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ELASTIC-PLASTIC CRACK TIP CHARACTERIZATION IN RELATION TO R-CURVES

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ABSTRACT

Relative to estimates of onset of rapid fracture in service structures, R-curves provide more help than is furnished by single measurement point fracture tests. Corrections to the linear analysis values of K (and G) for the crack tip plastic zone, as used in R-curve determinations, are equivalent to using a characterization of the plasticity type, for example, J. R-curves obtained during general yielding, using J or δ for characterization, appear to be the same as would be obtained with larger specimens where the crack tip plastic zone is enclosed by an elastic field. For structural steels at temperatures and strain rates such that substantial fibrous crack extension occurs prior to onset of rapid fracture, an R-curve toughness evaluation method is of definite interest for practical applications. Special attention must be given to the possible influence of loss of plane-strain constraint upon cracking events dominated by cleavage. In addition the lack of a plane-strain R-curve measurement technique is unfortunate.

INTRODUCTION

Recognition that significant amounts of slow-stable crack extension may accompany the loading of a fracture toughness test specimen became of interest during efforts to develop suitable toughness evaluations of the K_c type [1,2,3]. Refinements of crack extension resistance measurements during slow-stable crack extension, introduced by Heyer and McCabe [4,5], led eventually to a proposed standard method for resistance curve (R-curve) determinations [6]. The ideas which pertain to R-curves have assisted general understanding of fracture toughness measurements. A review of these ideas is of value in relation to current interest in the J-integral and crack(tip) opening displacement methods of characterization.

Relative to analysis and testing, the main ideas basic to R-curves are as follows:

1. When the characterization factor is G or K, computed using linear-elastic assumptions, the crack size used in such computations is the effective crack size. The effective crack size is the physical (visual) crack size increased by a computed plasticity adjustment factor, r_Y. Alternatively, in less arbitrary treatments, the effective crack size is estimated from observations of a compliance or a displacement ratio using a prior crack size calibration.

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2. The crack extension resistance is expected to increase toward a maximum value with increments of progressive crack extension in a manner which depends only upon crack extension and the "sharpness" severity of the fatigue pre-crack. When the "sharpness" condition is fixed, the influence of initial crack size, specimen dimensions (except thickness), and specimen shape are assumed to be small enough to be regarded as negligible.

3. R-curve investigations have employed only two-dimensional analyses of through-the-thickness cracks. With this restriction, the goal of the R-curve method is to provide information such that conditions for rapid extension of a prior crack of given size and location in a service component can be estimated. The preceding critical conditions are estimated by finding the position on the R-curve where (assuming appropriate loading conditions) the characterization factor computed for the pre-cracked service component begins to show a larger increase, per increment of crack extension, than pertains to the R-curve.

The information provided by the R-curve is more complete than that furnished by a one parameter fracture toughness evaluation method. As with most engineering methods, the basic ideas are oversimplified and imperfect. However, the R-curve method has provided satisfactory predictions of the critical load for onset of rapid fracturing in a number of trials [5,7] and the usefulness of the R-curve viewpoint for purposes of perspective does not appear to be questioned. The role of elastic-plastic aspects in relation to the R-curve method is of special interest in this discussion.

THE CRACK SIZE PLASTICITY ADJUSTMENT

The first of the three basic R-curve ideas noted above concerns use of the effective crack size in computations of the resistance parameter, K_R (or G_R). Very early in the development of linear fracture mechanics, it was recognized that testing methods would prefer use of specimens of minimum size. Thus methods for adjusting the linear analysis in compensation for finite size of the crack tip plastic zone received investigation [8,9]. The practice of this kind which remained of permanent interest consisted in a positioning adjustment, r_Y, of the crack tip which would place the crack tip for linear analysis purposes near the centroid of the crack tip plastic zone when the plastic zone is relatively small. Trials were, of course, extended to experiments in which conditions of general yielding were closely approached. In the case of the center-cracked and double-edge-notched tensile specimens, frequently used prior to 1965, measured values of K_c remained essentially constant when the average net section stress was less than 80 percent of the tensile yield strength [10]. The decrease of K_c usually observed when the experiments closely approached general yielding was regarded as more likely due to R-curve effects associated with slow-stable cracking than to inadequacy of the adjustment for the crack tip plastic zone. The experiments used high strength steel and aluminum alloy sheet materials for which variations of loading speed had little or no influence on test results.

The reason for the success of the crack size plasticity adjustment in the range where the crack tip plastic zones had substantial size was not obvious at that time. Beginning in 1960, attention was given to development of the characterization concept termed crack (tip) opening displacement and represented in this discussion by the symbol, δ [11]. It was then possible to make analytical comparisons of plasticity adjusted values of G to δ in terms of the expected relationship $G = \sigma_y\ \delta$. Figure 1 shows

results of this nature computed by Tada [12] assuming a center-cracked tensile specimen and strip plastic zones of the Dugdale type. Similar results had been found for the same specimen configuration using Mode 3 elastic-plastic solutions.

With the development of interest in use of J as the crack tip characterization parameter, the above results indicated that, for a wide range of crack sizes, an r_Y plasticity adjustment method could be used to estimate values of J prior to general yielding. J estimates obtained in this way were compared to experimental J determinations in reference [13]. The specimen types used were center-crack tension, compact tension, and notch-bend. The results were generally in support of the above conclusion.

As is well-known, the value of J can be interpreted as an energy release driving force for crack extension only when the material behavior is elastic. Using comparisons between R-curves with specimens of different lateral dimensions, Heyer and McCabe [4] concluded that use of a plasticity adjustment in computations of K (or G) was necessary in order to obtain R-curves which were nearly invarient functions of crack extension. In view of the close relation between J and the plasticity adjusted value of G, this conclusion is equivalent to saying that R-curves plotted in terms of J would be nearly invarient functions of crack extension. This idea deserves thought because the amount of crack extension during a typical R-curve determination is comparable to the nominal plastic zone size, $2r_Y$.

SLOW-STABLE CRACK GROWTH

For the center-cracked and single-edge notched tensile specimens used in early fracture toughness testing, K tended to increase with increase of the crack size. Since observations of slow-stable crack growth prior to rapid fracturing were commonly observed, it was clear that the resistance to crack extension increased with forward motion of the crack. It was recognized that natural roughening of the fracture surface beyond the fatigue "sharpened" initial crack would increase resistance to crack extension even in the flat tensile mode of separation. In addition there were the side boundary shear lips. Load transfer from the central tongue of flat tensile separation to side obundary regions which would sever later by oblique shear, tended to delay onset of rapid fracturing. With high yield strength steel and aluminum alloys, slow-stable crack growth and continued increase of K_R could often be observed after the separation process became entirely oblique shear. Clear demonstrations that a sufficient amount of slow-stable growth would reveal a maximum limiting value of K_R were not available for metallic plates prior to the careful investigations by Heyer and McCabe [2,3]. In these observations, an extension increment, Δa_p, of the physical (visual) crack size by an amount roughly equal to the nominal plastic zone size, $2r_Y$, during increase of K_R to 95 percent of the maximum value seemed to be typical.

In subsequent research, McCabe [14] has extended R-curve studies into the region of general yielding of the net ligament. A compact-tension specimen modified for wedge loading near the crack line was used. Preliminary loadings of identical specimens into the general yield range were used to obtain hinge-point data for purposes of δ calculations and load vs load-displacement data for purposes of J determinations. The comparisons of principal interest here employed a 6.4 mm thick plate of 2024-T351 aluminum and a 2 mm thick plate of medium strength structural steel. For the aluminum alloy, it was possible to compare R-curves obtained prior to general yielding (using large specimens) to substantial portions of small specimen

R-curves obtained after general yielding using δ and J methods of characterization. For the more ductile steel specimens only the R-curves in terms of δ and J could be compared. To assist comparisons, the data was plotted in terms of K_R using the relations

$$E\,K_R^2 = J = \sigma_Y\,\delta \tag{1}$$

where σ_Y is the average between the yield strength and the ultimate tensile strength. Figure 2 shows the aluminum alloy results. The two low values from the δ estimate correspond to a net ligament size which is intermediate between 25 δ and 50 δ.

For the 2 mm thick steel sheet (345 MPa yield strength) the R-curve region prior to general yielding was not large enough for usefulness. Within the range of new ligament size greater than about 35 δ, there were no significant differences between the R-curves based upon δ calculations and those based upon determinations of J. Two specimen sizes were used. Both were too small to show the approach of K_R to a limiting maximum value.

During initial portions of the R-curve when Δa_p is relatively small, the increase of K_R with crack extension is rapid. From comments by McClintock and Irwin [15], the plasticity adjusted value of G, for this region, does not equal the total plastic flow work rate. However, the adjusted G should equal J and the relative smallness of Δa_p is advantageous for use of the J and δ concepts. As Δa_p becomes substantial in comparison to $2r_Y$, it seems possible that residual stress influences due to prior plastic deformation from regions behind the crack tip may have important effects. From the information available, influences of this kind may exist. However, it would appear that such influences, if significant, are dominated by the region of largest strains near the crack tip sufficiently so that a substantial portion of the R-curve can be determined using small specimens and plasticity methods of characterization.

The information from reference [14] discussed above pertains to testing conditions for which strain rate does not have a large influence and the cleavage-fibrous transition is not involved. For such conditions it is clear that the resistance to slow stable crack extension can be represented usefully in terms of values of J computed without allowance for by-passed regions of prior plastic strain.

ONSET OF RAPID FRACTURING

From the above discussion, it is clear that some questions of physical interpretation of the characterization parameter develop during substantial amounts of slow-stable crack extension. These are due to possible influences of regions of prior plastic strain behind the advancing tip of the crack. Aside from whatever answers may be provided from future research, it is possible to employ a model type viewpoint with regard to estimates of the conditions for onset of rapid fracturing for a given prior crack in a service component. From this viewpoint, an exact physical interpretation of the crack tip stress-strain field implied by a value of J may not be required. One can assume that the crack tip stress-strain field for the crack in the service component will respond to slow-stable extension in nearly the same manner as occurred during the R-curve determination.

When an R-curve testing method is not available, the implications of this fact on toughness evaluations is important. A familiar example is furnished by K_{Ic} measurements of plane-strain fracture toughness. It is not easy to describe a feasible technique for plane-strain R-curve determinations. Observation of K (or J) during the first stages of development of a crescent of flat-tensile separation ahead of a straight fatigue pre-crack represents the closest approach to such measurements as is visible at the present time. As a result, K_{Ic} measurements center attention on the start of crack extension rather than onset of rapid fracturing. Of course, concepts such as "plane-strain at the crack tip" and "start of crack extension" cannot be taken literally and must be defined by somewhat arbitrary rules for testing control purposes. Examples of such rules are furnished by ASTM method E399. J_{Ic} testing methods also center attention on start of crack extension. Although investigations toward development of a standard J_{Ic} testing method are still in progress, it is apparent that a literal interpretation of "start of crack extension" will again be avoided.

Small specimen methods based upon elastic-plastic analysis are of considerable interest for application to structural steels with a yield strength less than 700 MNm^{-2}. The strain rate sensitivity and cleavage-fibrous transition properties of such materials must be considered in making fracture toughness evaluations. In addition, for heavy section applications, evaluation of the relevant fracture toughness should simulate conditions of plane-strain at the crack tip. The testing problem is simplified if the start of crack extension is dominated by cleavage. Beginning with the pioneering trials by Begley and Landes [16], a number of favorable correlations have been shown between J_{Ic} test results using small specimens and K_{Ic} test results from large specimens at testing temperatures where the amount of fibrous crack extension was quite small prior to development of rapid cleavage fracturing.

Despite the high levels of fracture toughness involved, extension of toughness evaluations into a temperature range where significant amounts of fibrous separation occur prior to onset of rapid fracturing is desirable for a number of reasons. Two of the difficulties in accomplishing this with fracture mechanics type testing are (1) the probable importance of degree of plane-strain constraint upon onset of cleavage fracturing and (2) the plane-strain R-curve behavior. These two aspects are in the "future research" area at present. A rough guess at the magnitude of the potential plane-strain R-curve effect can be made based upon observations by Shoemaker and Rolfe [17]. Using 25 mm thickness notched-bend specimens of A302-B steel and a slow loading speed, they observed two instances of crack arrest after about 7 mm of sudden cleavage crack extension. The illustration of the crack arrest event at -158°C shown in Reference [17] indicated no significant difference in crack front curvature between the fatigue pre-crack and the arrested crack. The K necessary to restart the arrested crack was 24 percent larger than the K for the start of the run-arrest segment. This was due to natural roughening of the fracture surface during an extension much larger than any estimate of the plastic zone size. If one assumes 200 MPa•m$^{1/2}$ as a toughness level for A533B steel at a temperature high enough to permit $2r_Y$ of fibrous crack extension prior to onset of rapid fracturing, the plane-strain value of $2r_Y$ would be about 11 mm and the increase of crack extension resistance during this crack growth seems likely to be larger rather than less than 24 percent. In evaluating nuclear vessel integrity safety factors for various imagined accidents, the R-curve increase of fracture toughness could be rather large and of considerable practical importance.

Currently, it is evident that elastic-plastic methods of characterization in terms of J are soundly established and provide large advantages over the characterization parameters associated with linear analysis. Successful trials of J and δ, using ductile sheet materials, lead one to consider extension of R-curve work so as to include certain plane-strain, high-toughness situations which are of practical interest. There seems to be a need for inventive ideas as to how this might be done.

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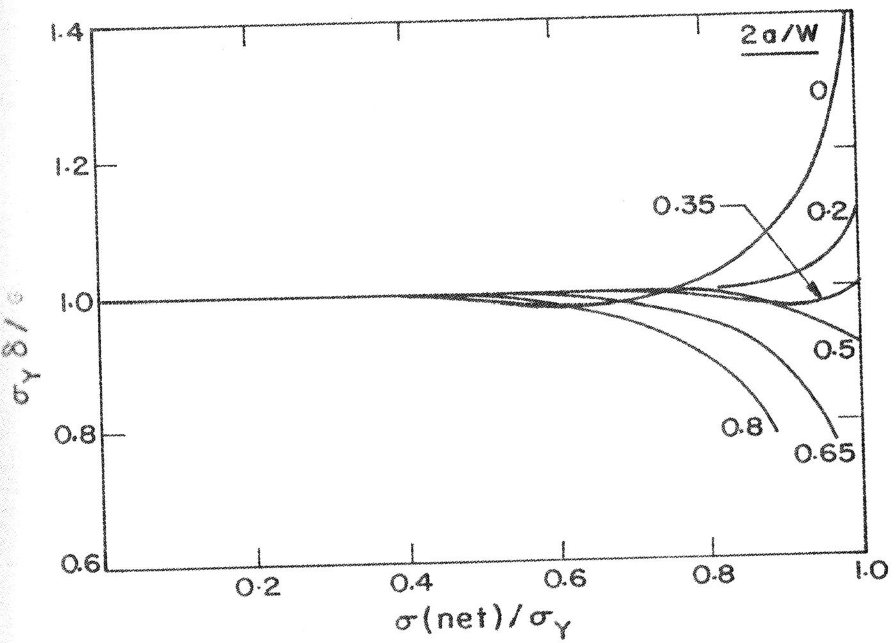

Figure 1 Ratio of $\sigma_Y\delta$ to the r_Y adjusted value of G as a function of the average net section stress, σ(net), divided by σ_Y. Curves are shown for a series of relative crack sizes, 2a/W, as indicated, where 2a is the length of the central crack and W is the width of the tensile specimen. Redrawn from reference [12].

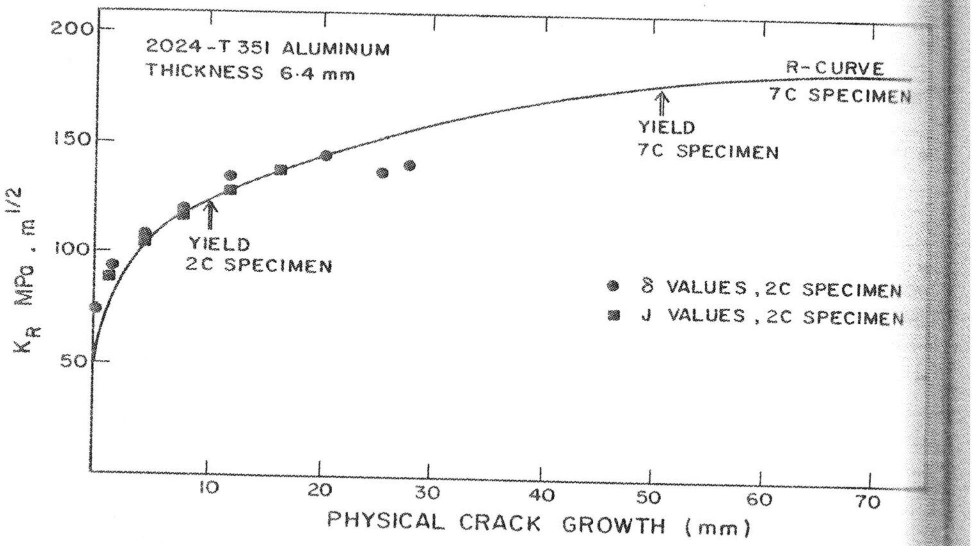

Figure 2 Solid curve shows values of K_R from a crack-line loaded compact tension specimen (W = 356 mm). Also shown are values of K_R from δ and J determinations using small specimens (W = 102 mm). Redrawn from reference [14].