Fracture 1977, Volume 1, ICF4, Waterloo, Canada, June 19 - 24, 1977

SOME ASPECTS OF NON-LINEAR FRACTURE MECHANICS

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ABSTRACT

The paper discusses different aspects of the use of the J-integral criterion for prediction of crack growth initiation. In this context selected investigations performed at the Royal Institute of Technology in Stockholm are reviewed. These investigations largely support the J-criterion in as far as it is found to be geometry independent. The thickness requirement proposed by ASTM is also verified. In J_{Ic}-testing, determination of the critical point i.e. the onset of crack growth is found to be the most crucial part. In the studies reported an electrical, high frequency current method has been used and found to give excellent results when compared to other methods. Comparative studies between different laboratories and with different methods of evaluation have been made. Scatter in the J_{Ic}-values measured are 20-25% which is considered satisfactory at present.

INTRODUCTION

Materials used in modern structures often have extremely high toughness but still are susceptible to crack growth from crack-like defects and stress-raisers. However, stable and unstable growth of cracks in these materials occur first after considerable plastic deformation. A criterion for crack growth has to take into account the non-linear effects caused both by plastic deformation and by stable crack growth.

In this paper only the initiation of crack growth will be considered and thus the only non-linear effect to be taken into account is that caused by non-linear stress-strain relations. The J-integral (Rice [1]) has been proposed to be a valid crack-growth criterion under these circumstances (Broberg [2], Begley and Landes [3]). The validity of such a criterion has also been verified in some experimental investigations (e.g. Landes and Begley [4]). Further work is, however, needed before the generality of the J-criterion is established and before all principal questions concerning experimental evaluation of critical J, i.e. J_{Ic}, and its application in structural design are solved.

THE J-INTEGRAL

For a two-dimensional problem the J-integral is defined (Rice [1]) as follows:

$$J = \int_C (Wn_1 - \sigma_{ij} n_j u_{i,1}) ds \qquad (1)$$

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where C is a path from the lower to the upper crack surface. The crack is in the plane $x_2 = 0$ with its front along a line x_1 = const. Further, ds is an element of arc length along C, n_j is the outward normal to C, u_i is the displacement vector, σ_{ij} is the stress tensor and W is the elastic non-linear strain energy density. The application of an elastic function W to elastic-plastic problems will be commented upon further below.

Another definition of J also due to Rice [5] relates it to the elastic potential energy U:

$$J = -\frac{\partial U}{\partial a} \tag{2}$$

Here a is crack length and a two-dimensional body of unit thickness is considered so that U is defined

$$U = \int_A W dA - \int_{C_T} T_i u_i ds \tag{3}$$

Here C_T is that part of the two-dimensional body of area A where tractions T_i are prescribed.

For a linear elastic material the crack extension force G_I according to Irwin [6] is

$$G_I = -\frac{\partial U}{\partial a} = \frac{(\kappa+1)}{8\mu} K_I^2 \tag{4}$$

Thus from equation (2) it is seen that J is a generalization of G. In equation (4) μ is the shear modulus, $\kappa = (3-4\nu)$ for plane strain and $\kappa = (3-\nu)/(1+\nu)$ for plane stress with ν being Poisson's constant. K_I is the stress intensity factor. It is recognized that non-linear elastic constitutive relations may be used to describe elastic-plastic deformation in a unique way as long as unloading does not occur and a deformation theory of plasticity is applicable.

On application of the J-integral to crack problems for elastic-plastic bodies it is understood that unloading from the plastic state must not occur. Therefore, the J-integral is expected to be applicable only to the initiation stage of crack growth.

Under these circumstances there are several fundamental reasons why a criterion

$$J = J_{Ic} \tag{5}$$

should be applicable for prediction of crack growth initiation:

(a) As indicated above, J is a generalization to the non-linear case of Irwin's crack extension force G_I, which is the fundamental measure of crack tip loading in linear elastic fracture mechanics.

(b) As shown by McClintock [7] the state of stress and strain at a crack tip is directly related to J for a non-linear material with a stress-strain relation of the form $\sigma = \sigma_0 \varepsilon^n$.

(c) The J-integral may in the non-linear case (Strifors [8]) by means of a virtual work consideration be interpreted as an apparent crack extension force. Strifors then uses a virtual displacement field: $\delta u_i = \partial u_i/\partial x_1$ which models a virtual crack extension. The virtual work equation then transforms to a generalized J-integral. The procedure is equivalent to replacing $\partial/\partial a$ in equation (2) by $\partial/\partial x_1$ and C_T in (3) by C (the entire boundary).

(d) The relation equation (2) may be considered as an energy balance for a body containing a crack. Then J represents the fracture energy connected with unit growth of the crack. This becomes clear by replacement of W in equation (3) with (W^e+W^p) where W^e is elastic and W^p plastic strain energy density. Then equation (2) takes the form

$$J = \frac{\partial}{\partial a}\left[\int_{C_T} T_i u_i ds - \int_A W^e dA - \int_A W^p dA\right] \tag{6}$$

The first term represents the work done by the external forces due to crack growth, the second the increase of elastic strain energy.

One remark with respect to the definition of J seems appropriate. Numerical computations by several investigators indicate that J according to equations (1) and (2) differ considerably for the non-linear case. It would be logical to refer to $-\partial U/\partial a$ as a generalized crack extension force and not as J. Below, however, both the J-integral and $-\partial U/\partial a$ are referred to as J.

In addition to the above fundamental arguments supporting the J-integral as a relevant criterion for initiation of crack growth there are several features which makes the J-integral attractive as a measure of crack tip loading. It is relatively easy to determine both experimentally and by means of FEM computations.

The J-criterion equation (5) is applicable to structures of much smaller dimensions, especially thickness, than is the corresponding criterion in linear elastic fracture mechanics. The relation between the required thicknesses in the two cases is approximately 10 $_y$/E. This implies a possibility to use J_{Ic} data determined from small specimens to compute valid fracture toughness values K_{Ic} through the relation:

$$J_{IC} = \frac{(1-\nu^2)K_{Ic}^2}{E} \tag{7}$$

These K_{Ic}-values may then be used in LEFM-computations.

APPLICATION OF THE J-INTEGRAL IN A CRITERION FOR CRACK GROWTH INITIATION

General considerations

Application of equation (5) in material testing for determination of J_{Ic}-values requires a) determination of J as a function of displacement and b) determination of critical J or critical displacement at onset of crack growth. As indicated in the previous sections of this paper there are several ways of determining J as a function of loading parameters. It can be done using equation (1) in connection with FEM-calculations or equation (2) in connection with experiments as proposed by Landes and Begley [4]. In addition several approximate, analytical relations have been proposed to determine J from experimental data (Bucci, Paris, Landes, Rice [9] and Rice, Paris, Merkle [10]).

In this context is is essential to study the agreement between different methods. Here only results obtained utilizing equations (1) and (2) will be compared. Other investigators, e.g. Keller and Munz [11], have studied also the approximate methods for J computations.

It should be pointed out that several investigations along the lines indicated above have been made and are under progress in laboratories all over the world. Below a selection of investigations carried out at the Department of Strength of Materials and Solid Mechanics at the Royal Institute of Technology, Stockholm, will be discussed.

Path independence of the J-integral

For the case of a hyperelastic material (corresponding to linear or non-linear elasticy), it is straightforward to show that the J-integral is path-independent, i.e. J attains the same value irrespective of the choice of integration path C between the end points. The proof hinges on the existence of a single-valued strain energy density function W, defined as

$$W = \int \sigma_{ij} d\varepsilon_{ij} \qquad (8)$$

Path-independence can, however, not be assessed in the incremental plasticity case, where W is generally a function of the loading history.

This might seem a serious drawback as regards application to elastic-plastic problems, since the potential usefulness of the J-integral is greatly dependent on its path-independent property. Still, results from several investigations indicate that approximate path-independence prevails, even when incremental plasticity formulation is employed, for many cases of practical interest.

The same conclusion is indeed drawn from numerical work based on FEM technique performed at the Royal Institute of Technology (Markström and Carlsson [12] and Markström [13]).

Elastic-incrementally plastic material properties (strain hardening as well as non-strain hardening) were simulated and several realistic fracture mechanics specimen configurations - Wide Plate and Compact specimens with varying crack length-to-width ratii - were studied for both plane strain and plane stress conditions.

In [12], variation in J as computed along different integration paths (some of which intersected the plastic zone) was found to be less than $\pm 1\%$, provided the total strain energy density was included (and not, as has sometimes been advocated, merely the elastic contribution). In [13] the corresponding figure was somewhat larger: $\pm 3\%$, probably due to the relatively coarser element mesh employed.

This favourable behaviour regarding path-independence is presumably explained by the fact that, in many practical situations, the loading history around the crack tip closely approaches proportional loading, in which case path-independence can again be theoretically established under

the restriction that unloading must not occur anywhere in the material volume considered.

In this context it is worthwhile to stress again that the J-integral appears unsuitable for studies of propagating cracks, since crack propagation necessarily implies partial unloading.

Alternative evaluation of J

Besides the formal definition (1) of the J-integral it is proposed that J may be alternatively evaluated according to (2). This later approach appears especially attractive in connection with experimental work, as it devices a way to compute J values from experimental load-deflection data for a series of specimens with varying crack lengths but otherwise identical.

The equality between expressions (1) and (2) can readily be demonstrated for the linear elastic case. However, the question arises, whether this equality is still preserved in situations of finite specimen size and inelastic material characteristics.

Specific attention was focused on this point during interpretation of results from experimental [14] and numerical [13] studies at the Royal Institute of Technology.

Here, systematic variations which could not be explained on basis of neither experimental nor numerical scatter were observed. One example is shown in Figure 1. J data from the experimental investigation were calculated according to (2), whereas in the numerical study equation (1) was employed. In [13], the geometry and material characteristics of the specimens used in [14] were modelled as closely as possible.

Further processing of numerical data from [14] is currently in progress. Preliminary results tend to indicate that, in certain cases, evaluation according to [13] and [14], respectively, yields J values differing by a significant amount. If this observation should remain valid, due consideration will be required in both theoretical and experimental work.

Experimental determination of J_{Ic}

In order to apply the proposed crack growth criterion (5) the "material property" J_{Ic} has to be determined. This can only be done experimentally.

In principle, evaluation of J_{Ic} data from test results is a very simple procedure, provided two sets of information are available:

1. a J calibration for the actual specimen, preferrably in the form

$$J = J(\Delta) \qquad (9)$$

   where Δ is specimen deflection.

2. an accurate estimate regarding the critical point, e.g. Δ_{cr} - the specimen deflection at which crack growth starts.

In experimental work, the main difficulties are usually encountered in connection with determination of critical points. Frequently, small errors in Δ_{cr} might induce relatively large errors in J_{Ic}, thus tending to increase scatter in the experimental results.

Several methods for detection of critical points have been proposed, some of them successfully used in certain experimental applications. Unfortunately, observation of current crack length at the specimen surfaces is generally not sufficient, because crack growth usually starts in the center portion of the specimen.

Apart from the direct inspection of fracture surfaces for traces of crack growth (used in so-called R-curve method) the more commonly used techniques involve interpretation of test records (load-deflection or load-crack opening displacement curves) or monitoring some independent quantity (ultrasonics, acoustic emission, holography, potential drop, impedance, eddy-currents) during testing.

At present, no single method can be generally recommended; the choice of detection technique must be judged on the basis of experience and with respect to the specific situation. Two major difficulties, common to most methods, are, attainment of sufficient resolution (a relative detection capability of the order 10^{-3} is usually needed) and separation of effects caused by inelastic processes within the specimen.

Here, a remark in connection with determination of critical points from test records is appropriate: it is generally *not* true that crack growth commences when the load passes through a maximum, which has occasionally been assumed - consciously or inconsciously.

The R-curve method, which involves loading of several specimens to different load levels and observing the corresponding amount of crack growth after subsequent fracturing, is not attractive from a practical point of view due to the requirement of multiple tests to obtain each single J_{Ic} value.

Moreover, the method as usually practiced (and included in proposed standards for J_{Ic} testing) involves a moment of ambiguity: the J_{Ic} value is not evaluated at zero crack extension but at a value Δa_o,

$$\Delta a_o = \frac{J}{2\sigma_y} \qquad (10)$$

The purpose is to exclude the contribution to measured crack extension from crack tip blunting. The blunting effect indeed corresponds to $J/2\sigma_y$ for the Dugdale model, but may well be different for realistic test specimen configurations.

As regards the necessary J calibrations, these may be obtained in several ways. Very accurate J-deflection relations can be computed using FEM technique. Such computations for elastic-plastic cases are, however, quite expensive. Frequently, reasonably good estimates can be obtained from theoretical considerations. Solutions for linear-elastic and rigid-plastic cases are presently available for a variety of configurations. An approximation to a real elastic-plastic case can be achieved by superposition of these two idealized cases, eventually also incorporating an Irwin-type plasticity correction. An alternative method for calculation of J is based on the relation (2). In evaluation, test records from specimens with different crack lengths but otherwise identical are utilized.

In general, as evident from (2), integration of the load-deflection relations followed by a differentiation with respect to crack length is required. This involves rather tedious calculations, unless some automated data processing technique is employed. Note, however, the exceptional case of the ASTM three-point bend specimen, where the following expression happens to yield good estimates throughout the entire elastic-plastic region [10]:

$$J = \frac{2U}{B(W-a)} \qquad (11)$$

Here, W is specimen width.

The latter method seems particularly attractive from a practical point of view, since evaluation is based on quantities measured during testing of the actual specimen rather than on an idealized mathematical model.

Some of the points elaborated above have been studied at the Royal Institute of Technology [14]. Here experimental J_{Ic} determinations were carried out on two medium-strength steels, using four different specimen configurations.

Two independent electrical methods for detection of crack growth were simultaneously employed: one (HF) measuring the electrical impedance across the crack (c.f. Figure 2), the other (EC) based on eddy-current principles. This technique greatly reduced the possibilities of errors in interpretation of test results and was shown to yield consistent results for all specimen types studied.

J calibrations were obtained on basis of (2). Special equipment for collection and processing of test data was designed and the numerical work was computerized. Difficulties originally experienced in connection with the numerical differentiation process were resolved by application of special smoothing and fitting techniques. Input to the processing equipment consisted of analog load-deflection test data stored on magnetic tape. As output, tabulated J-deflection data for each specimen were obtained.

Dependence of J_{Ic} on specimen geometry, size, and thickness

As previously discussed, a basic requirement for successful application of the J-integral in a fracture criterion is that the quantity J_{Ic} be a true material constant. This implies that experiments on different types and sizes of specimen should yield identical J_{Ic} values. There is experimental evidence that this is indeed the case for many situations of practical interest, even for large variation in specimen geometry.

As an example, in [14] no significant variation in J_{Ic} with specimen type or size could be observed, (c.f. Figure 3). This study comprised 40 compact and Wide Plate specimens, all with different crack lengths, of two medium-strength steels. Specimen widths ranged from 0.04 to 0.09 m (only the in-plate dimensions were varied).

Regarding the variation of J_{Ic} with specimen thickness, it is commonly believed that J_{Ic} is virtually thickness-independent for thicknesses greater than a certain (material dependent) value B_{cr}, conveniently expressed in the form

$$B_{cr} = \alpha \frac{J_{Ic}}{\sigma_y} \qquad (12)$$

where α is some non-dimensional constant.

A valid J_{Ic} determination would thus require specimen thickness exceeding B_{cr}. In linear elastic fracture mechanics a similar limitation exists. However, the numerical value of B_{cr} would hopefully be substantially lower in the former case.

Experimental work in progress at the Royal Institute of Technology is intended to study the influence of thickness on J_{Ic} for selection of steels. Preliminary results for one of the materials indicate a transition thickness in the range 0.005-0.010 m, which implies an α-value of the order 20-40. For comparison, the ASTM limit for valid K_{Ic} determination is about 0.25 m for this steel.

Scatter in J_{Ic}

Like other experimentally determined quantities, J_{Ic} is subject to certain scatter. In addition to inherent material scatter due to local variations on a micro-structural level, imperfections in the experimental technique will contribute to the total scatter. This contribution is certainly of significance in most practical cases, but can in principle be reduced by refinements in testing and evaluation procedures. In this respect, determination of onset of crack growth would be expected to be the critical point.

It is reasonable to expect scatter in J_{Ic} of about twice the magnitude observed in K_{Ic} tests. Thus, though seemingly large at first glance, a figure of $\pm 25\%$ scatter in J_{Ic} would not be unlikely even under well-controlled testing conditions.

A recent investigation regarding the statistical variation in J_{Ic} data is reported in [15]. A large number of three-point bend specimens were tested at two different laboratories and several alternative methods of evaluation were employed. No significant variation in J_{Ic} mean values due to either evaluation procedure or laboratory chosen could be observed. However, somewhat surprisingly, the HF technique (previously mentioned) for detection of incipient crack growth gave considerably less scatter than the R-curve method. In the former case the figure (standard deviation) was $\pm 20\%$, and in the latter, about twice that much. Also, it is of interest to note, that in the study reported in [14], for both materials studied all data points fell within the limits $\pm 25\%$ referred to the respective mean values, regardless of specimen type and dimensions.

CONCLUSIONS

The J-integral criterion for initiation of crack growth is supported by a number of investigations at the Royal Institute of Technology. The relatively large scatter that has been obtained in J_{Ic} may be explained partly by deviations between numerical and experimental J-determinations, partly by difficulties in determining the critical point for onset of crack growth. When these aspects have been studied further it is believed that accuracy in J_{Ic} determination can be improved.

Aside from the scatter in the results it is surprising that tests at different laboratories and with different methods of evaluation give J_{Ic}-values that are in such good agreement.

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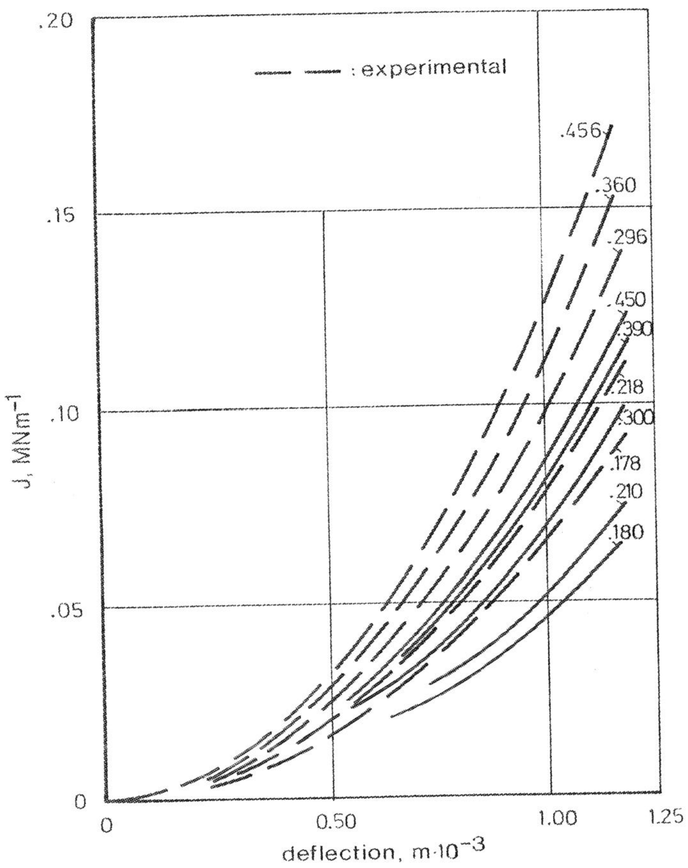

Figure 1 J versus specimen deflection, comparison of numerical and experimental results. Domex 400 steel, 0.900 m wide Center Cracked specimen. Numbers denote total crack length.

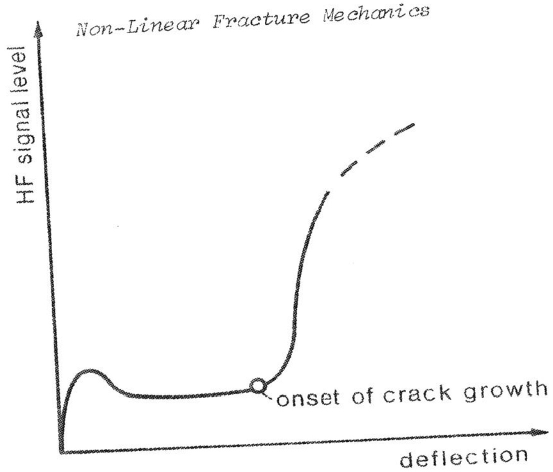

Figure 2 Determination of critical point from HF versus specimen deflection test record.

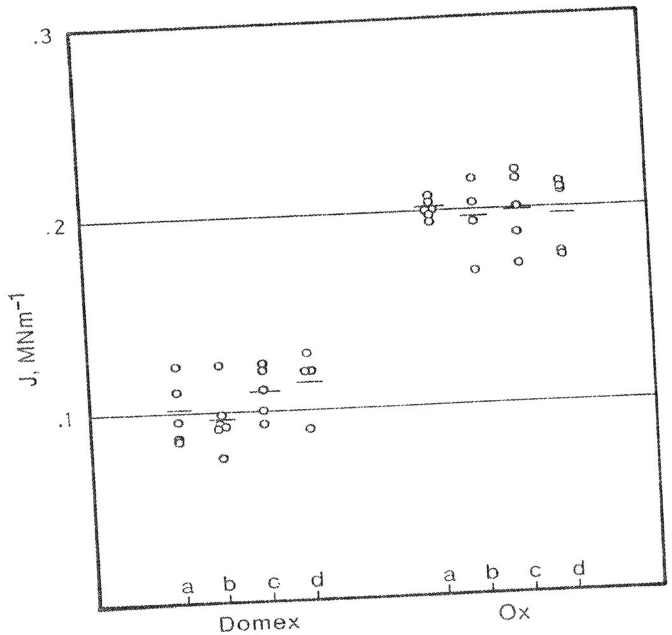

Figure 3 J_{Ic} data for individual specimens, steels Domex 400 and Ox 802.
a: 0.900 m wide Center Cracked specimen
b: 0.900 m wide Double Edge Cracked specimen
c: 0.200 m wide Compact specimen
d: 0.040 m wide Compact specimen