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A NEW METHOD FOR CALCULATION OF NOTCH- AND SIZE-EFFECT IN FATIGUE

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INTRODUCTION

In understanding of the microscopic aspects of fatigue, much progress has been obtained in recent years. This, however, has not brought any new method for the calculation of fatigue life and fatigue endurance limit of notched and unnotched specimens, which depend on specimen size and shape. For this purpose, the following three assumptions about the microscopic fatigue behaviour of materials are made in this paper:

a) There are flaws statistically distributed in the specimen.

b) The largest of these flaws propagates during fatigue loading and forms the final crack.

c) No flaw-extension takes place, unless the size of the flaw exceeds a certain value.

The first two assumptions formulate a method for the calculation of fatigue life in the stress-range, where fracture always occurs. The third assumption describes additionally the behaviour near the fatigue limit.

FLAW DISTRIBUTION PRIOR TO FATIGUE LOADING

In technical materials, different kinds of flaws, such as inclusions, grain-boundaries, etc., can initiate a fatigue crack. To describe the importance of each one, it is assumed, that each flaw is equivalent to a crack of length a. For fatigue fracture, only the size a_o of the largest flaw in the specimen is important. For this largest flaw, the following distribution function was derived by Frechet [1]:

$$F(a_o) = \exp\left[-c_1\left(\frac{1}{a_o}\right)^{c_2}\right] \tag{1}$$

c_1, c_2 are constants, depending on the material. When an unnotched specimen is loaded with the stress σ, the largest flaw causes a stress intensity factor K_o of:

$$K_o = \sigma\sqrt{a_o} \ . \tag{2}$$

Substitution of equation (2) into (1) gives the distribution of largest stress intensity factors:

$$F(K_o) = \exp\left[-c_1\left(\frac{\sigma}{K_o}\right)^{2c_2}\right] \ . \tag{3}$$

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When notches and other stress-raisers are present, stress varies in the specimen. A (comparison) stress can be defined at each point of the specimen by:

$$\sigma = \sigma_{max} \cdot g(x, y, z) \ , \tag{4}$$

where σ_{max} is the maximum stress in the specimen. Then, for notched specimens, the distribution function equation (3) is:

$$F(K_o) = \exp\left[- \frac{c_1}{A_1}\left(\frac{\sigma_{max}}{K_o}\right)^{2c_2} \int g(x, y, z)^{2c_2} dA \right] . \tag{5}$$

The integral describes the influence of specimen size and stress distribution. It is to be taken over specimen surface, as cracks always initiate there. A_1 is a reference area.

The distribution given by equation (5) is equal to that of cracks of length a_o

$$a_o = \left(\frac{K_o}{\sigma_{max}}\right)^2 , \tag{6}$$

which is located at the point of maximum stress σ_{max}. The distribution of the largest flaws in notched specimens is then given by:

$$F(a_o) = \exp\left[- \frac{c_1}{A_1}\left(\frac{1}{a_o}\right)^{c_2} \int g^{2c_2} dA \right] . \tag{7}$$

FLAW-EXTENSION DURING FATIGUE LOADING

It is assumed now, that the largest flaw extends due to fatigue loading according to the Paris-Erdogan relation [2]:

$$\frac{da}{dN} = c_3 K(a, \sigma)^{c_4} \ . \tag{8}$$

Initial flaw size a_o and the number of cycles to failure N_f are then connected by:

$$N_f = \frac{1}{c_3} \int_{a_o}^{a_c} \frac{da}{K^{c_4}} , \tag{9}$$

where a_c is the critical crack length. The distribution of the number of cycles to failure $F(N_f)$ for a given stress level σ_{max} can be calculated from the distribution of largest flaws $F(a_o)$ and vice versa, using equation (9).

CALCULATION OF STRENGTH NEAR THE FATIGUE LIMIT

Many materials show a fatigue limit, below which no fracture occurs. When the specimens are loaded on a stress level just above this fatigue limit, some of them break, and some do not. It is therefore assumed, that flaw-extension takes place only, when the stress intensity factor at the largest

flaw, according to equation (5), exceeds a threshold K_{th}. Then the probability P of the value of K_o being greater than K_{th} is:

$$P(K_o > K_{th}) = 1 - \exp\left[-c_5^* \left(\frac{\sigma_{max}}{K_{th}}\right)^{c_6} \int g^{c_6} dA \right] . \tag{10}$$

This is equal to the probability, that fracture will occur after a finite number of cycles at a (cyclic) stress level σ_{max}. The distribution function of maximum stresses σ_{max} in the specimens, which lead to fracture, is then given by:

$$F(\sigma_{max}) = 1 - \exp[-c_5 \sigma_{max}^{\ c_6} \int g^{c_6} dA] \ . \tag{11}$$

The influence of notches on the behaviour near the fatigue limit is sometimes described by the fatigue strength reduction factor K_f, which is defined as:

$$K_f = \frac{\text{mean fatigue strength of unnotched specimen}}{\text{mean (nominal) fat. str. notched specimen}} \ . \tag{12}$$

When the fatigue strength has been determined at an unnotched specimen with a surface area A_1, then the fatigue strength of a notched specimen with the stress distribution $g(x, y, z)$ is given by:

$$K_f = K_t \left(\frac{\int g(x, y, z)^{c_6} dA}{A_1}\right)^{1/c_6} \tag{13}$$

K_t is the theoretical stress concentration factor.

EXPERIMENTAL RESULTS

These proposed methods are now used to compare fatigue life and fatigue limit of unnotched, slender notched and sharp notched specimens, according to Figure 1. The three different types of specimens were cyclically loaded in such a way, that the maximum stress at the notch root is equal. At a (cyclic) stress level of σ_{max} = 2000 N/mm^2 a cumulative frequency of cycles to failure for the three types of specimens was obtained as shown in Figure 2. Though the maximum stress in all kinds of specimens is equal, the life of the differently shaped specimens is quite different.

With the results for specimen shape 1, fatigue life predictions were made for the specimens of shape 2 and 3. For this purpose, the cumulative frequency of the largest flaws a_o was calculated from the cumulative frequency of cycles to failure N_f, using equation (9). The parameters c_1, c_2 were then obtained by curve-fitting of equation (7) to the cumulative frequency of a_o. Substitution of the integral for the specimen of shape 1 by that of shape 2 and 3 gives the distribution function of the large flaws in those specimens. From this, the distribution of cycles to failure can be derived by means of equation (9). These predictions are shown in Figure 2, and have a good correlation to the test results.

The behaviour near the fatigue limit was studied, using Maennig's method [3]. Here the cumulative frequencies of fracture of a specimen are determined on two different stress levels. The results for the three types of specimens are shown in Figure 3. The scale is plotted in that way, that the function equation (11) gives a straight line. From the two different cumulative frequencies of fracture of specimen shape 1, the two constants c_5, c_6 of the distribution function equation (11) can be determined. Substitution of the integrals in equation (11) gives the distribution functions of the fatigue strength of the specimens of shape 2 and 3. Figure 3 shows a good correlation between these predictions and the test results. Though the fatigue lives of the specimens 2 and 3 are considerably different (see Figure 2), the fatigue strength is nearly equal - according to the theory.

CONCLUSIONS

It was shown in this paper, that the size- and the notch-effect are based on the same phenomenon, when specimens are compared at equal maximum stress. The changes in fatigue life and strength are caused by the changes in the distribution of the largest flaws and - for finite life - in the propagation of these flaws.

It is believed, that the proposed method for the calculation of life and fatigue limit can be considerably simplified.

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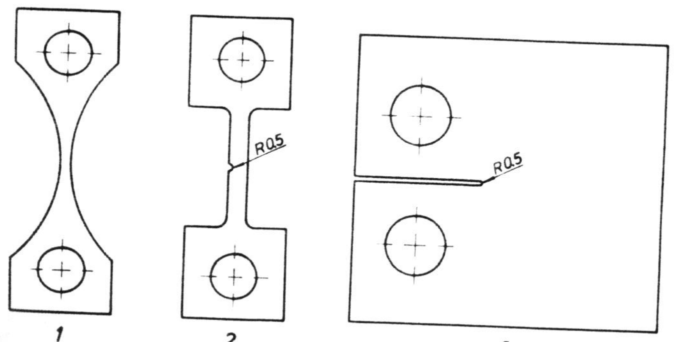

Figure 1 The Three Different Specimen Shapes

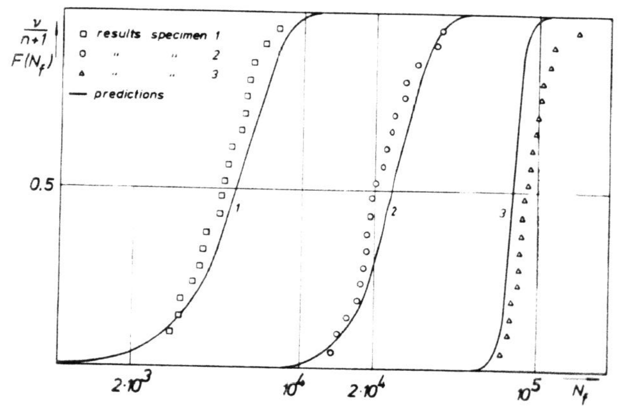

Figure 2 The Distribution of Fatigue Life

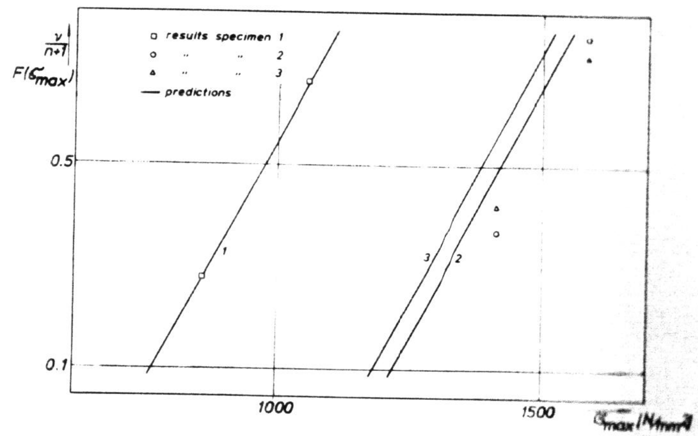

Figure 3 The Distribution of Fatigue Strength Near the Fatigue Limit