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DYNAMIC CRACK PROPAGATION AND ARREST IN PLATES, PIPES, AND PRESSURE VESSELS†

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INTRODUCTION

Conventional fracture mechanics carries the analysis to the end of stable growth and assumes the onset of unstable propagation ends the useful life of the structure. However, there are situations when the events following the onset of fast fracture must be analyzed. This is the case when economical designs cannot preclude initiation in all circumstances, but where unchecked, catastrophic extension is intolerable. Particular examples include LNG ship hulls, arctic pipelines, and nuclear pressure vessels. In these cases a second line of defense--an assurance that the crack will arrest--is needed.

Only very recently has a fundamentally sound basis for the treatment of rapid unstable crack propagation and arrest become available. To contrast with the conventional fracture mechanics approach, such an approach can be referred to as dynamic LEFM. This paper gives the theoretical basis of a dynamic fracture mechanics methodology, identifies relevant material properties, and describes several different practical applications that have so far been made to ensure crack arrest.

THEORETICAL BASIS FOR THE ANALYSIS OF DYNAMIC CRACK PROPAGATION AND ARREST

Although many actual problems may require fully elastic-plastic treatments, the process of rapid unstable crack propagation and arrest in structures can currently be discussed only in terms of linear elastic fracture mechanics (LEFM) concepts and parameters. A dynamic extension of LEFM recognizes four contributions to a propagating crack: elastic strain energy, kinetic energy, work done by applied forces, and the energy dissipated by crack tip flow and fracture processes [1,2]. The first three of these depend primarily on the crack length, the applied loads, and the geometry of the body containing the crack. The net change in these three components, per unit area of crack extension, is called the dynamic energy-release rate, or, equivalently, the driving force for crack extension. Giving this the symbol G, then, formally

$$G = \frac{1}{b}\left\{\frac{dW}{da} - \frac{dU}{da} - \frac{dT}{da}\right\} \quad , \tag{1}$$

where U is the strain energy, T is the kinetic energy, W is the work done on the structure by external loads, a is the crack length, and b is the plate thickness at the crack tip.

†Extended Abstract.

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Two generalizations exist in the evaluation of G for a fast propagating or arresting crack beyond those required in the static case. First, a kinetic energy contribution is present. Second, the relevant quantities must be evaluated from fully dynamic analyses, i.e., with inertia forces explicitly included in the equations of motion for the structure. Note that although equation (1) apparently represents G as a global quantity that must be evaluated by directly considering the entire structure, it can always be given a local crack-tip interpretation. In particular, by using the result obtained by Freund [3] and generalized by Nilsson [4], the dynamic energy-release rate can be directly connected to the dynamic stress-intensity factor K. For plane strain conditions, this relation is

$$G = \frac{1-\nu^2}{E} A(V) K^2 \quad , \qquad (2)$$

where A is a geometry-independent function of the crack speed V while E and ν, as usual, are the elastic modulus and Poisson's ratio, respectively. The function A monotonically increases from unity at zero speed to become unbounded at the Rayleigh speed. As a consequence of equation (2), it is immaterial whether one addresses the problem in terms of a dynamic energy-release rate and a corresponding critical crack-tip energy-dissipation rate, or in terms of a dynamic stress-intensity factor and its corresponding critical value.

A crack extension criterion follows from the principle of energy conservation. That is, the energy-release rate or driving force must be balanced by the fracture resistance R, which is the fracture energy of the propagating crack. Equivalently, the dynamic stress intensity must match the propagating crack toughness K_D. This statement means that rapid propagation is only possible when $G = R$ or, equivalently, when $K = K_D$. It follows that, for a propagating crack, arrest must take place when $G < R$ or $K < K_D$. Thus crack arrest occurs as the termination of a general dynamic crack propagation process, not as a unique event as suggested by the "crack arrest toughness" K_{Ia}-approach.

MATERIAL PROPERTIES

The material properties that enter into calculations of the energy release rate are the elastic properties, (simply E and ν for isotropic materials) and the density ρ. These quantities frequently appear in the form of characteristic elastic wave speeds, the bar wave speed $C_o \equiv \sqrt{E/\rho}$ and the Rayleigh wave speed C_R, which is the limiting speed for a crack in an elastic medium [5]. It should be noted that both the E- and ν-values for certain polymeric materials are loading rate sensitive. This feature complicates the analysis because loading rates vary with time and position in a structure with a propagating crack.

The propagating crack fracture energy (and the corresponding propagating crack toughness $K_D \equiv A^{-1/2}(V)\sqrt{ER/(1-\nu^2)}$ resistance to cracking) is taken as a material property, essentially independent of external geometry and applied load. When the starting flaw is a fatigue crack, the quantities R and K_D correspond to G_{Ic} and K_{Ic} at the onset of propagation and may display a transient variation with crack extension (the R-curve) until the fracture mode stabilizes.

The values of R and K_D for dynamically propagating cracks have been derived from dynamic LEFM analyses and measurements of the local strain [6], photoelastic fringes [7,8,9], the crack velocity [1,2,10,11], the crack

length at arrest, and more recently by the method of caustics [12]. These measurements draw attention to the velocity dependence of the dynamic fracture resistance--the so-called R - or K_D velocity curves. These curves display 2 characteristic features: (1) a minimum resistance, either R_m or K_m at zero or finite velocity, and (2) a very rapid increase in the resistance observed at velocities usually in the range $0.2\ C_o - 0.4\ C_o$--which are smaller than the Rayleigh speed($\sim 0.57\ C_o$). A common cause for the rapid increase, if it exists, is not established [13,14], and its bearing on crack branching is discussed in a separate paper at this conference [15]. It is clear that the rapid increase in resistance limits the crack velocity that can be attained in many materials.

As is the case for static toughness values, R or K_D depend on the temperature and fracture mode, the shear mode offering greater resistance than the flat (plane strain) fracture mode of propagation. Structural steels in relatively thin sections (e.g., 10mm- to 25mm-thick) which fracture with a full shear mode display high resistance values $R \sim 2 \cdot 10^6 Jm^{-2}$ ($K_D \sim 800\ MPa{\cdot}m^{1/2}$). Dynamic fractures in these materials are accompanied by large plastic zones extending $\sim$100mm from the crack, which invalidate dynamic LEFM analyses of small laboratory test pieces. In these cases estimates of R-values have been obtained from the DWTT energy [16] (drop weight tear test) which correlates with C_V, the 2/3 subsize Charpy shelf energy [17]

$$R \approx \frac{\text{DWTT Energy}}{A} = 4\,\frac{C_V}{A} \quad ,$$

where A is the appropriate fracture plane area.

Available dynamic LEFM material properties will be summarized in the full paper.

CONSIDERATIONS IN THE APPLICATION OF THE THEORY

The work of Freund [3] can be used to show that, for a crack propagating in an infinite plane, the crack-tip motion is closely governed by the relation:

$$K_S = K_D \left\{ 1 - \frac{V}{C_R} \right\}^{-1/2} \quad , \qquad (3)$$

where K_S is the *static* stress-intensity factor for the instantaneous crack length, applied loads, and component geometry of a propagating crack (K_D and C_R are as defined above). For very large bodies or short crack jump lengths and travel times, equation (3) will suffice. However, for situations in which stress waves are reflected onto the moving crack tip (e.g., from free boundaries, from the opposite end of an expanding crack), equation (3) is invalid. In the latter case, more realistic analyses taking the component geometry into account are required.

A laboratory test specimen that has been used effectively by Hahn et al, [1,2] and by Crosley and Ripling [18] is the double cantilever beam (DCB) specimen. A model for dynamic crack propagation in the DCB specimen has been given by Kanninen et al [19,20]. The starting point for the derivation is the equations of the theory of elasticity with inertia terms included. Because the "beam-like" geometry of this specimen can be exploited, however, not all of the equations need to be explicitly considered. An effective device which further simplifies the analysis is the introduction of cross-sectionally averaged dependent variables. The equations of motion and expressions for the external work, the strain energy, and the kinetic energy of the system can be obtained in terms of these variables. Substituting the energy quantities into equation (1) gives an expression for the crack driving force which involves only quantities evaluated at the axial position representing the current crack tip. Used together with a specified $K_D = K_D(V)$ behavior, the model then provides an efficient and accurate representation of crack propagation and arrest in the DCB specimen for a wide variety of specimen geometries and loading conditions. More importantly, the predictions of the model have been verified by extensive comparison with the experimental results.

Tests in the DCB specimen conducted by Hahn et al [1,2] use blunt initial crack tips. This procedure allows enough elastic strain energy to be stored in the specimen for the crack (which propagates as a sharp crack) to attain a high speed and yet arrest within the specimen. The measure of the bluntness is K_q, the apparent stress-intensity factor at the initiation of crack growth. Computational results in which K_q is arbitrarily varied show that the prediction of the crack arrest point given by equation (3) is a very considerable underestimate. That is, while the fully dynamic theory coincides with the infinite medium solution until the time of the first stress wave reflection, sizable differences occur thereafter. Detailed consideration of the results obtained with the model reveals that the arrest point calculated with equation (3) is reached at about the same point that the kinetic energy reaches a maximum. This indicates that return of kinetic energy to the crack tip (neglected in the infinite medium solution) provides a very significant contribution to the crack driving force.

Other analysis work addressed to the development of models for crack propagation in realistic structural geometries includes both one-dimensional beam models and two-dimensional finite-difference and finite-element solutions. Burns and Bilek [21] have devised a simple beam model to represent DCB specimens tested under impact loading. As Malluck and King [22] have shown, however, their results are virtually identical to those obtained by specializing the model of Kanninen et al described above.

Beam models, of course, are not applicable to SEN, compact tension, or other essentially two-dimensional configurations. To investigate the crack propagation/arrest process in these cases, numerical methods are required. A finite difference scheme has been adopted by Hahn et al [2] in order to facilitate a direct generalization of the crack driving force calculation in the DCB model. Shmuely and Peretz [23] also use a finite difference method but, in contrast to the type of criterion given by equation (1), they employ a critical crack tip stress criterion.

One advantage of the finite element method is that, with the use of special crack tip elements, the dynamic stress intensity factor can be used directly as the crack growth criterion. However, analyses conducted by Aberson et al [24] and by Yagawa et al [25], indicate that there are numerical difficulties connected with the manner in which the crack is allowed to advance that have not yet been resolved. The approach of Kobayashi et al [26] circumvents this by using only regular finite elements, but they are forced to adopt a somewhat artificial crack extension criterion. Further discussion of these various approaches is given in the complete paper.

CRACK PROPAGATION IN PRESSURIZED PIPELINES

Crack propagation in pressurized pipelines, as determined in full-scale tests, generally occurs at an essentially constant speed, and, when arrest takes place, it does so in a fairly abrupt manner. Typically, the ductile (or shear) crack speeds observed in full-scale tests range from 100 to 300 ms^{-1}, brittle crack propagation speeds from 600 to 1000 ms^{-1}. In the latter case, a reasonable correlation has been found [27] with the resonance wave speed peculiar to circular cylindrical geometries. This upper bound crack speed is $0.75C_0(h/R)^{1/2}$ where h denotes the pipe wall thickness and R is its radius. Note that for typical pipe sizes, this is considerably smaller than its analog for the plane, the Rayleigh wave speed.

Most experimental work has been focussed on obtaining empirical guidelines for the toughness necessary to insure crack arrest in the ductile regime [28,30]. These are based on a minimum value of the 2/3 size Charpy upper shelf energy. However, while a decisive comparison of the different relations might seem possible with the experimental results, this is not the case. The difficulty lies in the fact that no experiment has yet been able to specifically determine a value of $(C_V)_{min}$ for given operating conditions. An experiment can determine only whether a crack propagated or whether it arrested. Consequently, while qualitative comparisons are possible, direct quantitative verification is not. A theoretical analysis appears to offer the only way to resolve this dilemma.

Many investigators are currently developing pipeline fracture models; e.g., Freund et al [31], Erdogan and Ratwani [32], Shannon and Wells [33], Poynton et al [34]. A model being developed by Kanninen et al [16,27,35], however, may be one most firmly based on dynamic fracture mechanics concepts. Their development starts from the equations for a circular cylindrical shell, making four key assumptions. These are that (1) radial deformations predominate, (2) circumferential variations in pressure can be neglected, (3) the crack-opening displacement is equal to the circumferentially integrated radial displacement at any cross section in the cracked region and (4) that a plastic yield hinge is developed behind the crack tip. Further simplification is introduced by specializing to steady-state conditions. This leads to an ordinary differential equation that can readily be solved provided the steady-state pressure profile in the pipeline is known. This is obtained by assuming a predominantly axial flow problem and accounting for the change of pipe cross-sectional area plus gas leakage behind the crack tip. The final step involves the development of an expression for the crack-driving force G via equation (1).

In the model of Kanninen, et al, a relation $G = G(V)$ has been developed as a function of the pipe geometry and operating conditions. Then, using a value of the dynamic fracture energy from a drop-weight tear test, steady-state crack speeds are determined. These values have been compared with observed speeds in full-scale pipeline tests with quite reasonable agreement being obtained. Of more importance, the model predicts a maximum possible crack-driving force for any given set of operating conditions. It is therefore possible to estimate a minimum level of fracture energy for the pipe material that will preclude catastrophic crack propagation. Comparisons between the prediction of this value and the full-scale test results (based on DWTT upper shelf energy), show encouraging agreement.

DESIGN OF CRACK ARRESTER SYSTEMS

The general criterion for crack arrest can be expressed in terms of the dynamic driving force and minimum propagation resistance; either $G < R_m$ or $K_I < K_m$. These criteria suggest two different strategies for assuring arrest: (1) by inserting a stiffener in the structure that reduces G (or K) below the minimum values, or (2) by inserting a tough arrester in the path of the crack with an R_m (or K_m) that exceeds the driving force. A third strategy, interruptions of the crack path through redundant structure members is also viable. The choice of an arrester system probably rests mainly on economic considerations (e.g., cost of materials, installation and fabrication costs), and other design considerations peculiar to the particular application; rather than on the inherent capabilities of the different strategies.

In general, the appropriate values of G or K_I at arrest must be obtained from dynamic LEFM analyses with the boundary conditions properly taken into account. Estimates based on static analyses are valid in special cases: (i) for the infinite body when K_m corresponds with zero velocity, (ii) small cracks in large bodies and (iii) for relatively small crack extensions. In other cases, statically based calculations can be highly misleading with regard to the crack arrest capability of a given arrester system and structural configuration. The extent to which this is true cannot be determined at this time and, in fact, is a highly appropriate area for further research.

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