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A MODEL FOR FATIGUE CRACK PROPAGATION IN DEFORMABLE MEDIA WITH NONLINEAR CHARACTERISTICS

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INTRODUCTION

In the present status of fracture mechanics a wide proliferation is encountered regarding the analytical means for fatigue crack propagation assessment. The known relationships are essentially empirical or quasi-empirical in nature, containing material parameters with no particular physical meaning, parameters which can be estimated only by pertinent testing. It is desirable that fatigue crack propagation assessment be based on models reflecting the basic processes, thus minimizing the correlating parameters.

To this aim, in the following, a new model for fatigue crack propagation is derived based on the evaluation of the cumulative damage in the cyclic-plastically deformed region at the tip of an extending crack. The proposed model correlates the fatigue crack propagation rate with the applied cyclic stress, the geometry of the element involved and the cyclic-plastic properties of the material.

THE PROPOSED MODEL

In front of a propagating fatigue crack, due to the severe strain and stress concentration, the material undergoes a cyclic-plastic straining. For the purpose of the present analysis the material will be considered to have nonlinear cyclic true stress-strain characteristics of a power law form:

$$\Delta\sigma = \sigma_0 \Delta\varepsilon^n \tag{1}$$

where $\Delta\sigma$ is true stress range, $\Delta\varepsilon$ true strain, n the cyclic strain hardening exponent and σ_0 a strength parameter. Equation (1) was proved to described the cyclic plastic behaviour of a wide class of metallic materials [1].

The stress and strain distribution ahead of a crack in a nonlinear material can be estimated, based on the relationship between the effective stress and strain concentration factors K_σ and K_ε, considered for the nonlinear material governed by equation (2) and the elastic stress concentration factor K_{el}:

$$K_\sigma K_\varepsilon = K_{el}^2 \ . \tag{2}$$

This relation proposed by Neuber [2] for a special case of nonlinear media, was proved experimentally, [3], to be also valid for solids with nonlinear

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characteristic of a power law form (equation (1)). For the range of variation of stress and strain ahead of the fatigue crack tip, from equations (1) and (2) results the effective strain range $\Delta\varepsilon$:

$$\Delta\varepsilon = \Delta\varepsilon_N(\Delta\sigma_{el}/\Delta\sigma_N)^{2/(1+n)} \tag{3}$$

where $\Delta\sigma_N$ is the nominal stress range at the crack tip.

The elastic stress range distribution $\Delta\sigma_{el}$ in the direction r of crack propagation can be inferred based on the fracture mechanics approach [4] according to:

$$\Delta\sigma_{el} = \Delta K(2\pi r)^{-1/2} \tag{4}$$

where ΔK is the stress intensity factor range, which general expression has the form

$$\Delta K = \Delta\sigma_\infty[\pi a\beta(a;L)]^{1/2} \tag{5}$$

with $\Delta\sigma_\infty$ the applied stress range in a region remote from the crack; a, the crack length and $\beta(a,L)$ a correction factor pertaining to the geometry of the body in which the crack is propagating. Taking into account that the following relationship is fulfilled:

$$\Delta\sigma_N[1-(a/L)] = \Delta\sigma_\infty \tag{6}$$

where L is the length over which the crack propagates till the complete separation in the cross section of the stressed body, putting:

$$h(a,L) = [1-(a/L)][a\beta(a;L)]^{1/2} \tag{7}$$

from equations (3) to (7) the expression for the effective strain range distribution for a given nominal strain range $\Delta\varepsilon_N$, as function of the crack tip distance r in front of the propagating fatigue crack is obtained as:

$$\Delta\varepsilon = \Delta\varepsilon_N[(2r)^{-1/2}h(a;L)]^{2/(1+n)} . \tag{8}$$

The $\Delta\varepsilon$-r variation is schematically illustrated in Figure 1.

At this stage it will be considered as a basic physical assumption of the present model, that the distance over which the material is separated in a cycle corresponds to the distance from the crack tip over which the strain range exceeds the true residual fracture ductility ε_{FR}. In the representation illustrated in Figure 1, the crack growth for a cycle is considered to be twice the distance d_R determined by the intersection of the curve for $2\varepsilon_{FR}$ and that for $\Delta\varepsilon$ given by equation (8). The multiplication factor of two for the distance d_R appears from the approximate equilibrium condition at the crack tip. Thus:

$$da/dn = 2d_R \tag{9}$$

where d_R is the solution in r of the equation:

$$2\varepsilon_{FR}(r) = \Delta\varepsilon(r) . \tag{10}$$

But the residual ductility ε_{FR} is affected by the prior cumulative fatigue damage of the material due to the increasing strain as the fatigue crack tip is approaching. This process can be visualised as a material particle moving towards a stationary crack tip and thus entering a region with effective strain range distribution ahead of the fatigue crack (equation (8)). It results that the straining spectrum which determines the residual fracture ductility of the material particle is continuously increasing from cycle to cycle.

Low-cycle fatigue studies [5] reveal that the true residual fracture ductility resulted after sequences of cycles at $\Delta\varepsilon_i$ strain range levels to which N_{Fi} fatigue lives corresponds (i = 1, 2, ..., k) can be estimated by:

$$\varepsilon_{FR} = \varepsilon_F[1-\sum_i(N_i/N_{Fi})]^\alpha \tag{11}$$

where α is the fracture ductility corresponding to the singular loading determined in a fracture test or resulting from an adjusting procedure of the low-cycle constant strain range test data, according to a Manson-Coffin type relationship:

$$\Delta\varepsilon_i(4N_{Fi})^\alpha = 2\varepsilon_F \tag{12}$$

where α is a basic low-cycle fatigue parameter with a representative value of 0.5 for a wide class of materials. Taking into account that $N_i = 1$ because each strain range appears only once, from equations (11) and (12) we obtain:

$$\varepsilon_{FR} = \varepsilon_F\left\{1-[4\sum_i(\Delta\varepsilon_i)^{1/\alpha}\Delta r_i]/[(2\varepsilon_F)^{1/\alpha}(da/dN)]\right\}^\alpha . \tag{13}$$

Regarding the fatigue crack propagation as a continuous process from equation (13), the true residual fracture ductility of the material at a distance r from the crack tip results as:

$$\varepsilon_{FR} = \varepsilon_F\left\{1-\left[4\int_r^\infty(\Delta\varepsilon)^{1/\alpha}dr\right]\Big/\left[(2\varepsilon_F)^{1/\alpha}(da/dN)\right]\right\}^\alpha . \tag{14}$$

The integration in equation (14) is easily performed when equation (8) is considered.

From equations (9), (10) and (14) the fatigue crack propagation rate is obtained as a function of the nominal strain range:

$$da/dN = h^2(a;L)\left\{1+[2\alpha(1+n)]/[1-\alpha(1+n)]\right\}^{\alpha(1+n)}(\Delta\varepsilon_N/2\varepsilon_F)^{(1+n)} , \tag{15}$$

or, as function of nominal stress range:

$$da/dN = h^2(a;L)\left\{1+[2\alpha(1+n)]/[1-\alpha(1+n)]\right\}^{\alpha(1+n)}(\Delta\sigma_N/2\sigma_F)^{(1+n)/n} . \tag{16}$$

It is worthwhile emphasizing that the fatigue crack propagation rate as expressed in alternative forms by equations (15) or (16) is determined by geometrical, material and loading parameters explicitly stated, which is not the case for many propagation rate relationships so far proposed, which can be applied only after an "a priori" empirical correlation of the pertinent parameters for the material involved.

In equation (16) the stress intensity factor range ΔK can be made explicit:

$$da/dN = C(\alpha,n)\,[h(a;L)]^{(n-1)/n}\,[\Delta K/(\pi^{1/2}\,2\sigma_F)]^{(1+n)/n} \qquad (17)$$

where $C(\alpha,n) = \{1+2\alpha(1+n)/[1-\alpha(1+n)]\}^{\alpha(1+n)}$ is a parameter which depends on α and n is illustrated in Figure 2. It is apparent that equation (17) cannot be reduced to the well known Paris form due to the crack-length dependent parameter $h(a;L)$.

At this stage of the analysis it is appropriate to evince that the influence of cycle asymmetry can be accounted for by considering the corresponding counter part of equation (12) in the form proposed by Ohji, Miller and Marin [5]:

$$\varepsilon_{max} = 2\varepsilon_F/[4N_F(1-R)^{1/\alpha}+(1+R)^{1/\alpha}]^{\alpha} \qquad (18)$$

where $R = \varepsilon_{min}/\varepsilon_{max}$ is the strain ratio, with ε_{min}, ε_{max} the minimum and maximum true strain. Based on this relationships, following the same procedure as the one leading to equation (16), we obtain:

$$da/dN = 2^{-1/\alpha} h^2(a,L)\ (1-R)^{1/\alpha}+(1+R)^{1/\alpha}+[4(1-R)^{1/\alpha}+(1+R)^{1/\alpha}]\ \cdot$$

$$\cdot\ \alpha(1+n)/2[1-\alpha(1+n)]^{\alpha(n+1)}\ \cdot\ (\sigma_{Nmax}/\sigma_F)^{(1+n)/n} \qquad (19)$$

where the maximum nominal true stress σ_{Nmax} has been made explicit.

DISCUSSION

According to the proposed model the fatigue crack propagation rate depends on:

a) the geometrical function $h(a;L)$ which concrete analytical form derives from the analytical expression for the stress intensity factor K, corresponding to the shape and size of the material element involved and to the particular loading type imposed.

b) the cyclic-plastic properties of the material, as determined in a low-cycle fatigue test, i.e., the cyclic strain hardening exponent n and the Manson-Coffin coefficient α.

c) the fracture properties as expressed by the true fracture ductility ε_F or the true fracture stress σ_F determined in a monotonic static fracture test or by a correlation procedure from the low-cycle fatigue test data. It appears that due to the fact that ε_F and σ_F refer to a situation of tri-axial state of stress and strain at the crack tip, the pertinent low-cycle fatigue test or static test data should be determined in conditions of severe notch concentration.

d) the loading intensity expressed by the nominal strain range $\Delta\varepsilon_N$ or stress range $\Delta\sigma_N$ and the cycle characteristic as expressed by strain (or stress) ratio R.

In order to check the validity of the fatigue crack propagation rates derived from the proposed model, tests were performed with carbon steel of 52 type in the as received condition loaded in plane alternate bending.

The test specimens were 30 mm wide and 2 mm thick with a 2 mm long and 0.2 mm wide central slit. The crack propagation was measured under a microscope.

The experimental results obtained are plotted in Figure 3 in appropriate coordinates which enable a linear correlation to be made based on equation (16). The line slope determines the cyclic strain hardening exponent n while the ordinate intercept determines the true fracture stress σ_F.

This comparative analysis was extended to some known experimental results obtained by Frost [6] with a carbon steel with 0.22% C, subjected to cyclic alternate axial loading, and by Radhakrishnan [7] with a B-8 type stainless steel heated to 1050° C and oil quenched, subjected to fluctuating tensile loading. The analysis of these experimental results, as illustrated in Figure 3, evince the linear trend predicted by the theory, with resulting cyclic strain hardening exponents and true fracture stresses in line with representative values for the materials considered.

CONCLUSIONS

A model for fatigue crack propagation in solids has been derived based on the evaluation of the cumulative damage in the cyclic-plastically deformed region at the tip of an extending crack. The proposed model correlates the fatigue crack propagation rate with the nominal stress range, geometric factors as expressed by a function of instantaneous crack length, which concrete analytical form is given by the crack dependent part of the stress intensity factor, the nonlinear cyclic plastic and fracture properties of the solid. It may be remarked that the parameters involved in the present fatigue crack propagation model have a clear physical meaning, in terms of the basic plastic and fracture properties.

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ADDENDUM

In the following the detailed algebra is given concerning equation (15) and (17): Cumulative low-cycle fatigue damage relationships (equation (11) in the text):

$$\varepsilon_{FR} = \varepsilon_F (1-\sum_i N_i/N_{Fi})^{\alpha} , \tag{A}$$

Manson Coffin equation (equation (12) in the text):

$$\Delta\varepsilon_i (4N_{Fi})^{\alpha} = 2\varepsilon_F . \tag{B}$$

From (A) and (B) results:

$$\varepsilon_{FR} = \varepsilon_F \left[1 - \frac{4}{(2\varepsilon_F)^{1/\alpha}} \sum_i (\Delta\varepsilon_i)^{1/\alpha} N_i\right]^{\alpha} . \tag{C}$$

Because N_i appears only once, we have:

$$\varepsilon_{FR} = \varepsilon_F \left[1 - \frac{4}{(2\varepsilon_F)^{1/\alpha}} \sum_{i=1}^{N_i} (\Delta\varepsilon_i)^{1/\alpha}\right]^{\alpha} . \tag{D}$$

If Δr is the mean length over which the crack propagates in a cycle then:

$$\Delta r \rightarrow \frac{da}{dn}$$

and thus:

$$\sum_{i=1}^{N_i} (\Delta\varepsilon_i)^{1/\alpha} \Delta r_i \rightarrow \frac{da}{dn} \sum_{i=1}^{N_i} (\Delta\varepsilon_i)^{1/2} \tag{E}$$

From equation (D) and (E):

$$\varepsilon_{FR} = \varepsilon_F \left[1 - \frac{4}{(2\varepsilon_F)^{1/\alpha}} \frac{1}{da/dn} \sum_{i=1}^{N_i} (\Delta\varepsilon_i)^{1/\alpha} \Delta r_i\right]^{\alpha} \rightarrow \varepsilon_F \left[1 - \frac{4}{(2\varepsilon_F)^{1/\alpha}} \frac{1}{da/dn} \int_r^{\infty} (\Delta\varepsilon)^{1/\alpha} dr\right]^{\alpha} \tag{F}$$

Accounting for equation (8) in the text the integral in equation (F) is:

$$\int_r^{\infty} (\Delta\varepsilon)^{1/\alpha} dr = \frac{\alpha(1+n)}{1-\alpha(1+n)} (\Delta\varepsilon_N)^{1/\alpha} \left(\frac{h^2}{2}\right)^{\frac{1}{\alpha(1+n)}} r^{1 - \frac{1}{\alpha(1+n)}} . \tag{G}$$

At fracture:

$$2\varepsilon_{FR}(r \rightarrow d_R) = \Delta\varepsilon(r \rightarrow d_R)$$

with $\Delta\varepsilon$ from equation (8) in the text results:

$$2\varepsilon_F \left[1 - \frac{4}{(2\varepsilon_F)^{1/\alpha}} \frac{1}{da/dn} \frac{\alpha(1+n)}{1-\alpha(1+n)} \left(\frac{h^2}{2}\right)^{\frac{1}{\alpha(1+n)}} (\Delta\varepsilon_N)^{1/\alpha} d_R^{\,1 - \frac{1}{\alpha(1+n)}}\right] = \Delta\varepsilon_N \left(\frac{h^2}{2}\right)^{\frac{1}{n+1}} d_R^{\,-\frac{1}{n+1}} \tag{H}$$

Considering $2d_R \cong \frac{da}{dN}$ (see explication in the text - equation (9)), equation (11) can be put in the final form:

$$\frac{da}{dN} = h^2 \left[1 + \frac{2\alpha(1+n)}{1-2\alpha(1+n)}\right]^{\alpha(1+n)} \left(\frac{\Delta\varepsilon_N}{2\varepsilon_F}\right)^{(n+1)}$$

which corresponds with equation (15) in text.

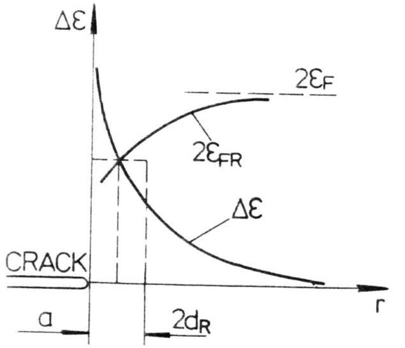

Figure 1

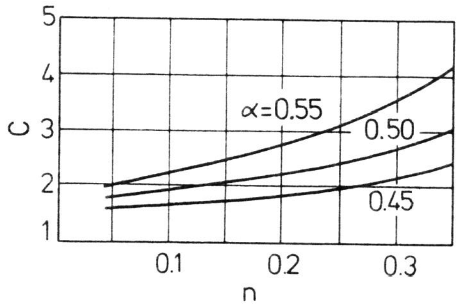

Figure 2

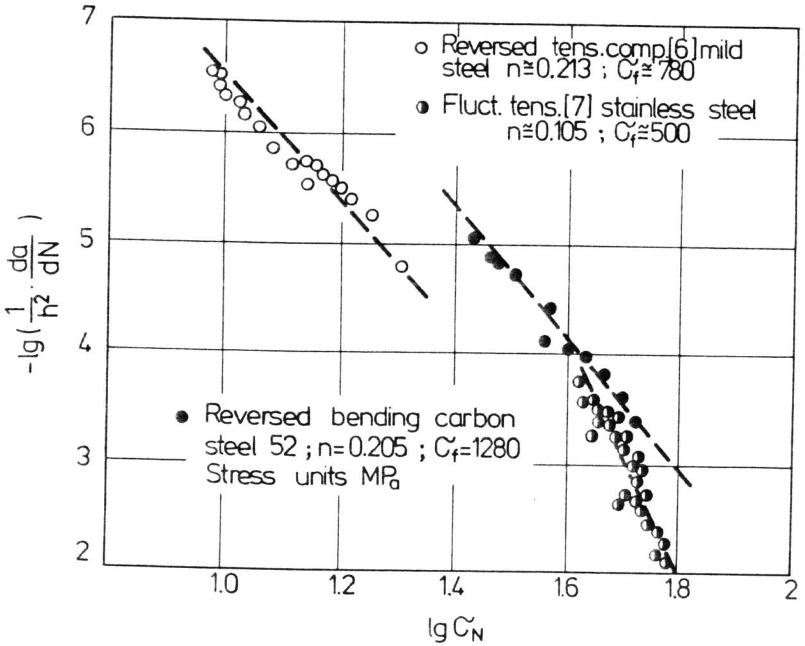

Figure 3