CRACK CLOSURE RELATED TO FATIGUE CRACK PROPAGATION

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INTRODUCTION

Crack growth due to varying load amplitudes has been the subject of much interest in the past few years. This has led to greater insight in the problem, but still a good prediction asks for many experiments, and is expensive. It has been the aim of this research to find a prediction method which requires minimum total cost of experiments and calculations. If possible program load experiments should be avoided and replaced by cheaper and faster calculations. As a basis for the calculations the material constants, derived from the stress-strain diagram, and the crack growth rate at a number of constant amplitude loads are assumed to be known. Wheeler [1] developed the concept of a retardation factor C_{pi} to be used when the load amplitude suddenly decreases. The crack length after r cycles at the low level, a_r, can be expressed as

$$a_r = a_o + \sum_{i=1}^{r} C_{pi} \left\{ f\left(\Delta K_i\right) \right\} \quad (1)$$

in which $$C_{pi} = \left(\frac{R_y}{a_q - a} \right)^m \quad (2)$$

R_y = length of current yield zone, a_q = length of greatest prior elastic plastic interface (see Figure 1). The exponent m has to be determined by experiments. An attempt is thus made here to avoid these experiments and to give the calculations a more physical basis. It is hoped that this will lead to a method that can be applied more generally.

PROPOSED METHOD

Crack closure [2] is assumed to be the main factor governing crack growth retardation and acceleration.

Other factors than crack closure may be of importance with regard to crack growth [3], but remain undiscussed. The change of stress condition in thickness direction and its influence on crack closure [4] should be considered in the future. So the crack growth will be governed by the effective ΔK, ΔK_{eff},

$$\Delta K_{eff} = S \cdot \Delta K \quad (3)$$

In which ΔK is calculated from the external load P and the geometry.

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$$S = \frac{P_{max} - P_{op}}{P_{max} - P_{min}} \qquad (4)$$

P_{op} = the load at which the crack opens when increasing the load from P_{min}.

Then the crack propagation rate is:

$$\frac{da}{dN} = C^* \ (S \ \Delta K)^{n^*} \qquad (5)$$

The material constants C^* and n^* should be based on ΔK_{eff} and constant amplitude loading. They are different from the usually applied material constants C and n, because crack closure will also occur at constant amplitude loading.

Determination of P_{op}

The determination of P_{op} is the most essential part in these considerations. This quantity could be calculated by means of an elastic-plastic finite element program, but this is expensive and time-consuming. In the following an estimation of P_{op} is proposed.

At P_{min} a residual plastic compressive area due to previous load cycles exists around the crack tip. When the crack has propagated into this area the crack surfaces will be pressed together below a certain load. Removing the stress from this compressive area would create an imaginary negative crack opening δ_1 at the real crack tip (see Figure 2). $a_s - a$ = length (X-direction) of residual plastic compressive area. R_{YCB} = twice the distance between crack tip and elastic-plastic compressive boundary, measured in Y-direction.

When increasing the load again a positive crack opening δ_2 at the real crack tip is created. The effective crack length used to determine δ_2 is put equal to a_s.

P_{op} is reached when

$$\delta_1 = \delta_2 \qquad (6)$$

Determining δ_1 is a complicated matter, but as a first approximation is taken:

$$\delta_1 = \frac{R_{YCB} \ \sigma_{0.2}}{E} \qquad (7)$$

(Hooke's law, uniaxial stress condition).

Substituting $\theta = \pi$, $r = a_s - a$, $\delta_2 = 2v$ into the known expression of the vertical displacement v in plane strain condition (see Figure 3)

$$v = \frac{K_I}{E}\left(\frac{r}{2\pi}\right)^{1/2} 2(1+\nu)\sin\frac{\theta}{2}\left[2 - 2\nu - \cos^2\frac{\theta}{2}\right] \qquad (8)$$

results in:

$$\delta_2 = \frac{K_I \ 8(1-\nu^2) \ (a_s-a)^{1/2}}{E \ (2\pi)^{1/2}} \qquad (9)$$

It is put:

$$K_I = \frac{P}{BW^{1/2}} \ f\left(\frac{a}{W}\right) \qquad (10)$$

W = test piece width.

With (6) it follows:

$$P = P_{op} = C \frac{R_{YCB}}{f\left(\frac{a}{W}\right) (a_s-a)^{1/2}} \qquad (11)$$

in which

$$C = \frac{\sigma_{0.2} \ B \ (2\pi W)^{1/2}}{8(1-\nu^2)}$$

It should be mentioned that the estimation of δ_1 and δ_2 has a great amount of arbitrariness and should be studied more carefully. However, it will be seen hereafter that qualitative results seem to agree with experiments.

RESULTS

Qualitative Results

Qualitative results of the above considerations are limited to two examples.

1) See Figure 4 for the assumed load pattern and residual compressive areas. Immediately after the first high amplitude load cycle (Figure 4c) R_{YCB} is still unchanged and a_s has grown in comparison with the situation of Figure 4b. This results in a sudden decrease of P_{op} (11). ΔK_{eff} increases (3,4) due to decreasing P_{op} as well as increasing P_{max}. So the crack propagation rate da/dN (5, Figure 4a) will increase. After the crack has grown for some cycles (Figure 4d), R_{YCB} has increased and $a_s - a$ has not changed in comparison with Figure 4c. So P_{op} gradually increases (11), ΔK_{eff} decreases (3,4) and da/dN decreases (5, Figure 4a). Figure 4e gives the stabilised situation at the high load amplitude.

2) See Figure 5. Immediately after the decrease of the load amplitude R_{YCB} and a_s have not changed (Figure 5b). So P_{op} (11) does not change, ΔK_{eff} decreases due to a decrease of P_{max} (3,4). This results in a sudden decrease of da/dN (5, Figure 5a). After some crack growth R_{YCB} has not yet changed, $a_s - a$ decreases, P_{op} (11) increases, ΔK_{eff} and da/dN decrease gradually. After some additional crack growth R_{YCB} will decrease and da/dN will increase (Figure 5d) until the stabilised situation has been reached (Figure 5e).

Quantitative Results

The described method is applied to a case of which some experimental results were available [1],[5]

Geometry: CTS - specimen
$B = 25.4$ mm, $W = 152.4$ mm
$a_o = 48.2$ mm
Material: T_i - 6Al - 4V
$m = 3.4$ (2)
$\sigma_{0.2} = 827$ MP_a, $\nu = 0.33$
Loading: One block contains:
50 cycles, $P_{min_1} = 0$ and $P_{max_1} = 50.9$ kN and subsequently
750 cycles, $P_{min_2} = 0$ and $P_{max_2} = 35.0$ kN

In order to get numerical results it is necessary to determine geometry and location of the plastic zone. Based upon research the boundary of Figure 6 has been chosen. With the help of simple computer programs the results of linear summation (Miner), Wheeler (m = 3.4) and the method described above are compared (see Figures 7, 8 and 9).

CONCLUSION

The results are promising. Other materials, geometries and loading programs should be experimentally checked to prove the value of the method proposed.

ACKNOWLEDGEMENT

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REFERENCES

1. WHEELER, O. E., Journal of Basic Engineering, Transactions of the ASME, 94, 1972, 181.
2. ELBER, W., ASTM STP486, 1971, 230.
3. SCHIJVE, J., Conference on Prospects of Fracture Mechanics, Delft, The Netherlands, June 24-28, 1974, Proceedings, 193.
4. LINDLEY, T. C. and RICHARDS, C. E., Materials Science and Engineering, 14, 1974, 281.
5. KATCHER, M., Engineering Fracture Mechanics, 5, 1973, 793.

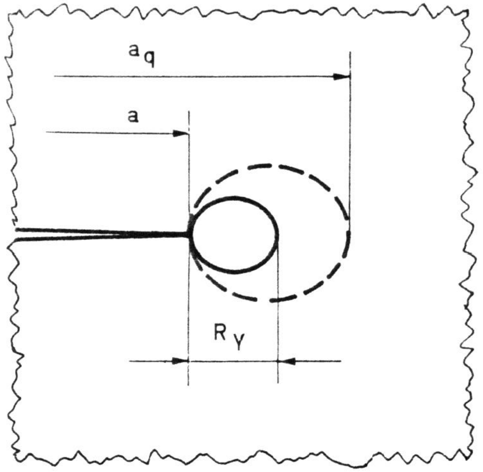
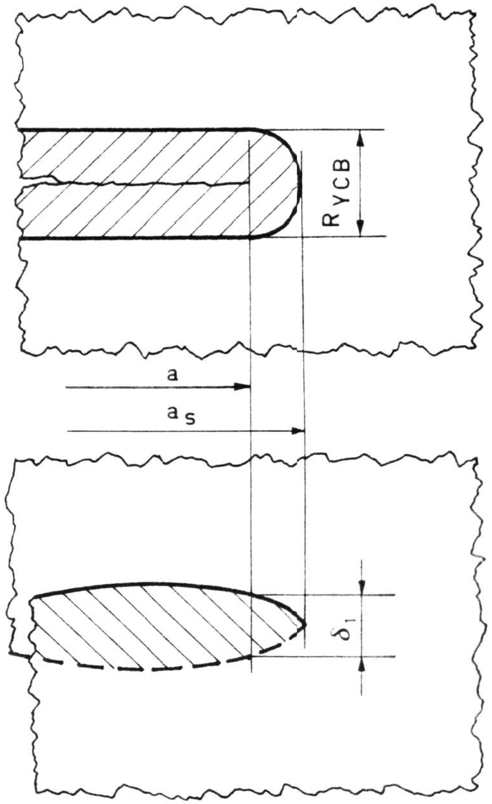


Figure 1

a



b

Figure 2 (a) Residual Plastic Compressive Area
(b) Unloaded \\\\\ Imaginary Overlap

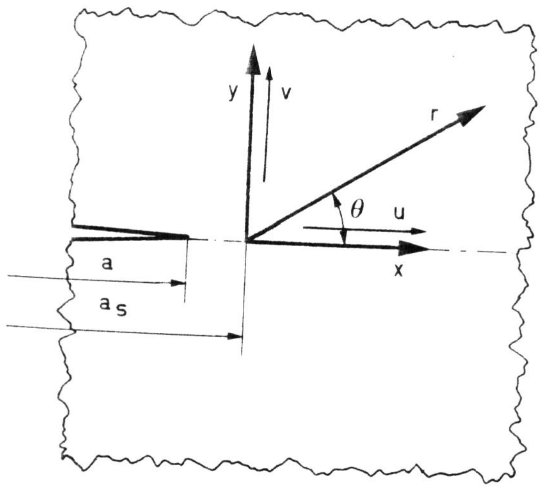
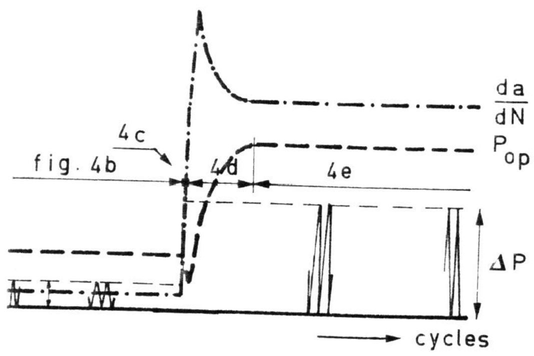
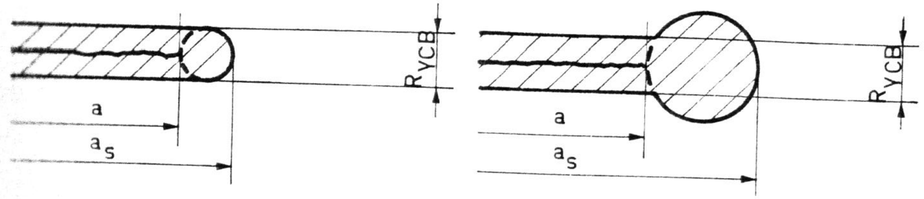


Figure 3

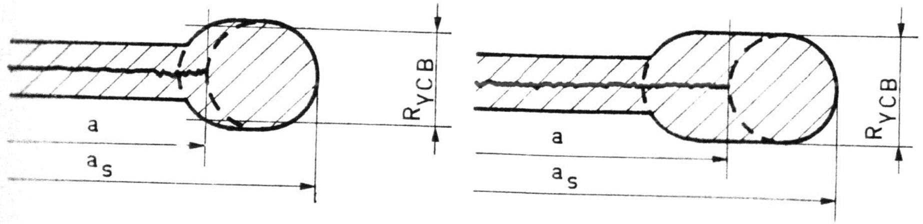


(a)



(b)

(c)



(d)

(e)

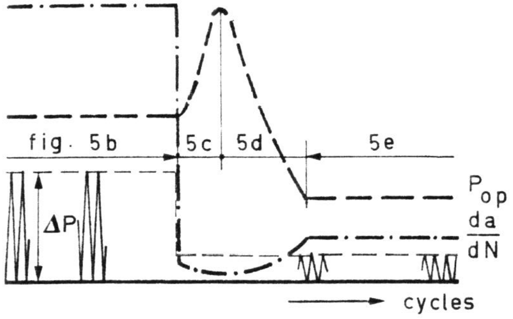
Figure 4 (a) - Load Pattern ΔP, Case 1
- Crack Opening Load P_{op}
- Crack Propagation Rate da/dN

(b) Residual Plastic Compressive Area at Low Load Amplitude Level

(c) Idem Immediately After the First High Amplitude Load Level

(d) Idem After Some Crack Growth

(e) Idem After Some Additional Crack Growth



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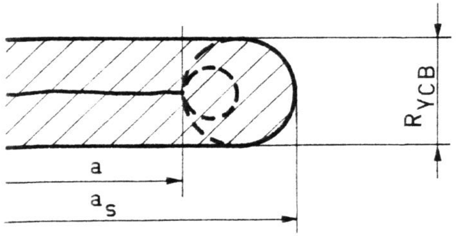

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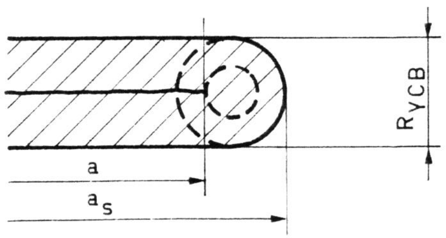
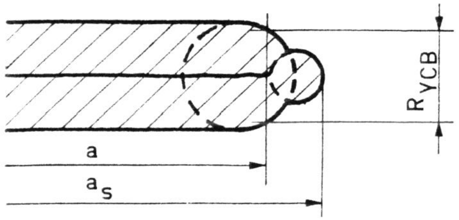
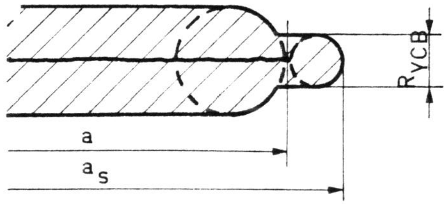


(c)



(d)



(e)

Figure 5 (a) - Load Pattern ΔP, Case 2
- Crack Opening Load P_{op}
- Crack Propagation Rate da/dN

(b) Residual Plastic Compressive Area Before and Immediately After the Amplitude Jump

(c) - (e) Increasing Crack Length

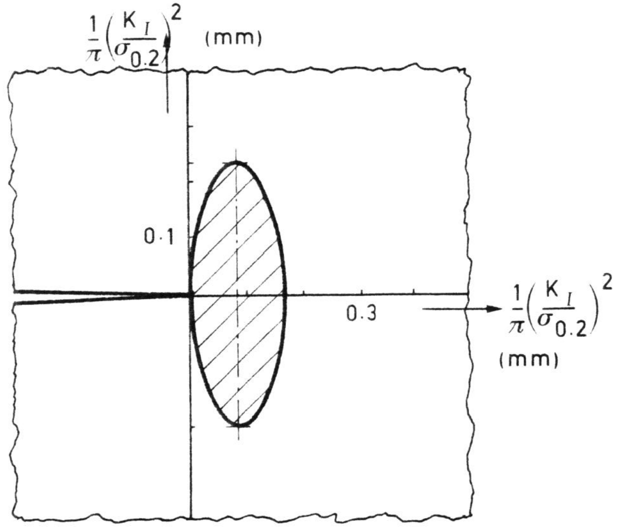


Figure 6 Plastic Zone Geometry and Location,
Ellipse $\left(\frac{X-0,088}{0,088}\right)^2 + \left(\frac{Y}{0,234}\right)^2 = 1$

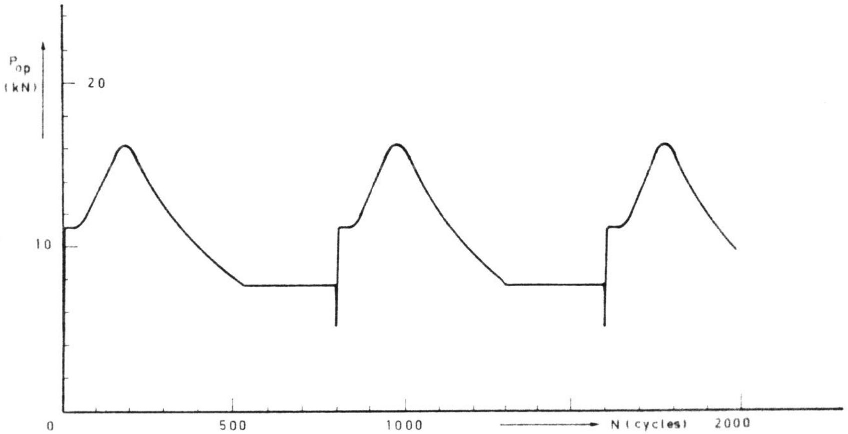


Figure 7 Crack Opening Load P_{op}

P_{max_1} = 50,9 kN at N = 0-50, 800-850, 1600-1650

P_{max_2} = 35,0 kN at N = 50-800, 850-1600, 1650-2000

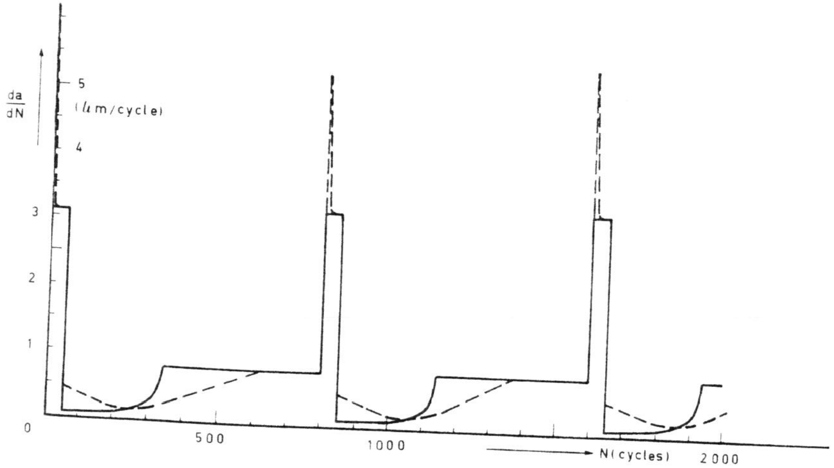


Figure 8 Crack Propagation Rate da/dN

— — — — Wheeler (m = 3,4)

———— Present Method

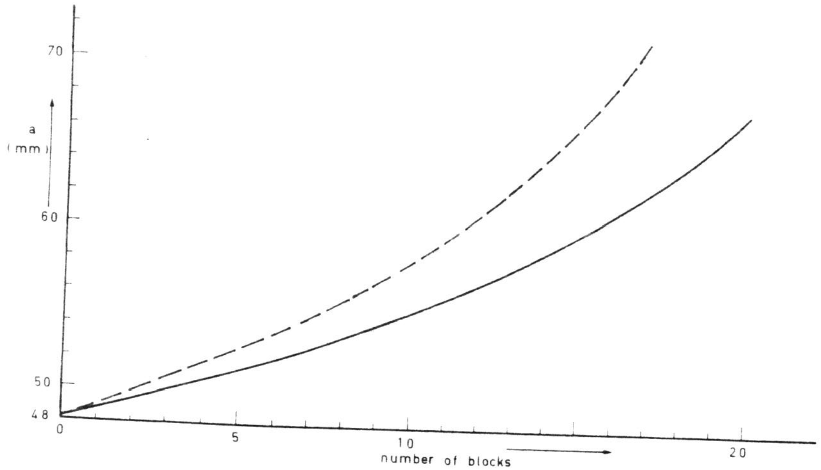


Figure 9 — — — — Miner

———— Wheeler and Present Method