

Figure 4 Shape of the crack as computed at different stages of loading

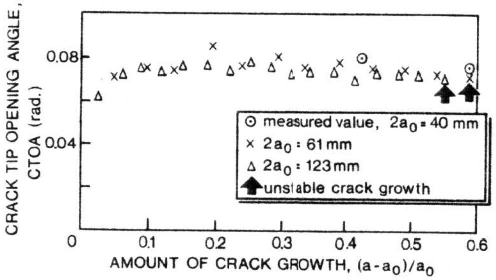

Figure 5 Computed and measured values of the crack tip opening angle in relation to the amount of crack growth

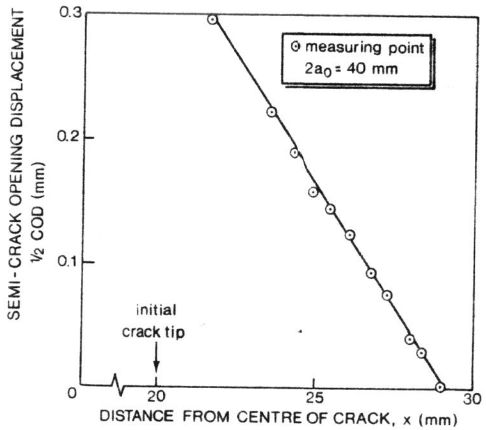

Figure 6 Shape of the crack as measured after 40 percent crack growth