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A DUCTILE FRACTURE CRITERION FOR METALS

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INTRODUCTION

Ductility is an important property in metal forming; also the safety of structures is influenced decisively by local ductility at the tips of notches and flaws. Ductile fracture generally initiates at the surface of the material under a plane stress state. Regarding the failure criterion there are still contradictory results under discussion.

FRACTURE CRITERION AND FRACTURE STRAIN

The best known criterion for ductile fracture of metals [1] is the shear stress criterion (Tresca):

$$\tau_{max} = (\sigma_1 - \sigma_3)/2 = \text{const.} = \tau_f. \tag{1}$$

In combination with several Mohr's envelopes of the major principal fracture stress cycles in the τ-σ- diagram it is often used in the modified form:

$$\tau_{maxf} = (\sigma_1 - \sigma_3)/2 = f[(\sigma_1 + \sigma_3)/2] \tag{2}$$

to take into account the influence of the mean stress [2] (neglecting σ_2) and that of the position of the intermediate principal stress σ_2 [1, 3]. For ductile fracture at the material surface and plane stress ($\sigma_3 = 0$) criterion (1) is identical with a normal stress criterion [4, 5]:

$$\sigma_1 = \sigma_{1f}. \tag{3}$$

Another fundamental proposal is based on a critical mean volume dilatation or volume strain which is identical with a mean stress criterion [5 - 7]:

$$\sigma_m = 1/3 \cdot (\sigma_1 + \sigma_2 + \sigma_3) = \sigma_{mf}. \tag{4}$$

Combining the failure criteria with the representation of the material flow curve $\bar{\sigma} = f(\bar{\varepsilon})$ using the equivalent stress for isotropic material (v.Mises):

$$\bar{\sigma} = \left\{1/2 \cdot [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]\right\}^{1/2} \tag{5}$$

and the equivalent natural (plastic) strain:

$$\bar{\varepsilon}_p = \left\{2/3 \cdot (\varepsilon_{p1}^2 + \varepsilon_{p2}^2 + \varepsilon_{p3}^2)\right\}^{1/2} \tag{6}$$

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the equivalent fracture strain (ductility) $\bar{\varepsilon}_{pf}$ for various stress states can be calculated. For plane stress ($\sigma_3 = 0$) criteria (1) and (3) lead to minimum ductility at $\alpha = \sigma_2/\sigma_1 = 0.5$ and equal equivalent fracture strains at $\alpha = 0$ and 1, whereas (4) leads to minimum ductility at $\alpha = 1$. Ductile surface failures in samples without stress or strain concentrations usually reveal a fracture surface inclination of approximately 45° which gives support to the validity of a shear stress criterion rather than a volume strain criterion.

A further criterion for ductile fracture may be deduced from the total plastic work,

$$W_p = \int_0^{\bar{\varepsilon}_{pf}} \bar{\sigma}\, d\bar{\varepsilon}_p ,$$

and the consideration that the maximum tensile stress, σ_1 has a strong influence on ductility [8, 9]. On this basis fracture should occur at constant values of the "tensile" work:

$$W_t = \int_0^{\bar{\varepsilon}_{pf}} \bar{\sigma}\, \frac{\sigma_1}{\bar{\sigma}}\, d\bar{\varepsilon}_p = W_{tf} . \tag{7}$$

Considering plane stress ($\sigma_3 = 0$), criterion (7) reveals equivalent fracture strains which vary only up to 15% in the range $\alpha = 0 \ldots 0.5$; but this is far too low when compared with all known experimental results. An empirical modification of (7) has been proposed therefore [10], leading to the criterion:

$$W_t^* = \int_0^{\bar{\varepsilon}_{pf}} \frac{2\sigma_1}{3(\sigma_1 - \sigma_m)}\, d\bar{\varepsilon}_p = W_{tf}^* . \tag{8}$$

Finally, a further proposal [11] is based on the statistical process of shear joining of voids:

$$K = (1 + \alpha)\sigma_1^2 = K_f . \tag{9}$$

Similarly to (4), criterion (8) leads to minimum equivalent fracture strain $\bar{\varepsilon}_{pf}$ at $\alpha = 1$, whereas after (9) equal ductility is given for $\alpha = 0.5$ and 1.

Besides these ductile fracture criteria some combined expressions are used to describe the influence of stress state on ductility. In these expressions the mean stress or octahedral normal stress $\sigma_{oct} = \sigma_m$ is related to any yield criterion, so for example σ_{oct}/τ_{oct} [12], $\sigma_m/\bar{\sigma}$ [13] or $\sigma_m/(\sigma_1 - \sigma_3)$ [3, 14], with the octahedral shear stress $\tau_{oct} = \sqrt{2}/3 \cdot \bar{\sigma}$. The position of the intermediate principal stress, σ_2, between σ_1 and σ_3, thereby has the function of a ranging parameter [3].

EXPERIMENTS AND RESULTS

Ten sheet materials were chosen from which to determine the ductility in plane stress, $\sigma_3 = 0$, and $\alpha = \sigma_2/\sigma_1 = 0 \ldots 1$, $\sigma_1 > 0$. Maximum normal stress σ_1 was arranged both longitudinal and transverse to the rolling direction of the material, hence characterizing longitudinal and transverse specimens. Stress ratios $\alpha = 0 \ldots 0.5$ were generated in three-point bend tests with samples of width to thickness ratio $w/t = 0.5 \ldots 10$ using a cylindrical mandrel. For $\alpha = 0.5 \ldots 1$ bulge tests with polished steel ball indentors on circular samples with various constraint releasing holes were used. The deformation type of both the bend and the bulge samples was bending superimposed on stretch. In all cases ductile surface fracture appeared without localized necking due to plastic instability, as has been observed in pure stretch of thin sheets [15 - 19]. Therefore, only softer strain gradients appeared adjacent to the fracture (e.g., Figure 1). Longitudinal and transverse surface strains just causing visible cracks could therefore be evaluated directly with an accuracy of ± 5% from microscopic measurements of photo grids or etched circular grids of finite (0.43 or 2.0 mm) gauge immediately at the failure point. Thus no error arising from an indirect evaluation method [20] or a gauge length effect [21] was possible. Also, to prevent any surface effects, the tensile surface of the samples had been polished before testing.

The fracture strains ε_{p1f} and ε_{p2f} were plotted in ε_{p1}; ε_{p2} - diagrams, Figures 2 - 7, in accordance with earlier work [1]. The theoretical failure curves in the ($+\varepsilon_{p1}$; $-\varepsilon_{p2}$) and the ($+\varepsilon_{p1}$; $+\varepsilon_{p2}$) quadrant were calculated for the two most promising and fundamental criteria, i.e., the shear stress and mean stress criteria, for comparison with the test results. The failure constants $\tau_f = \sigma_{1f}$ and σ_{mf} were estimated from the best verified plane strain ($\alpha = 0.5$) results from wide bend specimens. Besides (5) and (6) the condition for volume constancy:

$$\varepsilon_{p1} + \varepsilon_{p2} + \varepsilon_{p3} = 0 \tag{10}$$

was used, as well as the mean flow curves (Table 1) from compression tests on cylindrical samples taken at 0°; 45° and 90° to the rolling direction [22]. In those cases of tests performed in the total range $\alpha = 0 \ldots 1$, the equivalent fracture strains $\bar{\varepsilon}_{pf}$ were plotted also, Figures 8, 9.

From the results in Figures 2 - 9 the following conclusions can be drawn:

- Applying the test methods described to hot rolled steels reveals ductile fracture only at higher values of $\alpha = \sigma_2/\sigma_1$.
- The test results of cold rolled steels clearly demonstrate minimum ductility at $\alpha = 0.5$, especially for transverse samples. Hints of similar behaviour of some high strength sheet materials are given in [23]. It should be emphasized that the present results have been obtained without any localized necking of the samples, so that the imposed plane stress state $\alpha = \sigma_2/\sigma_1$ was acting up to surface fracture. In some other investigations [10, 11, 21, 24, 25] on the formability of thin sheet metals under stretch deformation, fracture is preceded by marked localized necking. It is known that after the initiation of necking the deformation tends to shift towards plane strain ($\alpha = 0.5$), so that stress states $\alpha > 0.5 \ldots 1$ are unlikely to be effective during the second portion of the deformation and at fracture. Therefore fracture strains evaluated under stretch of thin sheets are possibly not comparable with the present results.
- Comparison of many test results with the theoretical failure curves of the two simple criteria mentioned above demonstrates that for $\alpha = 0 \ldots 0.5$, i.e., for $\varepsilon_{p2}/\varepsilon_{p1} = -0.5 \ldots 0$, the shear stress criterion in most cases matches fairly well the effect of the stress state; this is based on more results from transverse than from longitudinal samples. In this range there seems to be no significant influence of mean stress on fracture shear stress. Smaller deviations from the criterion are possibly due to uncertainty of the flow curves used, mainly for large strains and/or plastic anisotropy.

- In the range $\alpha = 0.5 \ldots 1$, i.e., for $\varepsilon_{p2}/\varepsilon_{p1} = 0 \ldots 1$, the limited number of available test results is arranged between the curves of the shear stress and of the mean stress criterion, indicating an additional influence of mean stress on fracture shear stress. Notwithstanding, the influence of stress state on ductile fracture, even in this range, seems to be better represented by the shear stress than by the mean stress criterion.
- Considerable scatter of the test results must be attributed to small strain hardening at large strains and to the special mechanism of ductile failure initiation.

In summary it appears that, at least for a plane stress state at the material surface and without significant stress or strain concentration, the shear stress criterion can be applied for ductile fracture and fracture ductility; this is in accordance with the common failure appearance of fracture surface inclination.

The influence of mean stress on fracture shear stress seems to be negligible in the range of $\alpha = \sigma_2/\sigma_1 = 0 \ldots 0.5$, whereas for $\alpha = 0.5 \ldots 1$ there is an effect. For the development of a microscopic hypothesis of ductile fracture based on void formation and growth in shear bands [26] the shear stress criterion is sufficiently reliable. On the other hand, for triaxial stress states and strain gradients which restrain the formation of shear bands, the mean stress criterion may be more suitable [5].

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Table 1 Mean Flow Curves of the Materials Investigated

Material	Condition	Flow curve $\bar{\sigma} = A\,\bar{\varepsilon}^n$ A [MPa]	n [%]
Fine grained structural steel Q St 52-3 Cu 3 (DIN 17 100)	Hot rolled 6.5 mm gauge	880	0.22
	Cold rolled from 6.5 to 5 mm gauge	820	0.09
	Cold rolled from 6.5 to 4 mm gauge	820	0.03
Structural steel R St 37-1 (DIN 17 100)	Hot rolled 15 mm gauge	720	0.17
	Cold rolled from 15 to 10 mm gauge	750	0.04
	Cold rolled from 15 to 6 mm gauge	780	0.04
Flat bar steel C = 0.06 % (DIN 1652)	Cold drawn	760	0.05
Soft flat profile of Al Mg 3 w (DIN 1725, Bl. 1 and DIN 1745, Bl. 1)	Pressed	340	0.17
Sheet of Cu Zn 39 Pb 2 F 44 (DIN 17 660 and DIN 17 670, Bl. 1)	Cold rolled	720	0.10
Sheet of Ti 99.5 (DIN 17 850 and DIN 17 860)	Cold rolled	1040	0.11

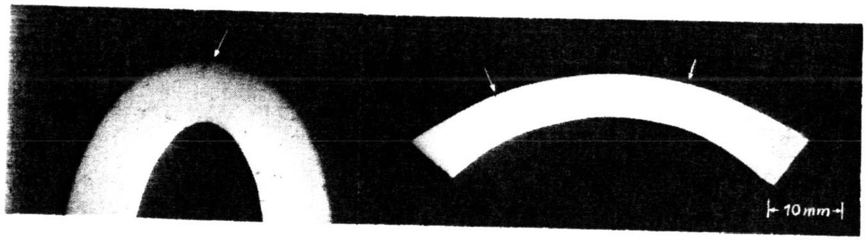

Figure 1 Middle Section of Bend Sample (Left: α = 0.5) and Bulge Sample (Right: $\alpha \approx$ 1); Cold Rolled Structural Steel. Arrows Indicate Small Surface Cracks

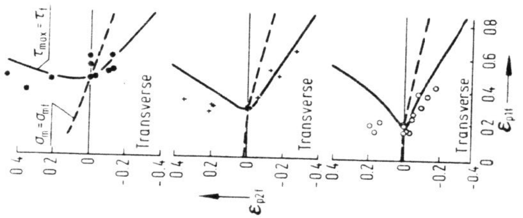

Figure 2 Fine grained structural steel Q St 52-3 Cu 3 (DIN 17 100)

- • Hot rolled 6.5 mm gauge
- + Cold rolled from 6.5 to 5 mm gauge
- o Cold rolled from 6.5 to 4 mm gauge

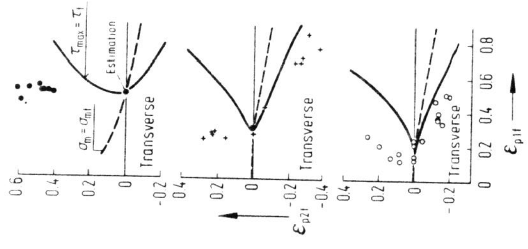

Figure 3 Structural steel R St 37-1 (DIN 17 100)

- • Hot rolled 15 mm gauge
- + Cold rolled from 15 to 10 mm gauge
- o Cold rolled from 15 to 6 mm gauge

Fracture strains at plane stress states for comparance with failure criterions

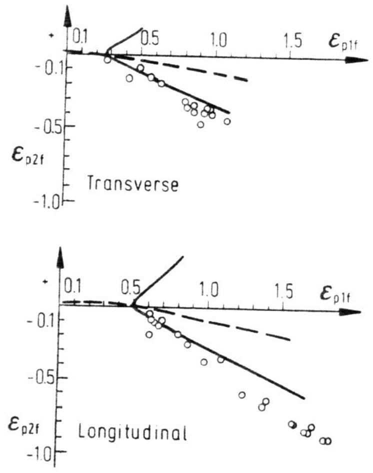

Figure 4 Cold drawn flat bar steel, C = 0.06% (DIN 1652)

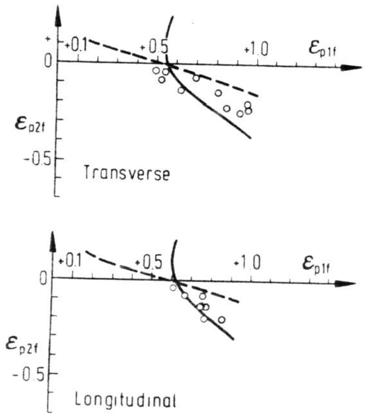

Figure 5 Soft flat profile of Al Mg 3 w
(DIN 1725, Bl.1 and DIN 1745, Bl.1)

Fracture strains at plane stress states for comparance with failure criterions.

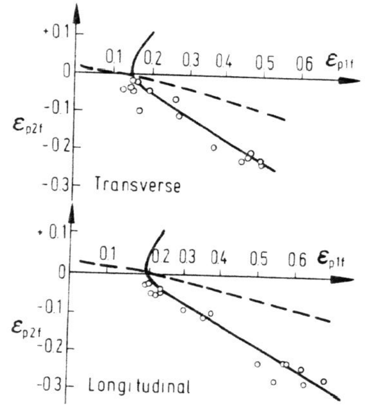

Figure 6 Cold rolled sheet of Cu Zn 39 Pb 2 F 44
(DIN 17 660 and DIN 17 670, Bl.1)

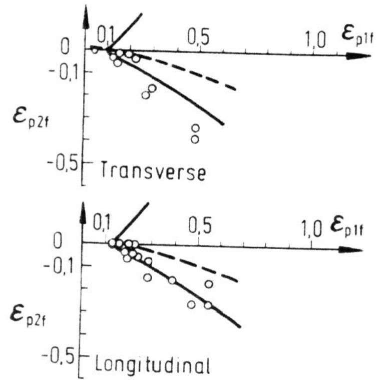

Figure 7 Cold rolled sheet of Ti 99.5
(DIN 17 850 and DIN 17 860)

Fracture strains at plane stress states for comparance with failure criterions.

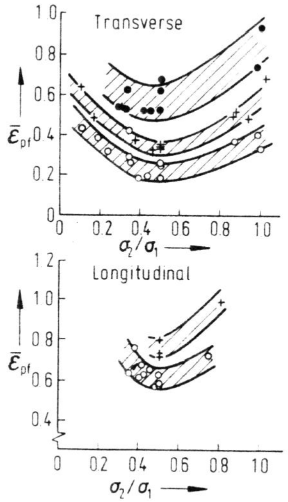

Figure 8 Confer Figure 2

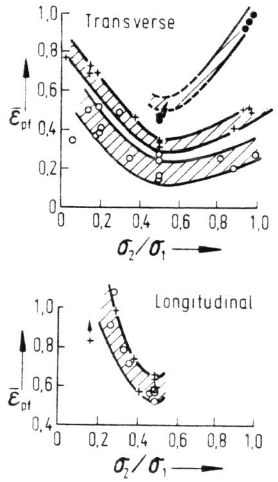

Figure 9 Confer Figure 3

Influence of stress state on equivalent fracture strain