Fracture 1977, Volume 1, ICF4, Waterloo, Canada, June 19 - 24, 1977

DEVELOPMENT OF FATIGUE CRACKS IN REAL STRUCTURES: APPLICATIONS TO AIRCRAFT DESIGN

J. Nemec*, J. Drexler** and M. Klesnil***

ABSTRACT

The development and propagation of fatigue cracks in real structures - with particular attention being paid to actual large-scale structures - have been thoroughly studied for the last four years in Czechoslovakia. In this respect, the presented paper deals with some specific fields of the mechanical structure design. First, the authors specify these fields from the point of view of fracture mechanics using a generalized probabilistic model to describe the state variations with time of a complex cracked structure. Secondly, the authors state the two main fields of interest based on the following generalized model:

- *that one of using the probabilistic description of the structure cracking behaviour to estimate the reliability that no crack will occur in the primary (fatigue-sensitive) substructure within a priori accepted time interval, i.e. to estimate the safe life of the structure,*

- *that one of discussing some physical features of two crack propagation modes being paid interest to in this paper, namely the "plastic" and "stepwise pop-in" ones, thereby, synthetizing the probabilistic and fracture mechanics approaches into an useful engineering tool.*

1. INTRODUCTION

The results of recent research into the cracking behaviour of actual large-scale mechanical structures under fatigue test, support the concept that cracks develop systems whose characteristics are random in nature with respect to time and space, [1],[2],[3]. As the basic ones - using the system engineering assessment - for describing the cracking state of a complex structure, one can consider the total number η of cracks in exposed substructure, and the propagation rates, $\rho = da/dN$, of the corresponding most developed cracks in dependence of a given number of programme testing units N. A graphical interpretation of such characteristics is given in Figure 1 which resulted from a test conducted to obtain an aircraft safe life certification.

In Figure 1, the hierarchy of participating on the total number of cracks in individual structure components may be arranged as follows accounting

*Academician, Czechoslovak Academy of Sciences, Praha, Czechoslovakia.
**Doc. Ing. CSc., Leading scientific worker at the Aer. Res. and Test Inst., Praha, Czechoslovakia.
***Prof. Ing. DrSc., Scientific worker at the Inst. Phys. Met. of Czechoslovak Academy of Sciences, Brno, Czechoslovakia.

for their fatigue resistances: skin 6, stringers 2, stiffeners 4, webs 5 and spars 3. In this hierarchy, structural components can play a "warning" role for primary substructure such as spars or fittings. Looking at Figure 1, one can state that for N between 4,5.10^4 and 5,5.10^4, skin cracks rapidly increased in number giving a "warning" signal preceding crack occurrence in stringers. Skin-, stringers-, stiffeners- and web crack growth pointed out the proximity of a critical crack in a spar for N = 7,1.10^4. The last one could be repaired before serious damage being brought to the wing structure as a whole.

The course of the overall structure, curve 1, reveals an explicitly non-linear character of the growth in number of cracks between individual repairs (stop-holes, overlapping). Thereby, the propagation of unrepaired representative cracks was very slow. It should be noted that the above example of an aircraft structure relates to the s.c. "safe-life" design concept which may be taken as typical for a large set of mechanical structures.

The observations made so far on aircraft cracking behaviour under fatigue test present, in general, two basic problems to be solved when investigating crack systems in real bodies: the first one is related to investigation of the crack system as a whole for a given set of external boundary and next-by conditions, the second one is related to crack development and propagation in the critical zone of the body. In this relation, the authors will deal with the two fields of interest pointed out in the Abstract.

2. A GENERALIZED PROBABILISTIC MODEL OF THE STRUCTURE CRACKING BEHAVIOUR

In further considerations the authors shall take the total number of cracks, η, and representative propagation rates, ρ, as characteristics of a state vector describing the cracking behaviour of a mechanical structure. Following the first aim, the authors shall construct a generalized probabilistic model of the phenomena under investigation. First assume some properties of the state vector of a cracking structure, [4],[5]:

1. The total number of cracks, η, and the representative propagation rate, $\rho = da/dN$, in a r-th structure subsystem or element are random components of a state vector $\chi[\eta,\rho;t]$, where generally η, ρ take discrete values η_{ir}, ρ_{jr} ($i = 1,2,\ldots,\omega$; $j = 1,2,\ldots,\xi$; $r = 1,2,\ldots,\zeta$). Thereby, the time

$$t = 1/f.N \quad (f = \text{Konst}) \tag{1}$$

is a continuous determined parameter, f being the average loading frequency.

2. η_{ir} and ρ_{jr} are independent random variables of POISSON type, i.e. for corresponding passage probabilities of the state vector components

$$(\eta_{ir} \to \eta_{kr}) \;,\; (\rho_{jr} \to \rho_{\ell r})$$

within a sufficient small time interval $\Delta t \to \emptyset$, thus

$$\ell im\ \text{Prob}[\eta_{ir}(t) \to \eta_{kr};\ (t + \Delta t)]/_{\Delta t \to \emptyset} = \emptyset \tag{2}$$

$$\ell im\ \text{Prob}[\rho_{jr}(t) \to \rho_{\ell r};\ (t + \Delta t)]/_{\Delta t \to \emptyset} = \emptyset \tag{3}$$

3. The state vector $\chi[\eta,\rho;t]$ of a structure subsystem or element may be characterized by finite memory relative to the total number of cracks η_{ir} and the representative propagation rate ρ_{jr}; i.e., any state of the cracking structure is in function of the preceding one, [6].

4. The state vector $\chi[\eta,\rho;t]$ of a structure subsystem or element is time dependent; i.e., the total number increment

$$\Delta\eta_{ikr} = \eta_{kr} - \eta_{ir}$$

between two successive states as well as the propagation rates

$$\Delta\rho_{j\ell r} = \rho_r - \rho_{jr}$$

depend on time.

From methodological point of view, the cracking state analysis of a mechanical structure is concerned with nonhomogeneous, nonstationary, differenciable, stochastic processes with discrete states and finite memory, [7].

Resulting from the 1. and 2. properties, the probabilistic model of the cracking structure can generally be presented in form - using the symbol N (number of programme testing units) rather than t, see equation (1),

$$\text{Prob}_{i,k\wedge j,\ell}[N_1,N_2] = \text{Prob}_{i,k}[N_1,N_2]\ \text{Prob}_{j,\ell}[N_1,N_2] \qquad N_1,N_2 \geq \emptyset \tag{4}$$

as interpreting the statistical independence of both the crack propagation rate and the total crack number.

Properties 2., 3., and 4. yield following expressions for $\text{Prob}_{i,k,r}[N_1,N_2]$ and $\text{Prob}_{j,\ell,r}[N_1,N_2]$:

$$\text{Prob}_{i,k,r}[N_1,N_2] \equiv \text{Prob}[\eta_{ir}(N_1) \to \eta_{kr}(N_2)] =$$

$$= \frac{1}{(\eta_{kr}-\eta_{ir})!}\,[\bar{\eta}_{\bullet\bullet r}(N_2-N_1)]^{(\eta_{kr}-\eta_{ir})}\exp\{-\bar{\eta}_{\bullet\bullet r}(N_2-N_1)\} \tag{5}$$

$$\text{Prob}_{j,\ell,r}[N_1,N_2] \equiv \text{Prob}[\rho_{jr}(N_1) \to \rho_{\ell r}(N_2)] =$$

$$= \frac{1}{(\ell r-jr)!} \cdot [\bar{\rho}_{\bullet\bullet r}(N_2-N_1)]^{(\ell r-jr)}\exp\{-\bar{\rho}_{\bullet\bullet r}(N_2-N_1)\} \tag{6}$$

where

$$\bar{n}_{\cdot\cdot r} = \frac{1}{N_2-N_1}\cdot\int_{N_1}^{N_2}\bar{n}_{ikr}(x)\cdot dx \qquad (7)$$

$$\bar{\rho}_{\cdot\cdot r} = \frac{1}{N_2-N_1}\cdot\int_{N_1}^{N_2}\bar{\rho}_{j\ell r}(x)\cdot dx \qquad (8)$$

are the crack number passage rate and crack propagation passage rate, respectively, in the considered number of programme units interval $\langle N_1,N_2\rangle$. Equation (5) expresses the probability that there occurred $(n_{kr} - n_{ir})$ cracks in the test interval (N_2-N_1), $n_{ir}(N_1)$ denoting the initial cracking state. Similarly, equation (6) interpretes the probability that the propagation rate of the most developed crack in the r-th subsystem and in the same (N_2-N_1) interval will be ρ_{ir}, given the initial propagation rate ρ_{jr}.

So far, relations (5) to (8) refer to one particular substructure or structural element. The synthesis of them to obtain a general model of the whole structure cracking behaviour, however, presents some difficulties because of the dependency of the crack occurrences in individual subsystems previously discussed as "warning" effects. Before attempting a simplified treatment, consider curve 1 on Figure 1 as a representative example.

One can state that inspections to detect the fatigue crack and its propagation rate were performed at randomly chosen intervals during the fatigue test. When a crack is detected, the cracked component is - in only some cases - repaired or replaced so that the residual strength of the component is renewed. The reason for repairing a crack originates from over-all structure behaviour: if the total number of cracks $\sum_r n_{ir}$ is rapidly increasing in rate, repair should be carried out. This is the case of the curve 1 in Figure 1. By doing such, all the weak points are identified and repaired successively in agreement with the experimental certification philosophy. These repairs being obviously accounted for a series production and flying aircrafts, one can conclude that the n characteristic of the certified aircraft becomes nearly a linear function of N. This linear function is tangent to the original piece-wise exponential-like function. Hence, the first product term of equation (4) may be represented by a stationary POISSON probability model describing the occurrence of just $\sum_r(n_{kr}-n_{ir})$ cracks within the $\langle N_1,N_2\rangle$ testing interval in the structure as a whole, namely

$$\mathrm{Prob}[\{\sum_r n_{ir}(N_1)\} \to \{\sum_r n_{kr}(N_2)\}] = \mathrm{Prob}[\{\sum_r n_{ir}\}\to\{\sum_r n_{kr}\}; N_2-N_1 = N] =$$

$$= \frac{[\bar{n}\cdot N]^{\sum_r(n_{kr}-n_{ir})}}{[\sum_r(n_{kr} - n_{ir})]!}\cdot \exp\{-\bar{n}\cdot N\} \qquad (9)$$

Among all of the possible cracking state situations a complex mechanical structure can be in under programmed fatigue tests, two critical cases (both related to model (4)) are of special interest as to the aircraft design:

i) The first case is characterized by very slow or decreasing crack propagation rates in primary structure;

ii) the second case is characterized by a change in the total number of cracks. In this case the cracking in the structure

is governed by one or more cracks in the fatigue sensitive zone.

It seems to be adequate therefore to express the generalized cracking behaviour model employing equations (6) and (9) as follows

$$\mathrm{Prob}_{i,k\wedge j,\ell}[N_1,N_2] = \frac{[\bar{n}\cdot N]^{\sum_r(n_{kr}-n_{ir})}}{[\sum_r(n_{kr}-n_{ir})]!}\cdot\frac{[\bar{\rho}_{\cdot\cdot d}\cdot N]^{(\ell d-jd)}}{[\ell d-jd]!}\cdot\exp\{-(\bar{n}+\bar{\rho}_{\cdot\cdot d})N\} \qquad (10)$$

index r=d relating to the dominant crack in the fatigue sensitive structure.

3. ESTIMATING THE SAFE LIFE OF THE CRACKING STRUCTURE

According to what was mentioned above for the (i) case, for slow propagating cracks the probability (6) will tend to one because the propagation rate passage $j\to\ell/j=\ell$ is most probable. Hence the generalized model (10) reduces to equation (9). Now, introduce the notion of the limit cracking state $\chi_{\ell im}=\chi[n_{\ell im};N_{SF}]$ for the case (i): the occurrence of $\chi_{\ell im}$ means that once an over-all total number of cracks $n_{\ell im}$ has been achieved, the corresponding cycle number being $N=N_{SF}$, a dominant crack in the fatigue sensitive sbustructure was detected. The term N_{SF} is defined herein as the safe life estimation of the structure.

Investigating the probabilistic characteristics of the inverse process $N_\chi[n_{\ell im}]\equiv N_{SF}$ in respect to $\chi_{\ell im}$, the safe life estimation presents no more difficulties. Using the theory of POISSON processes, the probability of the safe life N_{SF} being greater than an *a priori* given value k by the equation is expressed using equation (9) by

$$\mathrm{Prob}[N_{SF} > k;(n_{\ell im}-\sum_r n_{ir})] \equiv R\,[N_{SF};(n_{\ell im}-\sum_r n_{ir})] =$$

$$= \frac{\bar{n}}{\Gamma(n_{\ell im}-\sum_r n_{ir})}\cdot\int_{N_{SF}}^{\infty}(\bar{n}\cdot x)^{(n_{\ell im}-\sum_r n_{ir}-1)}\cdot\exp\{-\bar{n}\cdot x\}\cdot dx \qquad (11)$$

A practical application of the equation (11) as derived from the generalized probabilistic model (10) is shown in Figure 2. The safe life estimation has been carried out for the airplane structure from Figure 1. Curve 1, Figure 1 yields the following input data:

$$\rho = 6;\quad \sum_{r=1}^{\rho} n_{ir} = 6;\quad n_{\ell im}=106;\quad N_1=2{,}1.10^4\quad N_2=7{,}1.10^4$$

$$\bar{n} = \frac{n_{\ell im}-\sum_r n_{ir}}{N_2-N_1} = \frac{100}{5{,}10^4} = 0{,}002$$

For these data and $R[N_{SF}] = 1{,}0$, the threshold value of the safe life estimation N_{SF} is 3.10^4 programme testing units, as given for $(n_{\ell im}-\sum_r n_{ir}) = 100$ in the upper part of the diagram.

4. FATIGUE CRACK DEVELOPMENT IN THE STRUCTURE

Consider now the case (ii), i.e. the cracking behaviour of the structure is governed by the only crack in the fatigue sensitive zone. The probabilistic model of the dominant crack is given by a special form of the generalized model (4), namely by

$$\mathrm{Prob}[N_{SF} > k; \rho_{jd}(N_{SF}) \rightarrow \rho_{\ell d}(N_{SF}+\Delta N] = \mathrm{Prob}[N_{SF} > k; (\eta_{\ell im} - \sum_r \eta_{ir})] \cdot \mathrm{Prob}[\rho_{jd}(N_{SF}) \rightarrow \rho_{\ell d}(N_{SF}+\Delta N)] \qquad (12)$$

The first product term $\mathrm{Prob}[N_{SF} > k_i(\;)]$ on the right side expresses the probability that at least one dominant crack will occur in the structure after safe life N_{SF}. The second term is the probability (6) for $N_1 = N_{SF}$, $N_2 = N_{SF}+\Delta N$; thus, only the first product term governs the determination.

The overall total number of cracks in the structure after development of the dominant crack is obviously

$$(\eta_{\ell im} - \sum_{r=1}^{\zeta} \eta_{ir}) + 1 \qquad (13)$$

1 being for the dominant crack. Since the value of equation (13) is generally greater than one, one has with negligible error

$$\mathrm{Prob}[N_{SF} > k; (\eta_{\ell im} - \sum_r \eta_{ir}) + 1] \doteq \Phi\left[\frac{\bar{\eta}.N_{SF} - (\eta_{\ell im} - \sum_r \eta_{ir})}{(\eta_{\ell im} - \sum_r \eta_{ir})^{1/2}}\right] \qquad (14)$$

Substituting both equations (6) and (14) for equation (12), the probabilistic model of the dominant crack in development is given as follows

$$\mathrm{Prob}[N_{SF} > k; \rho_{jd}(N_{SF}) \rightarrow \rho_{\ell d}(N_{SF}+\Delta N) = \Phi\left[\frac{\bar{\eta}.N_{SF} - (\eta_{\ell im} - \sum_r \eta_{ir})}{(\eta_{\ell im} - \sum_r \eta_{ir})^{1/2}}\right] \cdot \frac{[\bar{\rho}_{\bullet\bullet d} \cdot \Delta N]^{(\ell r - jr)}}{(\ell r - jr)!} \cdot \exp\{-\bar{\rho}_{\bullet\bullet d} \cdot \Delta N\} \qquad (15)$$

Physically, equation (15) presents the probability of increasing stepwise the propagation rate by a value of $(\rho_{\ell d} - \rho_{jd})$ in a given testing unit interval ΔN under the condition that the safe life N_{SF} has been exceeded.

It is now of outmost practical interest to analyse the physical nature of the propagation rate passages of the type $(\rho_{jd} \rightarrow \rho_{\ell d})$ for the final synthesis with the probabilistic model (15).

4.1 Fracture mechanics assessment

Fractography of structural fractures shows that failures practically never occur according to the idea of quasi-homogeneous materials. In real structures, failures do not result from the development of single cracks with distinct plastic zones and high internal stresses in the elastic

medium surrounding the plastic zones. These stresses act as the principal barriers against fatigue. According to such an idea, the stored energy in the root of cracks is proportionally high, the whole sumarized area of cracks being therefore low and the threshold stress intensity factor depending on the loading history. In other cases, the separation energy is often very low and depends very little on strain hardening intensity in plastic zones and on the level of internal stress barriers.

In real structures many microcracks and voids and many microbrachings and macroconnections develop [8]. In this case, cracks develop stepwise, the effects of growth break being given by the multiplication of the area of cracks. The probability of the fracture of bodies tends to be lower due to the increase in the number of the crackbranchings. The macrocrack looks for the system of weak points in damaged material under corrosion, temperature conditions and complex ageing effects. The development of cracks has the feature of a jump process of repeated instability. A longer life of a body is generally correlated with the whole area of cracks being larger. However, the average velocity of the failure process is usually higher than in the case of the "plastic" growth of a single one fatigue crack.

Between individual jumps, the low cyclic ageing of material in the small coherent region in the tip of the arrested crack takes place. Real failures often show a complex mixed nature of both types. Important roles are played by the stress state and by the size and shape of bodies. The two dominant fatigue mechanisms mentioned above, of the development of microstructural changes under cyclic straining are also connected with the material properties. Hence, we have to distinguish the case of low and high strength metals, for example. The stage of microcracks multiplication and growth in jumps are well known in hard and brittle materials.

4.2 Quantitative description of "plastic" fatigue crack growth (type I)

From such considerations, it follows that for the single crack with a developed plastic zone in simple-shaped bodies, the average velocity of its growth can be expressed using linear fracture mechanics. Interactions with other damaged places and defects are very weak and the energy dissipation in the neighbourhood of the crack is very low. Barriers are built due to internal stresses and induced stresses around the front of the crack. Such a process has been investigated herein employing flat steel specimens to derive a law of damage accumulation accounting for the loading history. The small scatter in the experimental results demonstrate the usefulness of this law in describing fatigue crack growth for special loading and geometrical conditions for such a fatigue fracturing process being satisfied.

Plastic zone was quite extensively studied both theoretically and experimentally. As expected, the experiments showed that there are two plastic zones (monotonic, cyclic ones) for assymetrical cycling, [9],[10]. The plastic zone size was also found to be in reasonable agreement with theoretically derived values. Dislocation substructure of the plastic zone corresponded with a high amplitude cycling with marked dependence on the distance from the crack tip, [11],[12]. Microstructural parameters were found to depend on the stress intensity factor, [13]. Very important results were obtained by X-ray measurement of residual stresses, [14],[15],[16], showing that the magnitude of residual stresses depends on the stress intensity factor. This is a strong argument for relating the conditions for a crack not to propagate also to the value of the stress intensity factor.

All the experimental results prove that the dependence of the crack rate on the amplitude of the stress intensity factor, K_a, is a sigmoidal curve on log-log plot. For low K_a values, the curve asymptotically approaches a threshold value K_{th}. For high K_a values, the crack becomes instable at the critical value K_c. In the middle part, the curve is described by the power law

$$v_I = \frac{da}{dN} = A_I \cdot K_a^{\beta} \equiv \rho_I \tag{16}$$

originally proposed by Paris, [17].

So far are all the results in agreement. The differences among different authors lie in the mathematical description of the sigmoidal curve, in that of the influence of the mean stress and mainly in the formulation of the threshold conditions of non-propagation. The following information is based mainly on results obtained in this field in Czechoslovakia.

The value of the stress intensity factor amplitude at which the crack stops after propagating on the level K_a will be assigned as the threshold value and denoted by K_{at}; hereinafter, equation (16) is given correction as follows

$$\rho_I = A_I(K_I^{\beta} - K_{at}^{\beta}) \tag{17}$$

Figure 3 presents one example of the experimental results: the crack propagated at the level K_a. Thereafter, the stress amplitude was suddenly decreased. The value of the stress amplitude at which the crack stopped yields then the corresponding value of K_{at}, [18],[19]. The dependence K_{at} vs K_a is in log-log plot linear. Analytically, it was expressed in the form

$$K_{at} = K_{th}\left(\frac{K_a}{K_{th}}\right)^{\alpha} = K_{th}^{\,1-\alpha} \cdot K_a^{\alpha} \quad , \tag{18}$$

where K_{th} is the basic threshold value, α is a material constant.

The threshold value is determined both by the intrinsic resistance of the material against crack propagation and by compressive residual stresses. The effect of the resistance of the material is markedly expressed because the higher the micropurity of the material, the higher is the basic threshold stress intensity factor K_{th}. The intrinsic resistance naturally does not depend on the history of cycling, but only on the material. This corresponds to the value K_{th}, this being the lowest possible value of the threshold intensity. Such an interpretation has been proved, e.g., by annealing of the specimen with fatigue crack: after annealing the threshold value was found not to depend on the cycling history and to be equal to the K_{th}. The magnitude of residual stresses increases with increasing value of the K_a. That is why also the threshold value K_{at} increases with the K_a.

Equation (18) is correct for symetrical cycling. For the asymmetrical one, a modification of equation (18) is needed. Characterizing the stress cycle asymetry by the parameter κ given by

$$\kappa = \frac{K_{max}}{K_a} \tag{19}$$

which is related to the more frequently used parameter R by the relation $\kappa = 2/(1-R)$ the threshold value results in

$$K_{at} = \left(\frac{K_{th}}{\kappa^{\gamma}}\right)^{1-\alpha} K_a^{\alpha} \tag{20}$$

where γ is a material constant.

Transient effects resulting from the sudden changes of the cycling parameters represent a typical example of the manifestation of the existence depending on the history of cycling. In the first approximation, these effects can be quantitatively described solely on the basis of the steady state fatigue crack propagation data, provided that the threshold conditions are taken into account, [15]. There are in principle three categories of the transient effects:

1. High-low block sequence of constant stress loadings which leads to a retardation. The same effect is caused by the decrease of maximum value of the stress intensity factor, while keeping the stress intensity factor amplitude constant, i.e. by decrease of the stress cycle asymmetry.
2. Low-high block loading sequence of constant stress loads was in several cases observed to cause an initial acceleration.
3. Single or multiple peak overloads have been found to retard the crack propagation.

For the description of these effects, assume that after a sudden change of the external cycling parameters K_a and κ, the residual stresses at the crack tip cannot be immediately changed in such a way as to correspond to new values of K_a and κ. Instead, they remain unchanged; it means, they correspond to the parameters of cycling K_{a1} and κ_1. Mathematically interpreting, the driving term after the change K_{a1}, $\kappa_1 \rightarrow K_{a2}$, κ_2 is given by parameters of the second level, while the resistance term (threshold term) is presented by the parameters of the first level. Thus, the crack rate, immediately after the change K_{a1}, $\kappa_1 \rightarrow K_{a2}$, κ_2 is given by the equation, see equation (17)

$$\rho_{trans.} = A_I[(K_{a2}\,\kappa_2^{\gamma})^{\beta} - (K_{a1}\,\kappa_1^{\gamma})_t^{\beta}] \tag{21}$$

which can be written into the form

$$\rho_{trans.} = A_I[(K_{a2}\,\kappa_2^{\gamma})^{\beta} - K_{th}^{(1-\alpha)}(K_{a1}\kappa_1^{\gamma})^{\alpha\beta}] \tag{22}$$

The validity of this equation has been verified on Czechoslovak steels for a wide range of κ values, [19].

Qualitatively, this equation accounts for all the observed types of transient effects. Quantitatively, it is possible to describe the course and the duration of the transient effects, by making a further assumption that the crack length increment corresponding to the duration of the transient effect is equal to the plastic zone size formed just before the sudden change of the cycling parameters. The detailed analysis along these lines was performed in [15]. One of the final results is shown in Figure 4 demonstrating the relation of extra number of cycles due to retardation effect calculated theoretically in respect to that one measured experimentally. The agreement is acceptable.

4.3 Stepwise development of cracks during the desintegration fatigue type process (type II)

In a lot of cases of fatigue fractures of real bodies many cracks develop in brittle materials and in materials with low value of stress rate $\frac{\tau_{max}}{\delta_{max}}$ especially. The repeated jumps of cracks with many branches occur and plastic zones with low cohesive strength are extremely small. Barriers have geometrical features, and the energy dissipation is high. Therefore this process has a stochastic character. Thus, a model of fracture statistics shall be applied herein. This model is necessary for a number of reasons. Experimental fracture statistics, interpreted with the aid of a sufficiently sophisticated theory and mathematical statistics, support the the optimum data processing.

The most frequently applied statistical theory of fatigue fracture is that one of the weakest link assuming quasi-brittle materials containing flaws with a special distribution of strength. But this theory is not closely related to the physical nature of the material, e.g. grain size or to macroscopic terms like K_{IC}. The dynamic nature of the crack development process due to repeated sudden jumps with many interactions is generally neglected. It is necessary to investigate the effect of the orientation of cracks to the force flow and the boundaries of the body and the real distribution law according to these relations. The probability of fracturing can be derived using the notion of the average propagation rate of cracks by desintegration type of process given by the relation

$$\bar{\rho}_{II} = A_{II}\left(\frac{K}{K_{IC}}\right)^4 \qquad (23)$$

In the basic interpretation, fatigue is a process of destroying of all barriers against fatigue failures. Contrary to stochastic systems, no deterministic system is known which is able to adapt to overloading without losing stability. Technical materials represent a complex system able to create necessary reserves against failures; therefore, the deterministic concept of adaptive behaviours cannot reflect the complex processes occurring in materials under loading.

So far, the physical model of damage cumulation laws has been discussed only as a necessary component of statistical approach to the estimation of the reliability of real structures. Since it is generally known that the life time of structures is governed first by the stepwise development of fatigue cracks, perhaps the synthesis of crack length distributions with those of residual lives of bodies on statistical basis presents up-to-date a most useful and actual trend in fracture statistics.

Having equation (23), the question arises as to how to connect this fictive propagation rate with the stochastic interpretation of the cracking process. The answer, however, has been already given by the model (15), where the average propagation rate $\bar{\rho}_{\cdot\cdot r}$ is to be substituted by the ρ_{II} expression (23). Thus, the expression

$$Prob[N_{SF}>k;\ \rho_{jr}(N_{SF})\rightarrow\rho_{\ell r}(N_{SF}+\Delta N)] = \Phi\left[\frac{\bar{\eta}.N_{SF}-(\eta_{\ell im}-\sum_r\eta_{ir})}{(\eta_{\ell im}-\sum_r\eta_{ir})^{1/2}}\right] \cdot$$

$$\cdot \frac{[A_{II}\cdot(K/K_{IC})^4\cdot\Delta N]^{(\ell r-jr)}}{(\ell r-jr)!} \cdot \exp\{A_{II}\cdot(K/K_{IC})^4\cdot\Delta N\} \qquad (24)$$

interprets in a very general form the physically built-up probabilistic model of the fracturing process of a mechanical structure as governed by a crack in primary, fatigue-sensitive structure.

The probabilistic model (24) presents a useful basis for planning inspection intervals for a structure suspected of cracking under test or operation. Given the agreed limit number of jumps (ℓ_{im}-j) characterizing the fracture mode and the appropriate propagation rate ρ_{IIa}, the probability distribution of testing or operational units ΔN_L before achieving the agreed limit state (the crack failure mode) can easily be derived. Such a distribution is that one of the inverse process of propagation rates passages as given by equation (6). Hence, we have

$$Prob[\Delta N_L>k_L;(\ell_{\ell im}-j)] =$$

$$= \frac{\bar{\rho}_{IIa}}{\Gamma(\ell_{im}-j)} \cdot \int_{\Delta_{NL}}^{\infty} (\bar{\rho}_{IIa}\cdot x)^{(\ell_{\ell im}-j)-1} \cdot \exp\{-\bar{\rho}_{IIa}\cdot x\}.dx \qquad (25)$$

Taking for "plastic" propagation mode (type I)

$$(\ell_{\ell im}-j) \gg 1$$

i.e., the number of jumps of the crack before achieving the limit state of the "plastic" failure mode is sufficiently large in respect to one, and introducing the dimensionless variable

$$\frac{\bar{\rho}_{IIa}\cdot\Delta N_L-(\ell_{\ell im}-j)}{(\ell_{\ell im}-j)^{1/2}} \qquad (26)$$

the following is obtained:

$$Prob[\Delta N_L>K_L;(\ell_{\ell im}-j)/N_{SF}>k;(\eta_{\ell im}-\sum_r\eta_{ir})] = Prob[(24)] =$$

$$= \Phi\left[\frac{\bar{\eta}\cdot N_{SF}-(\eta_{\ell im}-\sum_r\eta_{ir})}{(\eta_{\ell im}-\sum_r\eta_{ir})^{1/2}}\right]\cdot\Phi\left[\frac{A_{II}\cdot(K/_{IC})^4\cdot\Delta N_L-(\ell_{\ell im}-j)}{(\ell_{\ell im}-j)^{1/2}}\right] \qquad (27)$$

In the case of a step-wise "pop-in" crack (failure mode - type II) we have

$$(\ell_{\ell im}-j) = 1$$

Hence,

$$Prob[\Delta N_L>K_L;(\ell_{\ell im}-j)=1] \equiv \int_{\Delta N_L}^{\infty} \exp\{-\bar{\rho}_{IIa}\cdot x\} \cdot \bar{\rho}_{IIa}\cdot dx \qquad (28)$$

$$= \exp\{-\bar{\rho}_{IIa}\cdot\Delta N_L\}$$

and then

$$\text{Prob}[\Delta N_L > K_L; (\ell_{\ell im}-j)=1/N_{SF}>k; (\eta_{\ell im}-\sum_{r}\eta_{ir})] = \Phi\left[\frac{\bar{\eta}.N_{SF}-(\eta_{\ell im}-\sum_{r}\eta_{ir})}{(\eta_{\ell im}-\sum_{r}\eta_{ir})^{1/2}}\right] \exp\{-\bar{\rho}_{\Pi a}.\Delta N_L\} \quad (29)$$

Summarized, equations (27) and (29) express the probabilities that the inspection interval ΔN_L is sufficiently larger than an *a priori* agreed number value K_L.

For slow propagating cracks, the number of jumps becomes a large value before the limit state can be achieved and therefore

$$\Phi\left[\frac{A_{\Pi}.(K/K_{IC})^4.\Delta N-(\ell_{\ell im}-j)}{(\ell_{\ell im}-j)^{1/2}}\right] \Big/_{(\ell_{\ell im}-j)\to\infty} = 1 \quad (30)$$

In other words, the cracking behaviour of the structure is satisfyingly described by the first term in equation (27), thus coming to the i) case in section 3.

"Pop-in" stepwise cracks present most difficulties for inspections. Considering one-jump-crack only, one can easily derive from equation (29) that the inspection period for such substructures suspected for pop-in failure mode should be $\Delta N = \emptyset$ for optimizing the left-hand probability term, i.e. for achieving

$$\exp\left\{-\bar{\rho}_{\Pi a}.\Delta N_L\right\} \Big/_{\Delta N_L \to 0} = 1$$

5. CONCLUSION

In the present paper, the authors developed a synthetic approach to describe a cracking behaviour of a complex mechanical structure, based both on probabilistic and fracture mechanics assessments. In this respect, the primary interest has been paid to applications to aircraft structure design.

After deriving a generalized probabilistic model of the structure cracking behaviour, two special cases were considered. In the first one, the estimation of the safe life of an aircraft structure could have been shown based on test data of the only one structure specimen being available. The second one dealt with some physical aspects of two failure modes (the "plastic" and stepwise "pop-in" ones), the structure's life being governed by a dominant crack in the fatigue sensitive zone.

ACKNOWLEDGEMENT

The authors would like to express their acknowledgement to Dr. Sedlacek for his valuable discussions to feasible synthesis procedures as based on theory of stochastic processes of Poisson kind.

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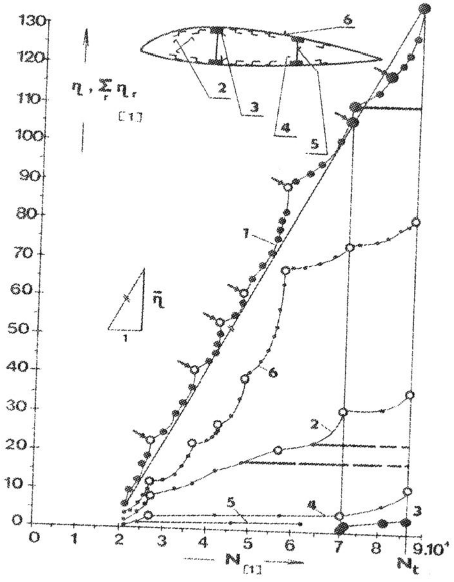

Figure 1 Total number of cracks η in a wing fatigue sensitive substructure vs number of programme testing units N, follow the legend in the text. Arrows with graph 1 indicate response to repairs of cracks in respect to the straight line approximation as compatible with the certification philosophy.

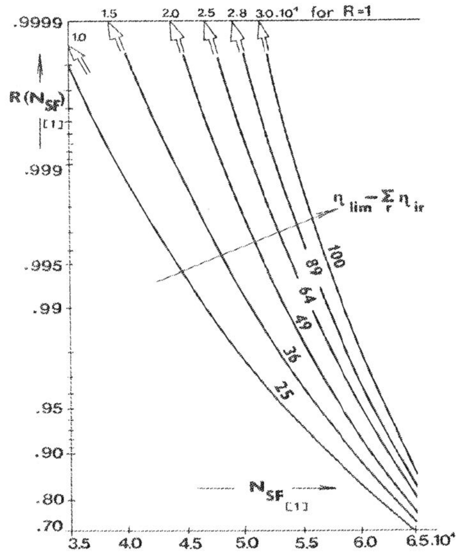

Figure 2 Probability R of no fatigue crack occurring in the primary structure (see Figure 1) for a given number of testing units N_{SF} (safe life).

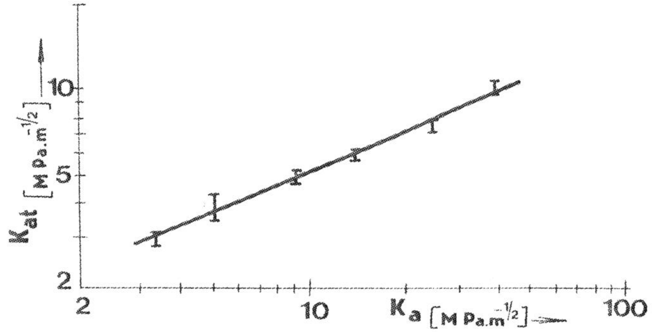

Figure 3 Threshold value of the stress intensity factor amplitude in dependence on the stress intensity factor amplitude. Cast steel: 13% Cr, 1% Ni, asymetrical cycling, κ=2.1.

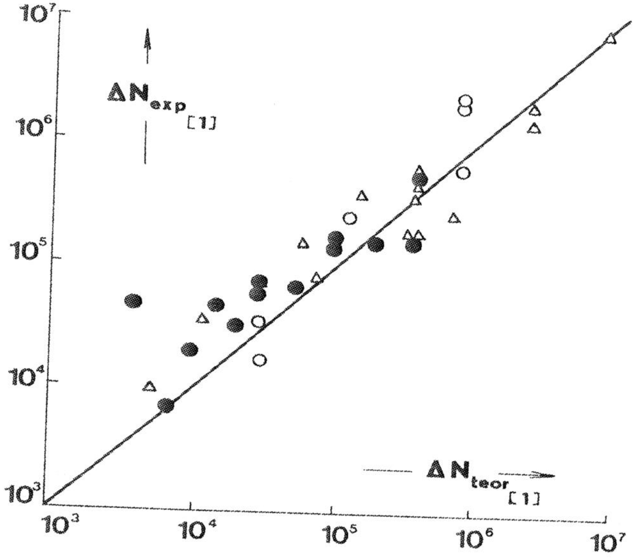

Figure 4 Relation between the theoretical and experimental number of extra cycles due to the transient effect. Carbon steel. o change of κ; Δ change of K_a; • single overload.