

A COMPUTER SIMULATION OF FATIGUE CRACK PROPAGATION
BASED ON THE CRACK CLOSURE CONCEPT

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INTRODUCTION

It has been pointed out by Elber [1] that crack closure plays an essential part in fatigue crack propagation, and many studies have been carried out subsequently regarding this point [2 - 8]. Most of them were experimental studies in which the crack opening stress was measured by means of various techniques, and the effective range of the stress intensity factor, ΔK_{eff} , was determined. But it is laborious to determine this experimentally, especially when the applied stress is not held constant. So, if there are some simple models which express the crack closure behaviour approximately, it is preferable to estimate ΔK_{eff} by these models. In this paper two such models are proposed, and the behaviour of fatigue crack propagation is simulated.

SIMULATION BASED ON THE MODEL 1

Modelling

This model is based on the following four assumptions.

(1) The displacement along the crack line is assumed to follow the BCS-model solution which has been proposed by Bilby, Cottrell and Swinden [9]. For a crack in an infinite plate as shown in Figure 1, the displacement, $v(x, a)$, is given by the equation

$$\frac{8\pi\mu}{(\kappa+1)b(\beta\sigma_y)} v(x, a) = \cos\theta \log \frac{\sin^2(\theta_2 - \theta)}{\sin^2(\theta_2 + \theta)} + \cos\theta_2 \log \frac{(\sin\theta_2 + \sin\theta)^2}{(\sin\theta_2 - \sin\theta)^2} \quad (1)$$

where θ and θ_2 are given by

$$x = b \cos\theta (|x| < b, |\theta| < \pi), \quad a = b \cos\theta_2, \quad \theta_2 = \frac{\pi}{2} \frac{\sigma}{\beta\sigma_y} \quad (2)$$

and μ and σ_y denote the shear modulus and the yield stress, respectively [10]. Further, the parameters κ and β are the elastic constant and the plastic constraint factor, respectively. They are given as

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$$\left. \begin{aligned} \kappa &= \frac{3-\nu}{1+\nu}, \quad \beta = 1.0 && \text{for plane stress} \\ \kappa &= 3-4\nu, \quad \beta = 2.82 && \text{for plane strain} \end{aligned} \right\} \quad (5)$$

where ν is Poisson's ratio.

When the applied stress is decreased by the amount of $\Delta\sigma$, the range of the displacement along the crack line, $\Delta v(x, a)$, is given by replacing the parameters ν , σ , and σ_y in equations (1) and (2) by [11]:

$$\nu \rightarrow \Delta\nu, \quad \sigma \rightarrow \Delta\sigma, \quad \sigma_y \rightarrow 2\sigma_y \quad (4)$$

(2) It is assumed that every point along the crack line has a maximum amount of residual plastic deformation that is received in its stress history (see Figure 2a). This plastic deformation is evaluated from the displacement inside the plastic zone in the BCS-model.

(3) The rate of the fatigue crack propagation is assumed to be given by the equation

$$\frac{da}{dN} = C \left(\Delta K_{\text{eff}} \right)^m = C (U \Delta K)^m \quad (5)$$

where C and m are material constants and U is the effective stress ratio defined by

$$U = \frac{\Delta K_{\text{eff}}}{\Delta K} = \frac{\Delta\sigma_{\text{eff}}}{\Delta\sigma} \quad (6)$$

When there is a central crack of length $2a$ in the infinite plate, the range of the stress intensity factor is given by

$$\Delta K = \Delta\sigma\sqrt{\pi a} \quad (7)$$

(4) The effective stress is evaluated by

$$\Delta\sigma_{\text{eff}} = \sigma_{\text{max}} - \sigma_0 \quad (8)$$

where σ_0 is the stress at which the point r_p^* behind the crack tip opens or closes. The parameter r_p^* denotes the size of the cyclic plastic zone and it is given in the BCS model by

$$r_p^* = a(1 - \cos\theta^*), \quad \theta^* = \frac{\pi}{4} \frac{\Delta\sigma}{\beta\sigma_y} \quad (9)$$

This assumption combined with the assumption (2) is equivalent to the fact that the very crack tip opens or closes at σ_0 when the reversed flow at the crack tip is taken into account (see Figure 2b).

Calculation Method

The flow chart of the calculation is shown in Figure 3. At first, necessary data are read and initial values of a and N are set. Then the value of the $\Delta\sigma_{\text{eff}}$ is calculated for each stress cycle. It is determined by solving the following equation for σ_0

$$\Delta v \left(a - r_p^*, a \right) = v \left(a - r_p^*, a \right) - v_p \left(a - r_p^* \right) \quad (10)$$

where

$$\begin{aligned} v(a - r_p^*, a) &: \text{the displacement of the crack line at} \\ & \quad x = a - r_p^* \text{ when } \sigma = \sigma_{\text{max}}. \\ \Delta v(a - r_p^*, a) &: \text{the range of the displacement at } x = a - r_p^* \\ & \quad \text{for } \Delta\sigma_{\text{eff}} = \sigma_{\text{max}} - \sigma_0. \\ v_p(a - r_p^*) &: \text{the amount of plastic deformation remaining at} \\ & \quad x = a - r_p^*. \end{aligned}$$

When the value of $\Delta\sigma_{\text{eff}}$ has been determined, the crack propagation rate is given by equation (5). Then the cycles are repeated until the crack propagates to the final length, a_f , when the calculation should be stopped and the crack growth curve can be obtained.

Simulation of Constant $\Delta\sigma$ Test

When the applied stress range and the stress ratio R ($R = \sigma_{\text{min}}/\sigma_{\text{max}}$) are held constant, the effective stress ratio U has a constant value in this model and it is independent of the crack length. Figure 4 shows the $U - \Delta\sigma/\sigma_y$ curves for $R = 0$, and Figure 5 shows the $U - R$ curves. These figures show that the results are in reasonable agreement with Elber's curve [1], i.e.,

$$U = 0.5 + 0.4R \quad (11)$$

Simulation of Decreasing ΔK Test

Figure 6 shows the result of the decreasing ΔK test which is carried out to find the threshold stress intensity factor, ΔK_{th} . In the analysis it is assumed that the crack will stop propagating when the COD at the crack tip is less than 5.0×10^{-7} mm. It is observed that the curves in the figure depend on the parameter R , while the same tendency is also observed in experiment [5].

Simulation of Variable Amplitude Test

When there is a sudden change in $\Delta\sigma$, an accelerating or retarding effect occurs for a certain period. Results are shown in Figure 7. These are in good agreement with the experimental results [12].

In these three cases, the model can express the essential features observed in the experiments. Thus the simulation technique is considered to be useful for the estimation of fatigue lives.

SIMULATION BASED ON THE MODEL 2

Modelling

This model is simpler than model 1. It is essentially based on the following three assumptions.

- (1) When the applied stress range is held constant, the effective stress ratio is assumed to follow Elber's equation, i.e. equation (11).
- (2) When the applied stress varies, there appears a transition period, ΔN_t , during which the effective stress range is assumed to be given by the

scheme shown in Figure 8. This transition period is assumed to continue until the cyclic plastic zone formed by the present stress range will pass over the cyclic plastic zone formed by the previous large stress range (see Figure 9).

(3) The rate of crack propagation is assumed to be given by equation (5) also in this model.

Simulation of Variable Amplitude Block Loading Test

Porter [13] has carried out a series of variable amplitude block loading tests. They can be simulated by this model. The load spectra are set in the same way as his experiments. The material constants used in the simulation are

$$\sigma_y = 588 \text{ MPa}, C = 1.0 \times 10^{-9} \text{ mm}^4/\text{N}, m = 2.0$$

Figures 10 and 11 show the results of the simulation compared with Porter's experiments. These figures show that the results of the simulation agree fairly well with the experiments. Thus it is considered that this model can express well the interaction effects of the varying loads, i.e. accelerating or retarding effects on fatigue crack propagation.

Simulation of Random Loading Test

This model can be applied directly to the random loading test. It is reported that the random loads applied to a real structural element may be described approximately by the Rayleigh distribution [14]. Thus it is assumed in the present simulation that the maximum stress, σ_{max} , follows the Rayleigh distribution, while the minimum stress, σ_{min} , is held constant. In the choice of the load spectra, the modal value, σ_{rm} , is fixed, while the standard deviation, σ_{rd} , is changed (see Figure 12).

Figure 13 shows the crack growth curves for various load spectra. The tendency of the accelerating and retarding effects is similar to the cases of the block loading tests in Figures 10 and 11. On the other hand, Figure 14 shows the results of the simulation calculated in three different ways. The curves called the interaction model in the figure are obtained by means of the present model. These two curves show the upper and lower limits respectively among ten cases of random loadings. The curve called the linear model is obtained using the assumption that there is no transition period in Figure 8, and the last curve is obtained by the calculation for the constant load spectra whose maximum value coincides with the root mean square, σ_{rms} , of the random load sequences. This curve overlaps with the curve of the linear model, while marked retarding effects are observed in the curves of the interaction model.

CONCLUDING REMARKS

Based on the crack closure concept, two models of fatigue crack propagation have been proposed. A computer simulation has been carried out for various kinds of loading conditions: constant $\Delta\sigma$ tests, decreasing ΔK tests and variable amplitude tests. Agreement with the experimental results is found to be fairly good. Therefore, these simulation techniques are considered to be useful to estimate the fatigue lives of actual structural elements.

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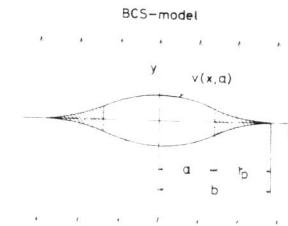


Figure 1 BCS - Model

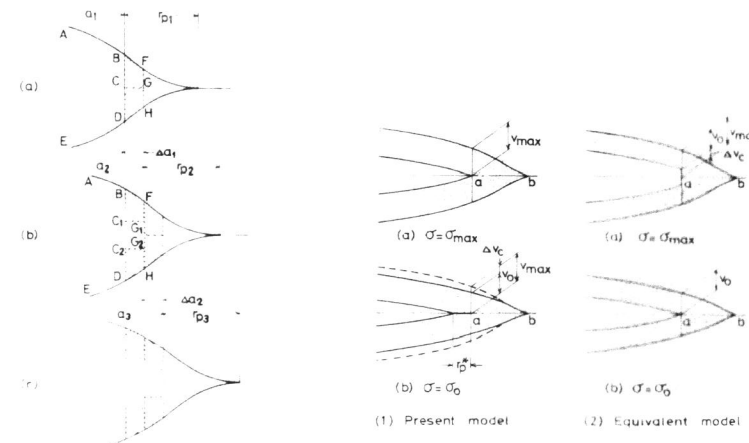


Figure 2a Residual Plastic Deformation Along the Crack Line - Figure 2b Relation Between Model 1 and Equivalent Model Line

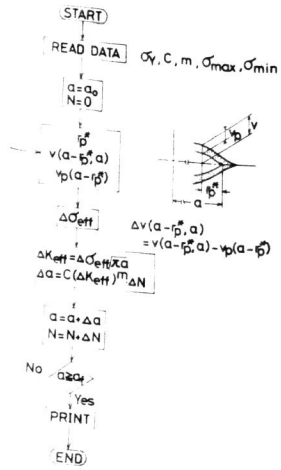


Figure 3 Flow Chart

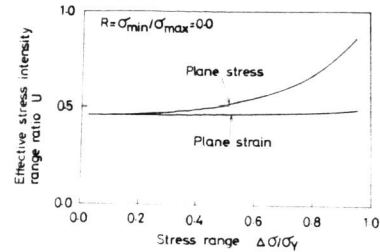
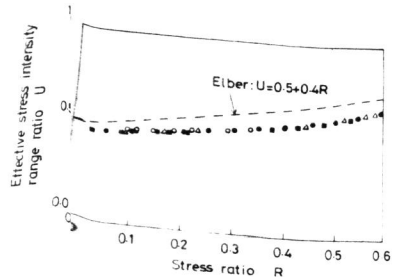


Figure 4 U - $\Delta\sigma/\sigma_y$ Curve



•	$\sigma_{max}/\sigma_y = 0.5$
○	$0.025 \leq \sigma_{min}/\sigma_y \leq 0.475$
◊	$\sigma_{min}/\sigma_y = 0.05$
◻	$0.075 \leq \sigma_{max}/\sigma_y \leq 0.525$
◼	$\sigma_{mean}/\sigma_y = 0.25$
◊	$0.025 \leq \Delta\sigma/\sigma_y \leq 0.475$
◻	$\Delta\sigma/\sigma_y = 0.25$
◼	$0.125 \leq \sigma_{mean}/\sigma_y \leq 0.575$

Figure 5 U - R Curve

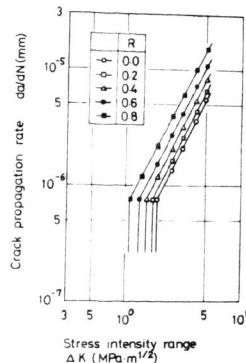
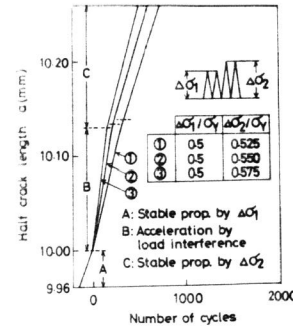
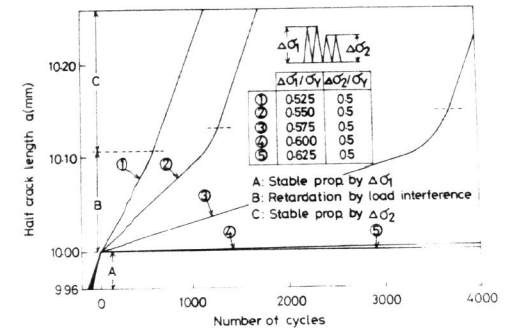


Figure 6 Decreasing ΔK Tests



(a) Accelerating Effects



(b) Retarding Effects

Figure 7 Crack Growth Curve for Variable Loading

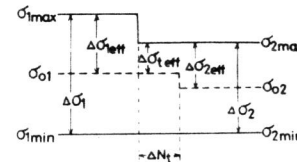


Figure 8 Transition Period

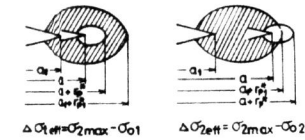
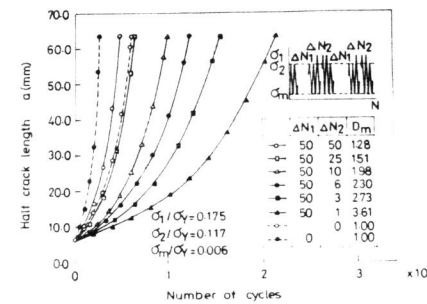
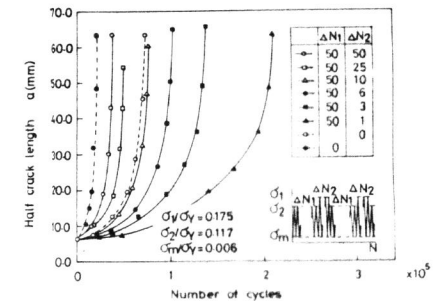


Figure 9 Effective Stress

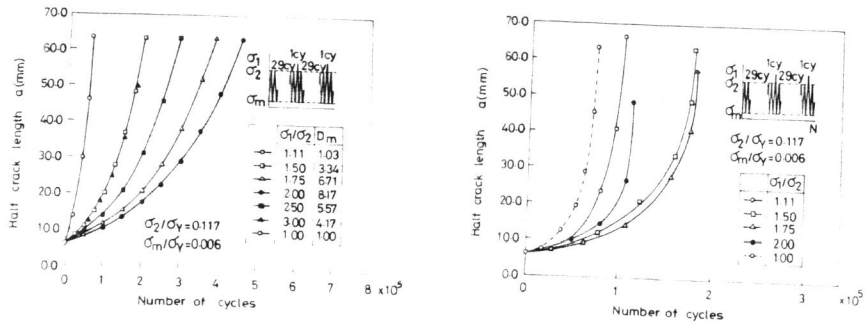


(a) Simulation



(b) Porter's Experiment

Figure 10 Crack Growth Curve for Block Loading



(a) Simulation

(b) Porter's Experiment

Figure 11 Effects of Single Peak Overload

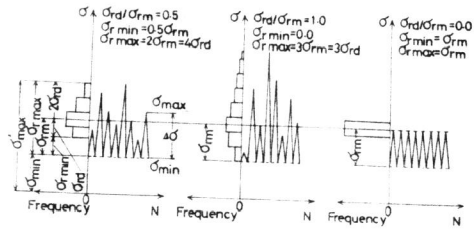


Figure 12 Random Load Spectra

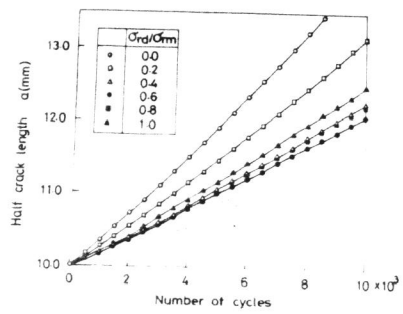


Figure 13 Crack Growth Curve for Various σ_{rd}

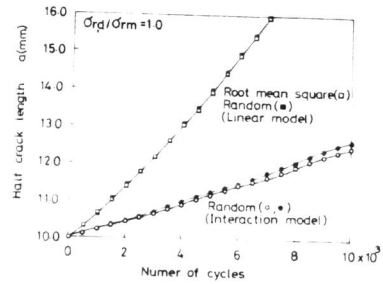


Figure 14 Crack Growth Curve Simulated by Various Models