

A COMPARISON OF METHODS OF CORRELATING CREEP  
CRACK GROWTH

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## INTRODUCTION

With the use of materials under increasingly arduous conditions at elevated temperatures increasing attention has been devoted recently to establishing the circumstances under which cracks could be extended by creep. Some experimenters [1-5] have claimed that creep crack growth rate  $\dot{a}$ , can be expressed in terms of stress intensity factor,  $K$ , in the form;

$$\dot{a} = D K^\beta \quad (1)$$

whereas others [6-8] claim better correlations with the nett section stress,  $\sigma_{\text{nett}}$ , remaining on the uncracked ligament or with a reference stress [9-10], i.e.

$$\dot{a} = F \sigma_{\text{nett}}^\alpha \quad (2)$$

where  $D$ ,  $F$ ,  $\alpha$  and  $\beta$  are coefficients which in general will depend on the material and test temperature. Values of  $\alpha$  and  $\beta$  reported range from 3 to 30 but for a particular material are usually close to the value of the stress sensitivity,  $n$ , of secondary creep strain rate,  $\dot{\epsilon}$ , in the creep law,

$$\dot{\epsilon} = C \sigma^n \quad (3)$$

where  $C$  is a temperature dependent material parameter. Generally, the data indicate that for relatively brittle materials creep crack growth rate correlates best with equation (1) and where substantial creep deformation is possible with equation (2). This is not surprising as creep will cause redistribution of the elastic stresses at the crack tip and for sufficient creep ductility and high enough values of  $n$  the stresses at the crack tip will approach the nett section stress.

It may be expected that because of the non-linear nature of the creep law non-linear mechanics should be more relevant than linear mechanics. Recently a number of authors [11-15] have attempted to extend the  $J$  contour integral concept used to describe the stress and strain distributions around a crack tip in a non-linear elastic material to the creep circumstance. For a non-linear material the numerical value of  $J$  can be obtained from the expression

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$$J = - \frac{1}{B_n} \frac{dU}{da} \quad (4)$$

where  $B_n$  is the thickness of the crack and  $U$  is potential energy. Although in the presence of plasticity  $J$  is no longer the energy potentially available to grow the crack its value can still be evaluated from equation (4). For a non-linear material obeying the work hardening law,

$$\epsilon = A \sigma^n \quad (5)$$

and for test pieces in which the primary mode of displacement is by bending it can be shown [14] that for a constant load  $P$ ,

$$J = \frac{P}{B_n(n+1)} \frac{d\Delta}{da} \quad (6)$$

where  $\Delta$  is the deflection at the loading points.

An analogy can be drawn between a material obeying the work-hardening law equation (5) and one obeying the creep law equation (3). It is possible to define a contour integral like  $J$ , in which  $\epsilon$  is replaced by  $\dot{\epsilon}$ ,  $A$  by  $C$  and  $\Delta$  by the displacement rate  $\dot{\Delta}$ , which will describe the state of stress and strain rate around a crack tip in a creeping material. This creep equivalent of the  $J$  contour integral has been called  $C^*$  by Landes and Begley [12] and  $\dot{J}$  by others [13,14] because it has the dimension of  $J$  divided by time. It is not, however,  $dJ/dt$  and to avoid any possible ambiguity it will be called  $C^*$  here. It can be evaluated in the same way as  $J$ , except with  $U$  replaced by a term  $\dot{U}$  which has the dimensions of power, i.e.

$$C^* = - \frac{1}{B_n} \frac{d\dot{U}}{da} \quad (7)$$

$$= \frac{P}{B_n(n+1)} \frac{d\dot{\Delta}}{da} \quad (8)$$

when bending displacements dominate. Some success has been achieved in characterising creep crack growth with this parameter. In most instances approximate proportionality between  $\dot{\Delta}$  and  $C^*$  was observed [12-15]. Since the state of strain rate around a crack tip varies according to [14],

$$\dot{\epsilon} \propto C(C^*/C)^{n/(n+1)} \quad (9)$$

this suggests that creep crack growth may be directly proportional to the strain rate at the crack tip as for most materials  $n \gg 1$  and  $n/(n+1)$  will be close to unity. An attraction of the  $C^*$  approach is that it is consistent with the  $K$  approach for creep brittle circumstances and with the nett section stress description when creep strains dominate and  $n \gg 1$ .

In the previous investigation of  $C^*$  by Nikbin et al [14] only one geometry of test piece was examined. In this paper, the work is extended to cover a range of geometries and a critical assessment is made of the  $K$  and  $C^*$  characterisations.

## EXPERIMENTS

The materials investigated were aluminium alloy RR58 and a 1/2%Cr, 1/2%Mo, 1/4%V steel. Details of their composition and heat treatment, and of the experimental procedure have been given previously [2,5,14]. In these series of experiments the aluminium alloy was tested at 150°C and the steel at 565°C. Displacements were measured automatically with a transducer and crack growth measurements made visually with the aid of a telescope. The geometries of test pieces used included the contoured DCB (C-DCB) geometry having a constant compliance with crack length used previously [2,5,14], parallel edge DCB (P-DCB), compact tension (CT) and double torsion (DT) shapes. Each specimen was provided with side grooves to control the direction of crack growth. Two thicknesses  $B$ , of the aluminium alloy and two thicknesses of steel samples, each with different notch depth ratios were tested. Compliance calibration experiments were performed on each geometry. Most of the creep cracking tests were carried out at constant load but in some cases load changes were made to investigate history effects.

## RESULTS

The following general observations can be made concerning the creep crack growth against time curves for both materials. In most instances at constant load, crack growth rate decreases progressively with time (and crack length) in the constant  $K$  contoured DCB test-pieces (as was reported previously [2,5,14]) decreased or remained approximately constant in the double torsion samples and increased in the remaining geometries. Comparisons of the data with  $K$  are shown in Figures 1 and 2. The symbols in the figures, which represent test-pieces with the same values of  $B$  and  $B_n$  but different shapes, show that although there is some correlation of the results within one geometry there is little agreement between the different geometries suggesting that  $K$  is not an adequate characterising parameter in these circumstances. This is emphasised by the observation that crack growth rate decreased and did not remain constant in the constant  $K$  tests although previously [5,14] this decelerating rate has been partially attributed to overageing. Comparisons of the results from specimens with the same geometry but different thicknesses and notch depth ratios indicate that at the same value of  $K$  crack growth rate increases with increase in thickness and side groove ratio suggesting that increase in degree of constraint increases creep crack growth rate.

Comparisons of the same data with analytical estimates (where these were possible) of  $C^*$ , made in the same manner as reported by Nikbin et al [14] are shown in Figures 3 and 4. The values of  $n$  used to calculate  $C^*$  were those which gave the best fit of equation (3) to the creep data and were respectively for the aluminium alloy and steel 10 and 5. Although there is better correlation of the data than there is with  $K$  for individual geometries of the same thickness and notch depths there is again lack of agreement between specimens of different geometries particularly for the aluminium alloy. For the same geometry crack growth rate again increases with degree of constraint at constant  $C^*$ .

## DISCUSSION

Because of the satisfactory correlations of the cracking data for one geometry with  $C^*$  shown in Figures 3 and 4, reasons for the discrepancies

between geometries were sought. In making the analytical estimates of  $C^*$  it was assumed that any elastic strains and displacements were small compared to the corresponding creep values. Checks of the experimental displacements however showed that this was not the case. In some instances the creep component of the deflection was found to be almost negligible and in no case was it appreciably greater than the elastic value. Consequently the assumption that creep strains were dominant is not valid.

An alternative approximate estimate of  $C^*$  which avoids the necessity of making the above assumption and which enables values of  $C^*$  to be obtained for a wider range of geometries than is possible analytically is as follows. The problem is one of estimating  $d\dot{\Delta}/da$  in equation (8).

For any circumstance where bending displacements dominate it may be expected that  $\dot{\Delta}$  can be written as

$$\dot{\Delta} = \frac{1}{B} f(a)g(P) \quad (10)$$

where  $f$  and  $g$  are functions. Therefore at constant load

$$\frac{d\dot{\Delta}}{da} = \frac{1}{B} d \frac{f(a)}{da} \cdot g(P) \quad (11)$$

Furthermore, if  $f(a)$  can be approximated over a limited range of crack lengths by a simple power law function  $f(a) = a^\eta$ , equation (11) becomes

$$\frac{d\dot{\Delta}}{da} = \eta \frac{\dot{\Delta}}{a} \quad (12)$$

and

$$C^* = \frac{\eta P \dot{\Delta}}{a B_n (n + 1)}$$

Equation (12) provides an approximate method of estimating  $C^*$  from the experimental data. It is analagous to estimating  $J$  for plasticity by the well known formula  $J = 2U/B(W - a)$  for deep notch three point bend test pieces where  $U$  is work done in this instance. Provided  $\eta$  does not vary appreciably (by more than a factor of about 2) for different geometries  $C^*$  will be proportional approximately to  $P\dot{\Delta}/aB_n$ .

Comparisons of the cracking data with  $P\dot{\Delta}/aB_n$  are shown in Figures 5 and 6. These figures show satisfactory correlations for all test-piece geometries with the same values of  $B$  and  $B_n$  indicating that the lack of agreement on Figures 3 and 4 was probably caused by inadequate estimates of  $C^*$ . No effect of constraint is apparent on Figure 5 for the aluminium alloy but Figure 6 shows that for the steel cracking rate is accelerated with increase in degree of constraint. The data on Figures 5 and 6 can be described satisfactorily by a straight line relationship giving

$$\dot{\Delta} \alpha \left( \frac{P\dot{\Delta}}{aB_n} \right)^\phi \alpha \left[ \frac{(n+1)C^*}{\eta} \right]^\phi \alpha (C^*)^\phi$$

where  $\phi = 0.86$  for the aluminium alloy and 0.82 for the steel. Both these values are close to the respective  $n/(n+1)$  values for each material adding weight to the possibility that creep crack growth rate may be directly proportional to the rate of straining at the crack tip, equation (9), suggested earlier [14].

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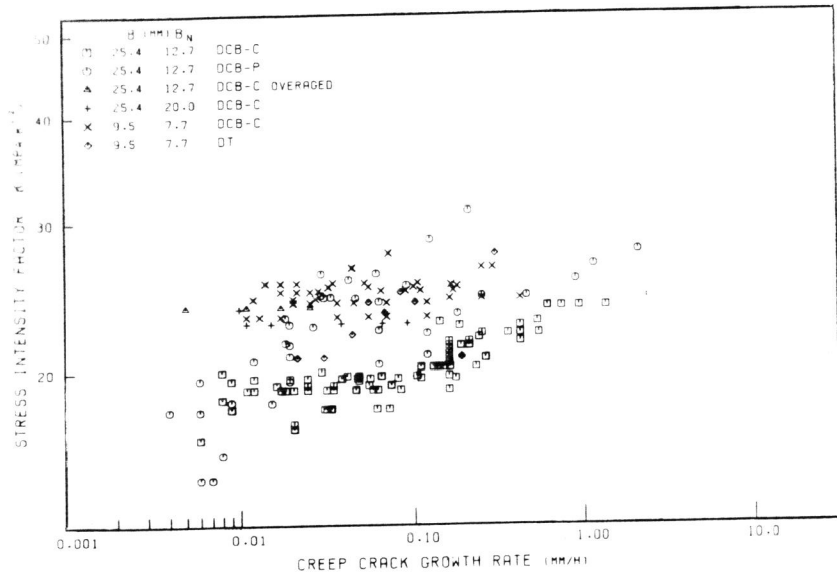


Figure 1 Correlation of creep crack growth with stress intensity factor K for aluminium alloy RR58 at 150°C

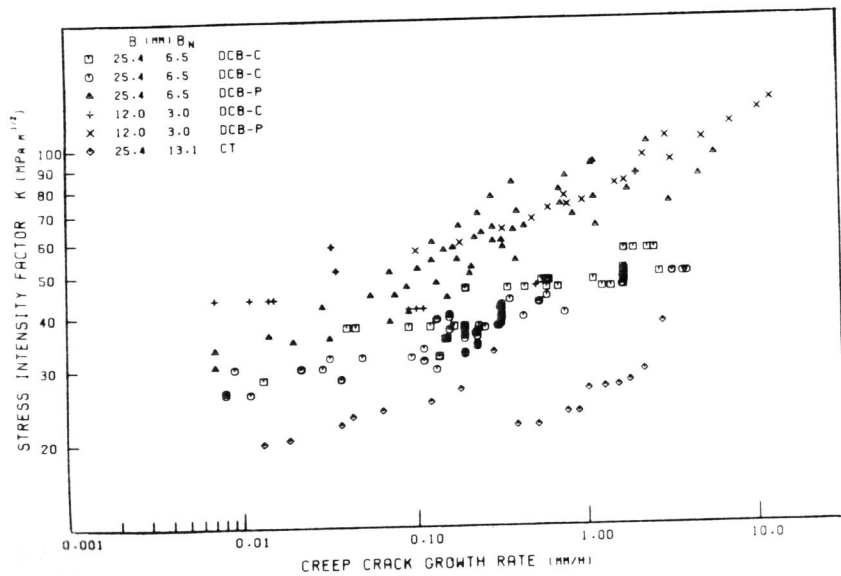


Figure 2 Correlation of creep crack growth with stress intensity factor K for 1/2Cr 1/2Mo 1/4V steel at 565°C

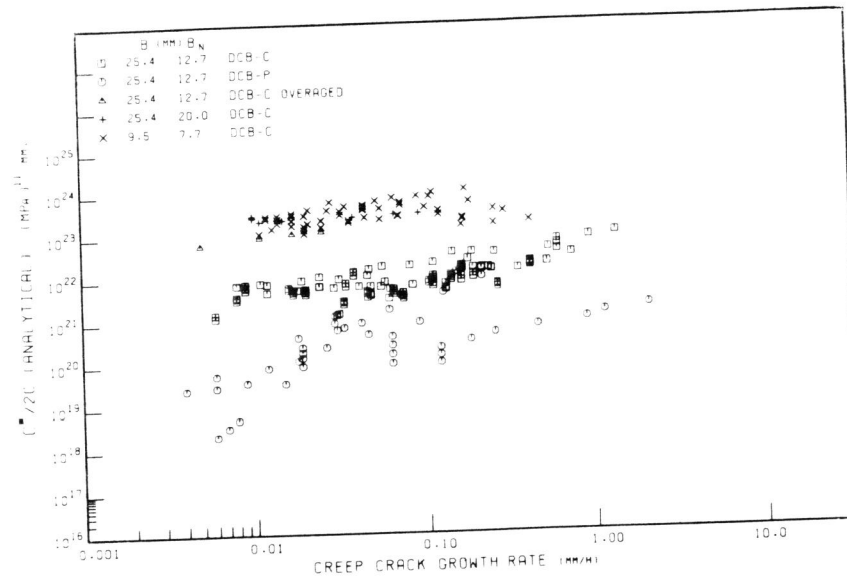


Figure 3 Dependence of creep crack growth rate on analytical estimate of C\* for aluminium alloy RR58 at 150°C

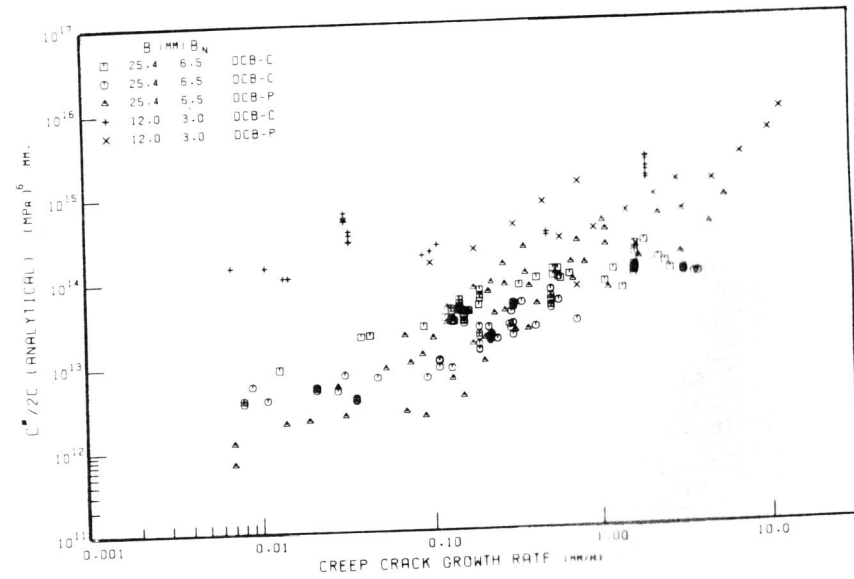


Figure 4 Dependence of creep crack growth rate on analytical estimate of C\* for 1/2Cr 1/2Mo 1/4V steel at 565°C

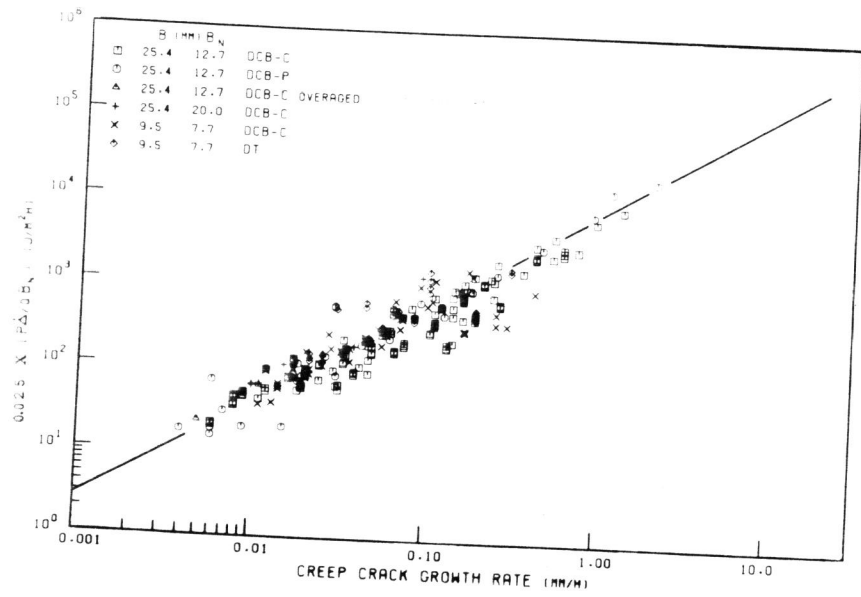


Figure 5 Comparison of creep crack growth rate with experimental estimate of C\* for aluminium alloy RR58 at 150°C

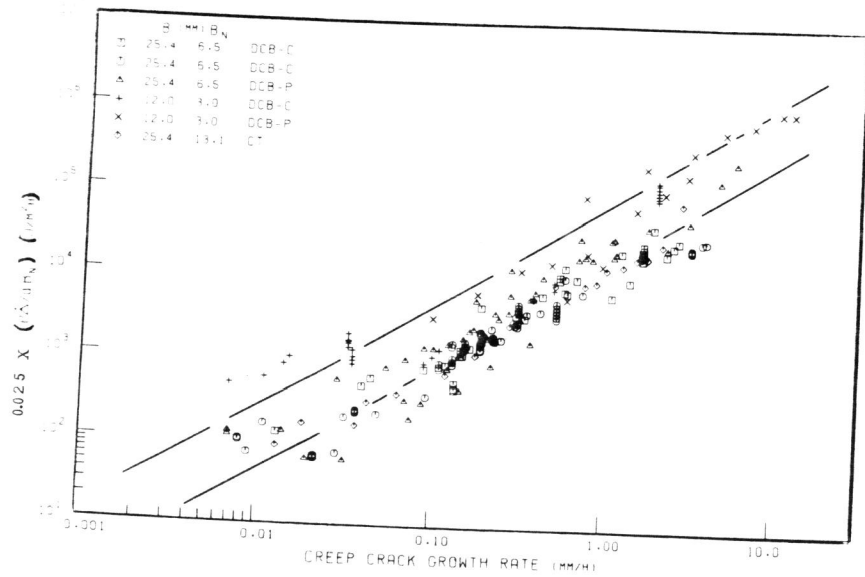


Figure 6 Comparison of creep crack growth rate with experimental estimate of C\* for 1/2%Cr 1/2%Mo 1/4%V steel at 565°C