

## ACCUMULATION OF LONGITUDINAL STRAIN UNDER CYCLIC TORSION

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## INTRODUCTION

Progressive deformation has been studied of a 0.45C steel under a fully reversed torsional strain superposing upon a steady tensile stress. The purpose of the study was to give information on the mechanical response of a commercial steel subjected to a rather simple history of cyclic loading such as a reversed plastic strain superposing upon a steady stress under a uni-axial or a bi-axial stress-strain condition.

The phenomenon of progressive deformation under conditions described above is familiar to engineers as cyclic strain-induced creep [1], which is characterized by the accumulation of an inelastic strain activated by plastic strain cycling along a superimposed steady stress. It has been shown that the accumulated strain grows to a large amount near the fracture ductility of materials and often it exceeds that.

A conventional procedure for testing the phenomenon of cyclic strain-induced creep is to impose on the material a cyclic plastic strain superposing upon a steady stress acting in a different direction, that is, under a bi-axial condition of a steady stress and a cyclic strain. Typical loading modes which have been engaged are a cyclic torsion superposing upon a steady tension or push-pull strain reversal superposing upon a steady torsion. Some fundamental features of the phenomenon have been reported under the loading modes just mentioned [2, 3].

Apart from the phenomenology, the phenomenon should be predicted by a change of a flow property of metals depending on the prior loading history experienced by the material. However, the usual constitutive equations in the mathematical theory of plasticity are not adequate to represent such behaviour of materials. Furthermore, constitutive laws currently proposed in various ways have not been successful in providing a full description of the history-dependent effect on plastic flow properties. The present study is intended to give useful information on the effect of cyclic loading history on the flow properties along the superimposed steady stress under the aforementioned loading condition.

## EXPERIMENT

A hollow cylindrical specimen (13 mm O.D., 10 mm I.D. and 50 mm G.L.) of a normalized 0.45C steel (JIS S45C, 350°C-2hrs., A.C.) was subjected to a fully reversed torsional strain superposing upon a steady tensile stress. A specimen was loaded by an electro-hydraulic closed-loop testing system with a dead-weight mechanism for loading the steady tensile stress. The test was made at room temperature under a steady tensile stress ranging

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from 0.15 to 0.45  $\sigma_y$  where  $\sigma_y$  denotes the tensile lower yield stress of the material. The torsional strain was imposed according to a triangular wave shape with an amplitude ranging from 0.01 to 0.028 and a strain rate ranging from 0.0001 to 0.003  $s^{-1}$ .

However, the effect of strain rate on the tensile flow behaviour was found to be so small that the authors could ignore the effect of the strain rate on the cyclic strain-induced creep in the present study.

#### BEHAVIOUR OF LONGITUDINAL STRAIN DURING A CYCLE OF STRAIN REVERSAL

The definition of the number of strain reversals  $N_r$  is shown in Figure 1. An even value of  $N_r$  means a period from the maximum peak to the minimum peak of each cycle of the torsional strain. An odd  $N_r$  means that from the minimum to the maximum.  $N_r = 1$  means the first 1/4 cycle. The definition of an incremental longitudinal strain (plastic)  $\epsilon_p$  and the torsional plastic strain  $\eta_p$  are also denoted in Figure 1.

The growth of the incremental longitudinal strain during each reversal  $N_r$  is shown in Figure 2 for different numbers of strain reversals, in relation to the growth of the torsional plastic strain. Examination of Figure 2 reveals that the growth of the incremental longitudinal strain decreases as the number of strain reversals increases. Two other noticeable features of the growth of an incremental longitudinal strain are observed on the figure. First, the  $\epsilon_p$ - $\eta_p$  curve deviates from a linear relation for  $N_r = 2$ , and the deviation is more noticeable as  $N_r$  increases. The straight line for  $N_r = 1$  gives the same relation during the first loading, that is the relation under the first monotonic torsional loading without any prior strain history. The second feature is that there appears a threshold of the torsional strain for  $N_r \geq 2$ , that is, the longitudinal strain growth for  $N_r = 2$  occurs after the torsional plastic strain exceeds a critical value which is shown as the off-set point on the abscissa.

Figure 2 is transformed in Figure 3 which gives a relation between the longitudinal plastic strain and the torsional plastic strain decreased by the threshold torsional plastic strain, denoted by  $\eta_p^*$ , in log-log plots. As shown in Figure 3, the plots can be fitted approximately by parallel straight lines with a slope of about 1.6 independent of  $N_r$ , although a slight transient state is observed for smaller values of  $N_r$ , where a slight deviation from the fitted straight lines is observed at the right or the left ends of the curves.

Consequently, the incremental longitudinal versus torsional plastic strain relation is summarized by the following equation,

$$\epsilon_p = \alpha(\eta_p - \eta_p^*)^\beta, \quad (1)$$

where,  $\alpha$ ,  $\beta$  and  $\eta_p^*$  are parameters reflecting the effect of a cyclic loading history. For the first monotonic loading,  $\beta = 1$  and  $\eta_p^* = 0$ . The exponent  $\beta$  shows rapid convergence to about 1.6 although transition is observed at smaller values of  $N_r$ , and the converged value of 1.6 is independent of the torsional plastic strain range and the steady tensile stress, so that the exponent  $\beta$  may be taken as a material constant in the present study, while the other parameters  $\alpha$  and  $\eta_p^*$  should be history-dependent.

#### PROPERTIES OF HISTORY - DEPENDENT PARAMETERS

Figure 4 shows a relation between the parameter  $\eta_p^*$  and  $N_r$ . It is seen that  $\eta_p^*$  is independent of  $N_r$ , but dependent on the torsional plastic strain range and the steady tensile stress. This implies that this parameter  $\eta_p^*$  can be determined by the stress-strain condition attained by the monotonic loading of a first 1/4 cycle on the virgin material.

The above result is otherwise interpreted as following. Figure 5 shows a relation between torsional stress  $\tau$  denoted in Figure 1 and torsional plastic strain in each strain reversal plotted on a semilog graph and the value of  $\eta_p^*$  for the described test condition is indicated on the figure. The straight lines fit the experiments on a zone of so-called transient hardening. Deviations of the experiments from the straight lines at the right side of the transient hardening zone indicates the initiation of a dominant plastic strain, that is, so-called steady hardening. It is observed that the value of  $\eta_p^*$  corresponds to the torsional plastic strain at the upper bound of the transient hardening zone.

In Figure 6, the parameter  $\alpha$  for different ranges of the torsional total strain  $\eta_t$  is related to  $N_r$ . The value of  $\alpha$  decreases as  $N_r$  increases, under the constant torsional strain range and steady tensile stress, while  $\alpha$  decreases as the torsional strain range and the steady tensile stress increases. Hence, the following relationship is derived between the parameter  $\alpha$  and the number of strain reversal  $N_r$ .

$$\alpha = fN_r^{-g}, \quad (2)$$

where,  $f$  and  $g$  are constants depending on the torsional plastic strain range and the steady tensile stress.

#### STEPWISE CHANGE OF TORSIONAL STRAIN

In order to reveal an effect of a loading history on the parameters  $\alpha$  and  $\eta_p^*$  of equation (1), the torsional strain range was increased or decreased in steps at a certain number of strain reversals. In the case of the stepwise increase test, an increase in the incremental longitudinal strain was observed quite similar to the case of a stepwise change of the steady stress. The behaviour of the growth of the incremental longitudinal strain and its change by increase of the torsional strain range takes place as follows. Equation (1) holds good and the exponent  $\beta$  keeps the same value of about 1.6 after the stepwise variation of the torsional strain range. However, the parameter  $\eta_p^*$  is changed immediately after the change of torsional strain range to a different value which is attained in the continuous test with the changed torsional strain range from the beginning of cyclic loading, although a few transient reversals are observed immediately after the change.

In the stepwise increase of the torsional strain range, the value of the parameter  $\alpha$  showed a complicated variation such that it converged to a higher value than that obtained in the continuous cycling test with the changed torsional strain range, when they are compared at the same number of strain reversals. However, if the value of  $\alpha$  is shown in relation to the accumulated longitudinal strain  $\Sigma\epsilon_p$ , the value of  $\alpha$  after the change of the strain range coincides with the value obtained in the continuous cycling test with the changed torsional strain range. This result shows

that the tensile strain accumulation is governed by the strain accumulated along the referred direction. In the course of the change of  $\alpha$  produced by the stepwise increase of the torsional strain range, a transient stage appears also for a few numbers of the strain reversal immediately after the change. This transient state may be due partly to a transition state occurring in the torsional plastic strain range after the change.

In a stepwise decrease of the torsional strain range, any noticeably information was not obtained because of the small values of the torsional and longitudinal plastic strain.

DISCUSSION

Equation (1) gives an important information as the first approximation for the effect of the prior loading history on the subsequent plastic deformation behaviour. That is, the effect of the loading history is evaluated by the three parameters  $\alpha$ ,  $\beta$  and  $\eta_p^*$  in equation (1). They show the effects of strain reversal, monotonic loading history and the accumulation of strain along the superimposed steady stress. The parameters  $\beta$ ,  $\eta_p^*$  and  $\alpha$  correspond respectively to the first, second and third strain history just mentioned. It is rather difficult at this stage to relate these parameters to physical properties of materials obtained in more usual tests, and more information is required for this purpose.

CONCLUDING REMARKS

From the test on 0.45C steel subjected to a reversed torsional strain superposing upon a steady tensile stress at room temperature, equation (1) has been derived to express the effect of the prior loading history, in which the parameters  $\alpha$ ,  $\beta$  and  $\eta_p^*$  represent the effect of accumulation of the strain under the steady stress, strain reversal and monotonic loading, respectively.

A constitutive law for modeling the cyclic-loading history and cyclic strain-induced creep should at least take into account the behaviour of materials represented by the three parameters in the above equation. Further investigations are required to establish the physical meaning of the equation.

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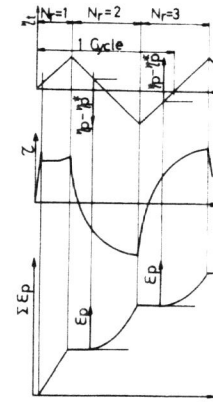


Figure 1 Definition of Number of Strain Reversal and Plastic Strains

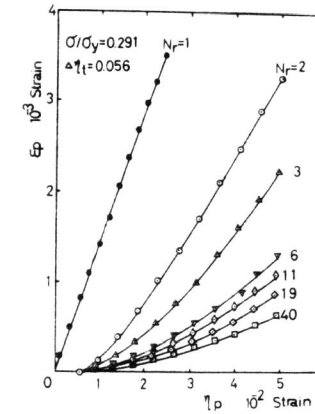


Figure 2 Relation of the Longitudinal Plastic Strain and Torsional Plastic Strain

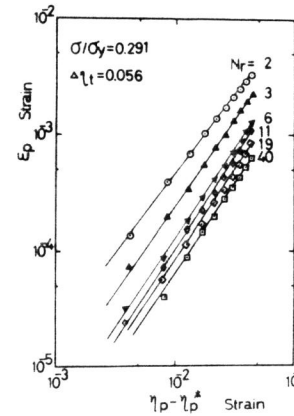


Figure 3 Longitudinal and Torsional Effective Plastic Strain Relation

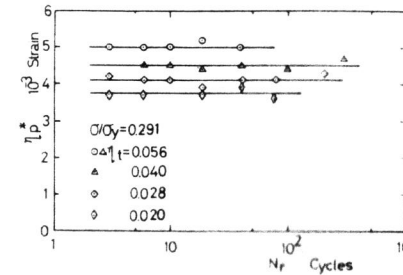


Figure 4 Threshold Torsional Plastic Strain

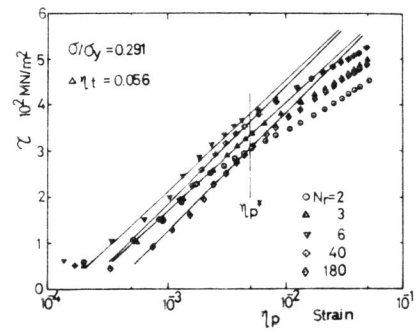


Figure 5 Torsional Stress-Plastic Strain Relation

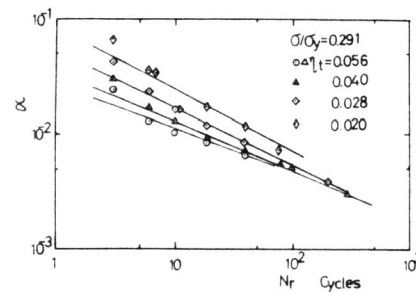


Figure 6 Relation of the Parameter  $\alpha$  and Number of Strain Reversal