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CRACKING AND FRACTURE IN COMPOSITES

G. A. Cooper* and M. R. Piggott**

ABSTRACT

The experimental and theoretical work which has led to our present understanding of the fracture of composites is critically reviewed. It is shown that processes contributing to toughness fall into three groups: (1) those which lead directly to energy dissipation during crack propagation; (2) indirect processes that co-operate to enhance toughness; (3) interactions between fibres and matrix which lead to a reduction in a property of fibres or matrix. The applicability of fracture mechanics to composite materials is also reviewed, and the concept of structural integrity is introduced.

1. INTRODUCTION

It is the purpose of this paper to review some of the recent progress which has been made in our understanding of the processes which lead to the tensile failure of fibre composites. The field of composite materials research has grown very rapidly in the last few years so that it is no longer possible to review the whole subject in a single publication, or even to cover such a restricted field as that of their mechanical properties, as was still possible only a few years ago. In the early days of the subject, the broad limits to the properties of composite materials were identified with the help of a handful of simple concepts: the rule of mixtures springs to mind as an obvious example.

Subsequently, as our understanding of composites has improved, more subtle effects have been detected and analysed. This is particularly true in the case of cracking and fracture of these materials. The importance of optimizing fibre pull-out for maximum toughness was noticed early on by Cottrell [1], and Kelly [2] and the potential toughening of easy splitting, and separation of the fibres was observed at the same time by Cook and Gordon [3].

Since then, numerous processes contributing to toughness have been identified. It has been our aim to select and compare some of these newer developments, starting with processes that can operate directly to cause energy absorption during cracking. Indirect effects are then discussed. By and large these fall into two categories; synergetic, or co-operative effects, and cohibitive effects, where one element reduces the potential contribution of the other. Finally, the applicability of fracture toughness concepts will be discussed, together with some experimental measurements of fracture toughness parameters.

*Institut CERAC S.A., Chemin des Larges-Pieces, CH-1024 Ecublens, Switzerland.
**Chemical Engineering Department, University of Toronto, Ontario M5S 1A4, Canada.

Creep failure and fatigue fracture, which have also been the subjects of a great deal of study, will not be included.

2. ENERGY ABSORPTION DURING CRACKING AND FRACTURE

In this section, we consider the micromechanics of the fracture process from the point of view of the various energy-absorbing mechanisms which may occur. Without discussing whether the concepts of fracture mechanics are applicable in all cases to composite materials (which is left to a later section), it is clear that any process which tends to absorb energy during fracture will hinder the failure process and thus be a desirable property. Processes discussed in this section will be limited to those that result from the components of the composite acting in concert. Thus, contributions directly attributable to the toughness of the fibres or matrix are excluded.

In what follows, the work of fracture, G, will be discussed, rather than what has often been called the fracture surface energy, γ, where $\gamma = G/2$.

2.1 Fibre pull-out

From the earliest days of the interest in composite materials, it was generally recognised as useful to surround brittle fibres by a tough, ductile matrix in order to increase the fracture resistance. The first mechanism to be put on a scientific footing was, however, fibre pull-out from crack faces normal to the fibre direction [2,4]. Figure 1 [5] shows a typical example of fibre pull-out.

The work done in pulling a single fibre out of the matrix over a distance ℓ against a constant shear stress τ at the interface was shown to be $\pi d\tau\ \ell^2/2$ where d = fibre diameter. If the fibres have a strength σ_{fu}, then the maximum length of fibre which can be pulled out is $\ell_c/2$, where $\ell_c = d\sigma_{fu}/2\tau$. Thus for fibres of length ℓ less than ℓ_c, all the fibres pull-out rather than breaking and the average work done in fibre pull-out is $\pi d\tau\ \ell^2/24$. So, writing s for the aspect ratio, ℓ/d, and V_f for fibre volume fraction, the total contribution of this process to the fracture work is

$$G_{fp} = V_f d\tau s^2/6 \qquad (1)$$

In the case where the fibres are of length greater than ℓ_c, only a fraction ℓ_c/ℓ of the fibres will be pulled out, while the rest will be broken. In this case the contribution to the fracture work will be

$$G_{fp} = V_f d\tau s_c^3/6s \qquad (2)$$

where s_c is the critical aspect ratio and is equal to $\sigma_{fu}/2\tau$, or σ_{fu}/σ_{mu} for fibres well bonded to ductile material matrices.

Fibre pull-out is not, however, confined to discontinuously-reinforced systems, as it will be found in general whenever fibres break adjacent to the plane of the matrix crack. This will occur, for example, when the fibres have a distribution of weak points along their length. Such a situation has been discussed by Cooper [6] who considered composites reinforced by continuous fibres whose strength was uniform except for the presence of weak points of constant (lower) strength uniformly spaced along the fibres. In general, if the weak points are severe and closely spaced, fracture will always occur at these points, leading to fibre pull-out, but as they become wider spaced or less severe, fibre breaks can occur in the

undamaged fibres at the plane of the matrix crack.

Pull-out effects have also been noted with continuous brittle fibres. Harris et al [7] observed that, with continuous carbon fibre reinforced epoxy resins, the pulled-out length increased when the adhesion between the fibres and matrix was reduced. The average length was equal to $\ell_c/2$, calculated using plausible values for the interfacial shear stress, τ. Phillips [8] and Sambell et al [9] have also observed pull-out effects with continuous carbon fibres, in glass and ceramic matrices.

Harris [10] considers that pull-out occurs because the fibres first debond, then fracture at an average distance from the crack face of $\ell_c/4$. The average distance from the crack face will clearly be controlled by the interface, and the average distance between large flaws in the fibres. Why it should come to $\ell_c/4$, however, was not clear.

Piggott [11] has put forward an alternative explanation, based on the deformation accompanying the stress concentration around the crack tip. This could break the fibres up into their critical lengths, if the deformed zone is sufficiently extensive. The effect is thus governed by the size of this deformed zone, and criteria have been derived which indicate that this process is favoured when the fibres have small breaking strains.

When debonding has taken place, pull-out can still require a considerable amount of work. The shear force at the interface, τ, is now governed by friction between the fibre and the matrix, thus:

$$\tau = \mu\sigma_r \qquad (3)$$

where μ is the coefficient of friction and σ_r the normal (radial) stress at the fibre-matrix interface.

τ is not constant along the fibres, so that the simple treatment of pull-out given above cannot be applied. Amirbayat and Hearle [12] have obtained results that indicate the μ is not constant, with embedded length. In addition, the value of σ_r is subject to considerable doubt.

Hadjis and Piggott [13] have attempted to measure μ and σ_r independently, by pulling fibres out of strained matrices. They found that epoxy and polycarbonate matrices contain large residual compressive radial strains. These residual strains, and the strains resulting from tensile forces applied to the composite, cause σ_r to be compressive at low fibre volume fractions. Thus the fibres and matrix can remain in contact, despite the fibre Poisson's contraction, and friction stresses can develop as a result of the interfacial friction during pull-out.

However, at high volume fractions, σ_r changes sign, and the stress becomes tensile, so that separation of fibres and matrix can take place, and τ becomes zero. Unfortunately, the value of V_f at which σ_r changes sign is not known. The stress analyses are only approximate, and do not appear to have been verified experimentally. They are discussed in detail by Holister and Thomas [14] who also derive equations for the case of short fibres. These equations, however, produce the unlikely result that the normal stress goes to infinity as the volume fraction becomes vanishingly small.

When fibres bridge a crack, σ_r will not be constant with distance along the fibre from the crack face. It will be affected by both the change in matrix stress, which increases with increasing distance from the crack face, and by the Poisson's shrinkage of the fibre, which decreases with increasing distance from the crack face, so long as fibre and matrix remain in contact. (Poisson's shrinkage effects provide a credible explanation for the unexpectedly low interfacial shears needed to account for Allred and Schuster's results [15]).

These shrinkage effects can also result in situations where the fibres can be pulled out indefinitely. This was first noticed by Piggott [16] and yields infinitely large values for the work of fracture. This has been examined in more detail by Kelly and Zweben [17]. Morley [18,19] has even proposed core and sheath elements where the core can pull out indefinitely. This latter process is likely to be limited to special applications where the loss in stiffness and strength, due to the presence of the core (a helical spring or kinked wire is proposed), is not important.

In addition, plastic flow of reinforcement wires can cause indefinitely large pulled out lengths. Morton and Groves [20] have observed that with annealed wires in epoxy resin, a wire which is too long to be pulled out at a stress below its yield stress deforms plastically, and in so doing contracts radially by an amount sufficiently great to free it from the surrounding matrix, thus permitting indefinite pull-out.

However, only a limited amount of pull-out can normally be considered to contribute usefully to the toughness, since the maximum amount of crack opening may be restricted by structural requirements. This will be discussed in more detail later.

2.2 Fibre debonding

A source of energy consumption which is normally associated with, but is distinct from, fibre pull-out, is that which occurs when the fibre-matrix interface fails. It is customary to discuss this in terms of systems in which the fibre-matrix bond is relatively brittle, and where the zone over which the failure of the interface occurs in small by comparison with ℓ_c. This is typically the case with resin or ceramic matrices, but less so with metals. The subject was first tackled by Outwater and co-workers [21-23]. They consider a long single fibre embedded in a block of matrix, debonded over a length x from the free surface. The stress necessary to continue to extract the fibre is given by

$$\sigma_f = \frac{4\tau x}{d} + \left(\frac{8E_f G_{iII}}{d}\right)^{1/2} \tag{4}$$

where τ is the sliding frictional stress in the region already debonded (discussed in 2.1 above). The second term in the expression is the energy required to break the bond, G_{iII} being the fracture surface energy of the interface in shear. In the case where $x = 0$, we have

$$\sigma_f = \left(\frac{8E_f G_{iII}}{d}\right)^{1/2} \tag{5}$$

which is the condition for debonding to begin. Rearranging the expression we see that for given material properties, there is a certain fibre diameter $d < 8E_f G_{iII}/\sigma^2{}_f$ for which the debonding stress is greater than the failure stress of the fibres. This implies that under these conditions the fibres can never be debonded from the matrix, and suggests that a crack propagating in the matrix would cut cleanly through the fibres, producing

a completely planar fracture with no fibre pull-out. Such a failure mode, of which an example is shown in Figure 2 is clearly to be avoided if at all possible: fortunately it does not appear to be very common with well-made composites.

2.3 Stress-relaxation and redistribution

Piggott [16] has shown that brittle fibres which break in the plane of a crack can also contribute to the work of fracture. This is by virtue of the work done in stressing them, and the subsequent loss of elastic energy due to stress relaxation after fracture. At the instant of fracture the total stored elastic energy in a fibre, neglecting that arising from elastic stress transfer, is,

$$U_{fb} = \frac{\pi d^3 \sigma^3_{fu}}{48\tau E_f} \tag{6}$$

and an equal amount of work has been dissipiated in the matrix, or at the interface, during the build-up of stress. Here τ is the matrix shear flow stress in the case of a well-adhering metal matrix, and the frictional shear stress for reinforced polymers. E_f is the Youngs modulus of the fibres. A correction factor for τ was derived for the case of significant Poisson's contraction of the fibres.

The failure and relaxation of the fibre involves a total dissipation of $2U_{fb}$, when account is taken of the matrix (or interface) work, and thus the contribution to the work of fracture comes to

$$G_{fb} = \frac{V_f d\sigma^3_{fu}}{6\tau E_f} \tag{7}$$

When polymers are reinforced by fibre bundles, these usually break in the plane of the crack, since their strength is generally uniform, and so the work of fracture must be attributed to stress relaxation rather than pull-out. Figure 3 shows an example of a failed fibre bundle at a fracture surface.

If the stress in the composite remote from the crack results in a significant fibre stress, σ_{fo} say, then σ_{fu} must be replaced by $\sigma^3_{fu} - \sigma^3_{fo}$ in the above equations. Clearly this correction will only be important when the composite stress is near its unnotched breaking strength.

Elastic stress transfer can also involve a significant amount of strain energy redistribution. Piggott [16] obtained

$$U_{fe} = \pi d^3 \tau E_f (1+\nu_m) \ln\{2\pi/\sqrt{3}\, V_f\}/4E_m \tag{8}$$

for the strain energy of the fibre in regions adjacent to the crack where the interfacial shear stress is less than the yield stress, or debonding stress of the fibre-matrix interface. Here E_m and ν_m are respectively the Youngs modulus and Poisson's ratio of the matrix. The corresponding strain energy matrix is very much smaller. These strain energies were considered to be recoverable, and hence not to augment the toughness. However, they can involve significant amounts of stored energy.

2.4 Oblique fibres

Cracks propagate obliquely to fibres when the fibres are randomly dispersed in the matrix, and can also do so when the fibres are aligned, if the crack is suitably constrained. When fibres cross cracks obliquely, extra energy absorbing mechanisms can be identified in both fibres and matrix.

Piggott [24] has shown that the tensile stresses which can be supported by brittle fibres such as glass, are reduced when a flexural stress is superimposed on the tensile stress. On the other hand ductile wires, such as steel, are not so weakened - at least when the flexure results from the wires crossing a crack obliquely in a polymer matrix.

As a consequence, short metal wires crossing cracks obliquely can contribute the normal pull-out work described in section 2.1, together with an extra, and potentially large, amount of work as they are dragged around the corners formed by the intersections of the fibre holes and the crack faces. Hing and Groves [25] and Helfet and Harris [26] have derived similar, approximate, expressions for this, based on the plastic work done, first to bend the wire, and then to straighten it again. Hing and Groves' expression for the work done for a single fibre is

$$U_{fs} = \frac{\pi d^2 \ell \sigma_{fy}}{32} \tan \phi \qquad (9)$$

where ℓ is the pulled out length, σ_{fy} is the yield stress of the fibre, and ϕ is the angle between the fibre and the crack plane. Thus for unidirectional fibres, of aspect ratio s, the total contribution to the work of fracture is

$$G_{fs} = \frac{V_f \sigma_f d \ s \sin \phi}{32 \cos^2 \phi} \qquad (10)$$

for $s < s_c$. When $s > s_c$ some fibres break, and the corresponding value for G_{fs} is the above, multiplied by $(s_c/s)^2$. Hing and Groves derived the analogous expression for the random fibre case.

When the fibres are brittle, on the other hand, flexure of the fibres reduces their apparent strength. The fibres cause yielding of the matrix (at a shear stress τ_{my}) at the corners formed by the intersections of the fibre holes and the crack faces. Thus the smaller the matrix yield stress, the more the deformation of the matrix, and the less sharp the curve assumed by the fibres. We find therefore that the reduction in fibre strength for brittle fibres is [24]

$$\Delta\sigma_{fu} = 5.56 \ \tau_{my} \tan \phi \qquad (11)$$

This will reduce the pull-out work, since the critical pull-out length will be less for an oblique fibre, as a consequence of its reduced apparent strength. The flexure will also cause reduction of the stress relaxation work. Results have been reported by Piggott and Lee [27] which show that G_{fb} falls with increasing angle between the fibre and the crack plane normal in the way predicted by Piggott. Also, Piggott [28] showed that pseudo-randomly dispersed glass fibre bundles in epoxy gave the expected reduction in toughness.

The fibre will, however, do work on the matrix as it deforms the corners formed by the intersection of the fibre holes and the crack faces; see Figure 4. In ductile matrices, fibres thus do additional work which increases the stress redistribution term by a factor $(1+1.43 \ \varepsilon_{fm} \tan \phi)$, where ε_{fm} is the maximum tensile strain in the fibre. In brittle matrices the deformation takes the form of multiple fractures at the crack face and the amount of work is difficult to estimate [29].

2.5 Practical considerations

The relative magnitudes of the pull-out and stress redistribution work can readily be compared by considering the effect of aspect ratio, if we neglect the effect of fibre fractures away from the crack plane. Figure 5 shows the variation, with aspect ratio, of the sum, G_{fc}, of these two contributions to the work of fracture. G_{fc} is plotted as a proportion of its maximum value, G_{fm}, which occurs when $s = s_c$, and may be calculated from equation 1 with $s = s_c$.

It may be seen that for very strong silica fibres ($\varepsilon_{fu} = 0.1$) the maximum contribution from the stress redistribution term (when $s \to \infty$) is a little less than 50% of the maximum pull-out work ($s = s_c$). For carbon fibres the corresponding value is about 1%. However, the picture would be entirely different were the strain energy in the fibre in the elastic stress transfer region near the crack face to be included. This energy (equation 8) is very large for low modulus matrices (e.g., polymers).

The debonding energy does not appear to have been separately evaluated. This is evidently an area which would repay further study. However, Kelly [30] considers that the debonding energy is very small. We also consider that it is not likely to be a very significant factor, despite arguments to the contrary by Marston et al [31]. It should be remembered that Marston et al were not able to measure it, but used what they thought was a reasonable value, based on some work of Harris. However, Harris et al [7] demonstrate its relative unimportance in the case of reinforced polymers by calculating a G_{fp} which agreed with their experiments, without considering the debonding work. (They calculated the pull-out work using plausible values for τ). Similarly, with metal matrices, values of G_{fp} which agree quite well with experiment can be calculated from pull-out work, neglecting the debonding work. In this case the shear flow stress of the metal is used for τ.

Fibres crossing cracks obliquely can give large works of fracture. If ductile wires are used, and the crack propagates obliquely to the fibres in the most favourable direction ($\tan \phi \simeq 2$) we find that the maximum value of G_{fs} is about 2.7 times greater than the highest value for brittle fibres crossing cracks normally [24]. However, this is a somewhat artificial situation, and it is more reasonable to compare the random fibre case, which reduces the contribution to about 0.2 of the above. In any case, if matrix yielding allows the fibre to cantilever, rather than shearing, this reduces the plastic work in oblique fibres to less than that due to fibres pulling out normal to the crack face, except for fibre lengths much less than the critical length [29].

Indefinite pull-out, which implies infinitely large works of fracture, is a suggestion that still needs to be put on a proper quantitative footing. The present proposals all appear to be impractical because the pull out forces being considered are too small. Thus significant work is developed only at unacceptably large crack openings. This danger was pointed out by Piggott in 1970 [16] but seems not to have received any attention till now.

Since the fibres never break, this type of work is analogous to the strain energy at the crack tip rather than to surface energy or surface work effects. Suppose a crack has converging faces, at a constant angle 2θ, Figure 6. Let the fibres be pulling out at constant stress σ_f. For a crack extension da, the pull out work will be $2V_f a\sigma_f$ da tan θ, since $V_f a$ represents the total cross section of fibres contributing to the work, and 2da tan θ is the crack opening, and hence pull out distance for a crack extension da. (In addition there will be a second order effectdue to the extra fibres involved in bridging the newly extended crack. As da $\rightarrow$ 0 this can be neglected compared with the other term).

The rate of change in total energy for the system for an applied stress σ_c is thus

$$\frac{dU}{da} = -\frac{\sigma_c^2\, a}{\alpha\, E_c^*} + 2V_f\, a\sigma_f \tan\theta + V_m G_m \qquad (12)$$

where E_c^* is the appropriate composite elastic constant, α is a constant depending on crack shape, and G_m the work of fracture of the matrix.

Crack propagation occurs when $\frac{dU}{da} < 0$, and taking the limiting case, $\frac{dU}{da} = 0$, we find

$$\sigma_c = \left(\frac{\alpha G_m E_c^*}{a(1-\beta)}\right)^{1/2} \qquad (13)$$

where

$$\beta = \frac{2\alpha V_f \sigma_f E_c^* \tan\theta}{\sigma_c^2} \qquad (14)$$

If β is large enough a crack can never propagate, a situation discussed in detail for non-composite materials by Piggott [32]. However, unless β is 0.5 or greater the contribution of the pull-out work does not exceed that of the matrix. Thus infinite pull-out is only useful if the pull-out stress, σ_f, is large enough to make $\beta > 0.5$. Taking a typical example: $\alpha=1$, $V_f = 0.5$, $E_c = 100$ GPa, tan $\theta = 0.01$, letting the applied stress be half the unnotched failure strength, e.g., $\sigma_c = 0.5$ GPa we find that we need $\sigma_f = 0.12$ GPa for $\beta = 0.5$. This is very much higher than the pull-out forces envisaged so far for indefinite pull-out.

The same difficulty arises when other pull-out and crack bridging mechanisms are maximized to increase the work of fracture, though not to the same extent.

Cooper [33] has pointed out the advantage of increasing the fibre diameter, but this is principally because the pull-out length is thereby increased. In any case, this is not always possible, since many reinforcing fibres, including metal wires, and above all, ceramic fibres such as glass, alumina, silicon carbide.and boron, show a systematic decrease in tensile strength as the fibre size increases. In addition, because of manufacturing problems, it is sometimes technically impossible to maintain fibre properties at large diameters (in the manufacture of carbon fibres from PAN, for example, the precursor fibre must undergo a preliminary oxidation before pyrolysis, and because of the slow diffusion into the interior of the fibre, it is prohibitively expensive to produce fibres with a diameter greater than about 10 μm).

A further disadvantage to increasing the size of the reinforcing fibres is that they quickly become very rigid, and although this may be an advantage in maintaining alignment it means that they cannot be moulded to follow the contours of a sharply-curved part. (For this reason it has frequently been found that boron must be replaced by carbon fibres in curved structures). A compromise solution which has achieved considerable success is to group the fibres into bundles which, while sufficiently flexible to be moulded, nonetheless show some of the characteristics of single fibres during fracture. (This is a solution which has been reached in practice in many cases without the help of the materials scientist, as is shown by the widespread use of glass fibre as chopped-strand mat in resins, and partially opened asbestos in cement).

The subject has been treated both theoretically and experimentally by Fila et al [34] and by Mandell and McGarry [35]. Both groups describe work in which the toughness is enhanced by the use of fibre bundles, and can be controlled by control of the mechanical properties of the bundle-matrix interface. The fibre bundles broke in the plane of the crack (their strength was uniform along their length), and the fracture work depended on bundle modulus and interfacial shear force as predicted by Piggott [16] though the actual values were somewhat higher than predicted.

These methods of improving the toughness all involve increasing the crack opening. A criterion which takes this into account [16] is the ratio of the critical strain energy release rate to the distance apart of the crack faces required for the pull out or stress redistribution work to be done. For fibre pull-out, with the most favourable length, this comes to $V_f\sigma_{fu}/12$, while for fibre fracture in the plane of the crack it comes to $2V_f\sigma_{fu}/3$. Thus, on this basis, stress redistribution is nearly an order of magnitude more effective than pull-out. It can be seen that this criterion depends only on the fibre strength and volume fraction.

Note, however, that the application of this "structural integrity" criterion may only be appropriate in small structures and load bearing members. Large ones are likely to be able to withstand much larger crack openings. In addition, significant crack openings may be desirable in certain special cases. For example, for pressure vessels, it is desirable to have leakage before bursting.

In addition, many of the mechanisms proposed for obtaining long pull-out lengths involve weakening the interface [7,36]. This introduces another problem, namely that the transverse properties can be seriously diminished.

3. SYNERGETIC EFFECTS

This section describes contributions to toughness which result from one component of the composite enhancing a property of the other component.

3.1 Deformation and ductility

Unless both fibres and matrix are ductile, the composite cannot be expected to be ductile. However, the combination of brittle fibres with a brittle or ductile matrix can bring about interactions which result in non-linear stress-strain relationships, analogous to those of ductile metals. With brittle matrices the effects are due to multiple fracturing of the matrix, while with the ductile matrix, short fibres can cause curvature of the stress strain curve. Both these mechanisms result in the absorption of energy under rising load conditions.

3.1.1 *Single and multiple fracture*

When one or another of the phases in a composite fails, complete failure may or may not ensue. A classification based on this distinction leads to a definition of "single" and "multiple" fracture [6]. We consider a generalised composite (Figure 7), having phases "1" and "2" with breaking stresses σ_1 and σ_2 respectively, and with corresponding breaking strains ε_1 and ε_2. At low strains, ε, we can write

$$\sigma_c = V_1E_1\varepsilon + V_2E_2\varepsilon. \tag{15}$$

If both phases remain elastic, this equation will hold true until failure of the more brittle phase. If $\varepsilon_1 > \varepsilon_2$, we have

$$\begin{aligned} \sigma &= V_1E_1\varepsilon_2 + V_2E_2\varepsilon_2, \\ &= V_1E_1\varepsilon_2 + V_2\sigma_2. \end{aligned} \tag{16}$$

At this strain, phase 2 fails, and the load which was carried by this phase is thrown onto phase 1. This then fails if:

$$V_1\sigma_1 < V_1E_1\varepsilon_2 + V_2\sigma_2 \tag{17}$$

and the composite breaks by the formation of a single fracture surface. This is defined as *single fracture*. If phase 1 is sufficiently strong, however, failure of the composite will not occur, but as loading is continued, phase 2 will be progressively broken by a series of transverse cracks. Finally, when the load reaches the failure load of phase 1, complete fracture will occur when $\sigma = V_1\sigma_1$. This process is termed *multiple fracture*.

In the case of a fibre-reinforced material, either the fibres or the matrix may be the more brittle phase and single or multiple fracture may be found in either system (see Figures 8 and 9). In each case, however, only one of the two types of failure is of interest when dealing with composites which may be correctly termed "reinforced". In the case of ductile matrix composites, multiple fracture (of the fibres) occurs at concentrations below V_{min}, the minimum volume fraction for which the strength of the composite obeys the law of mixtures [37], and is characterised by the progressive breaking down of the fibres into lengths between the critical length and half this value. There are numerous experimental observations of this behaviour, in a variety of systems (e.g., [2,38,39]). At fibre concentrations in excess of V_{min}, single fracture is expected, and this is indeed the normal fracture mode for high strength ductile-matrix systems.

In the case of composites with a brittle matrix, inspection of Figure 8b shows that the roles are reversed, and that high strength composites which will be of practical interest, will fail by multiple cracking of the matrix, followed by failure of the reinforcing fibres. This failure mode is typical of reinforced ceramics of all sorts, from cement and plaster reinforced by glass [40-42] to glass and aluminosilicate matrices reinforced by carbon [9,43-45].

Resin matrix composites may exhibit both single and multiple fracture under different conditions. Typically, when reinforced by stiff fibres such as carbon or boron, single fracture is found, but when the fibre modulus is decreased, or the matrix is embrittled by, for example, cooling or shock loading, multiple fracture behaviour can be observed. Chaplin [46] has shown good examples of multiple resin cracking from the root of a notch

in polyester - 12% E - glass, and Cooper and Sillwood [47] used a cooling method both to embrittle the matrix and to produce cracking by differential thermal contraction in an epoxy resin reinforced by steel wires.

3.1.2 *Stress-strain curve with brittle matrix*

The stress-strain curve of a composite with a brittle matrix has been discussed in detail by Aveston et al [48,49] (see Figure 10). In general, there is an initial steeply-rising portion, before cracking has begun, when the stiffness of the composite is given by the "rule of mixtures"

$$E_c = E_mV_m + E_fV_f. \tag{18}$$

When the matrix cracking begins, the compliance of the composite begins to increase, and the slope of the stress-strain curve decreases. (The slope will fall to zero if the matrix has a constant failure strain, but in general the curve will still be rising slowly as the failure of the stronger regions will require a somewhat greater stress than those areas which are the first to crack). Finally, the matrix will be broken down into blocks of length between x and 2x, where

$$x = \frac{V_m}{V_f} \cdot \frac{\sigma_{mu}d}{4\tau} \tag{19}$$

(σ_{mu} and τ are the strengths of the matrix and interface in tension and shear respectively). When cracking is complete, the slope of the stress-strain curve will again increase. Under these conditions, the fibres will be sliding through the blocks of matrix under constant stress, and the modulus of the composite will then be E_fV_f, a value which will be retained until failure of the fibres.

3.1.3 *Stress-strain curve with a ductile matrix*

In general, for a ductile matrix, the stress-strain curve begins with both fibre and matrix elastic, is followed by a transition when one of the phases yields (but is more or less constrained by the unyielding phase), and finally, there may be a portion where both phases have yielded. If the fibres are brittle, one or more of these stages may be suppressed. When the fibres are continuous the failure process may be considered to begin when one of the phases yields. At this point, the stiffness falls from a value given (to a good approximation) by the rule of mixtures, equation (18), to a lower value. Bounds on this modulus have been obtained by Hill [50] but, in a non-work hardening matrix, they are very close to the simple value E_fV_f. If the matrix is capable of work-hardening, the situation is more complex. A simple approach would suggest

$$E_c = E_fV_f + \left(\frac{d\sigma_m}{d\varepsilon}\right)_\varepsilon \cdot V_m \tag{20}$$

where the expression in brackets is the tangent modulus of the matrix at the strain in question. In fact, it is often found that the matrix shows considerably greater strength; this is discussed in 3.2.1. If multiple fracture of the fibres occurs, the fibre contribution to the modulus, E_fV_f, will also be modified. It will not necessarily fall to zero since the fibres will show increasing strength as they become shorter.

If the fibres are short to start with, however, stress-strain curves deviate from linearity quite close to the origin, on account of the stress concentrations at the fibre ends (see section 4.2). Piggott [51] has shown that the behaviour at higher stresses is governed by the interfacial forces between fibres and matrix near the fibre ends. With metal matrices, well bonded fibres cause matrix flow at the interface near the fibre ends and stress-strain curves as shown in Figure 11 are predicted. The effect of increasing the aspect ratio is to bring the stress-strain curve closer to the "rule of mixtures" (equation (18)).

With polymer matrices the interfacial forces near the fibre ends are governed by frictional effects. The frictional shears are not constant along the interface, so that the stress-strain curves are expected to have a different shape from the metal matrix case. Figure 12 shows some examples. Again, high aspect ratio fibres give curves close to the rule of mixtures, although the actual aspect ratio for a 5% maximum deviation from equation (18) is expected generally to be greater for the frictional case. As pointed out earlier (section 2.1), our knowledge of the radial forces at the fibre surface is scanty; this is clearly an area which would repay further investigation, both theoretically and experimentally.

Note that large deviations from the rule of mixtures are predicted for aspect ratios in the region 30-100. These aspect ratios give the maximum pull-out work (section 2.1).

3.2 Beneficial matrix constraints

3.2.1 *Ductile matrices*

The matrix often contributes a greater stiffness and strength to the composite than expected, for example, from equation (18). This has been observed in conventional composites with small inter-fibre spacings [52-55] and also in eutectic structures [56,57]. The matrix stress-strain curves in the presence of fibres are shown in Figure 13. Theoretical explanations of the effect have been suggested. Kelly and Lilholt [54] consider the effect to be due to plastic constraint of the matrix induced by the presence of the unyielded fibres, while Neumann and Haasen [58] have proposed a model in which the increased yield stress is derived from the pile-up of dislocations at the fibre boundary. Tanaka and Mori's [59] explanation of the phenemenon is that at small inter-fibre spacings, the stress fields of the fibres overlap, and so the stiffness of the matrix should be increased in proportion.

One recent observation has further confused the question. Lee and Harris [60] in experiments on the copper-tungsten system, have observed that the increased apparent yield stress in the matrix did not decrease after yield had occurred in the fibres. This is in direct opposition to Kelly and Lilholt's experimental results, and if true sheds considerable doubt on the plastic constraint theories, since the constraint could not be maintained if the fibres were no longer essentially rigid. Lee and Harris explain their results by assuming them to be due to dislocation pile-ups. They obtain agreement using a Hall-Petch type relation, but only if they use as their parameter a cell size of 0.5 micron rather than the grain size. This, they propose, could have been derived from relaxation of the stress arising from differential thermal contraction on cooling from the fabrication temperature.

Whatever the explanation of the effect, however, it will probably only be of significance when the fibres are very small: Kelly and Lilholt estimated that in their case, an unyielded zone of only two microns around each

fibre would have explained their experimental results, and the presence of such a zone around any but the finest fibres is likely to pass unnoticed.

3.2.2 *Brittle matrices*

A very interesting feature of the multiple cracking process which occurs when the matrix is very brittle is that the fracture of the matrix can be perturbed by the presence of the fibres. The mechanism can be envisaged in the following manner. We consider the energy balance established when a crack in the matrix propagates across the composite (without the fibres breaking). As the crack propagates, several changes occur. Firstly, energy is required to produce the crack in the matrix. Secondly, the interface between the fibre and matrix will fail over a distance x on either side of the crack given in equation (19). This also consumes energy. Since the matrix is now unloaded in the plane of the crack it will relax back over the distance x, and will lose strain energy. Conversely the fibres still bridging the crack are made to carry an increased load, and will therefore increase in strain energy. In addition, since the matrix relaxes while the fibres stretch, there will be a differential movement between the two, resulting in the loss of energy to frictional processes. Finally, since the crack propagates under fixed load conditions, and the composite extends slightly (by the amount that the fibres stretch), the external forces will do work, and provide an input of energy to the system. Aveston et al [48] showed that these separate energies can be calculated and the energy balance can be evaluated exactly. Intuitively, however, one can see that all the terms in the balance involve the thickness of the relaxed zone, x, on either side of the crack with the exception of the energy required to propagate the matrix crack, which is just $G_m V_m$ per unit area of fracture surface. Since x is proportional to the fibre diameter and inversely proportional to the fibre concentration, one can see that by making the fibres sufficiently small, or by increasing their concentration, x can be made so small that there is no longer a sufficient release of strain energy to support the growth of the matrix crack (Figure 14). The criterion can be conveniently given as:

$$\frac{V_m d}{2V_f^2} > \frac{6\tau G_m E_f}{E_c \varepsilon_m^3 E_m^3} \tag{21}$$

for cracking. This shows that for it to be possible for the matrix crack to form at strain ε_m, the fibre concentration (V_f) must not be too high, and the fibre diameter must not be too small. If cracking is suppressed, then continued loading of the composite will increase the available strain energy, and equation (21) can then be rearranged to show that the matrix will fail at a strain:

$$\varepsilon'_m = \left(\frac{12\tau G_m E_f V_f^2}{E_c E_m^2 V_m d} \right)^{1/3} \tag{22}$$

Further increases in the fibre concentration or decreases in the fibre size will progressively increase the fracture strain of the matrix until, theoretically at least, it reaches the failure strain of the fibres, in which condition the stress-strain curve will be linear to the failure strain of the fibre, when both phases will fail simultaneously.

It is interesting to compare this behaviour with that of conventional metals when the yield stress is increased without affecting the work-hardening behaviour. This can be done, for example, by applying increasing degrees of cold work before machining the test specimens (Figure 15). The un-workhardened metal, or the composite with no matrix strength enhancement, shows elastic behaviour only to small strains, but a large extension to failure accompanied by irreversible, energy-consuming deformation. As the degree of prior work-hardening or matrix constraint is increased, the elastic limit is increased, but the extension to failure is decreased. Finally, if the metal is cold worked to or beyond its point of plastic instability, or the composite matrix is prevented from cracking until the failure strain of the fibres, the loading becomes essentially elastic until failure, and the only irreversible deformation will be found in the immediate vicinity of the fracture surface.

There have been many experimental observations of matrix crack inhibition during failure by multiple cracking. Phillips et al [44] have obtained results which show quantitative agreement with the theory outlined above, and there are many qualitative observations of the effect (see, for example, [61-63]). A good example [64] is shown in Figure 16, which clearly shows the increasing resistance to cracking of the matrix, and the progressively closer approach of the cracking stress to the ultimate strength as the fibre content is increased.

3.3 Modifications to fibre properties

3.3.1 *Ductile fibres*

The plastic constraint between fibres and matrix has a beneficial effect on the U.T.S. of a composite in which both phases are ductile. Here the U.T.S. is defined by the condition that $d\sigma_c/d\varepsilon = 0$, the point of plastic instability. In the case of the composite, we have

$$\frac{d\sigma_c}{d\varepsilon} = V_f\frac{d\sigma_f}{d\varepsilon} + V_m\frac{d\sigma_m}{d\varepsilon} = 0. \qquad (23)$$

Tyson [65] was the first to point out that since $d\sigma_m/d\varepsilon$ is still likely to be positive at the failure strain of the fibres, the failure strain of the composite will probably be greater than that of the fibres alone. This idea has been extended by Mileiko et al [66,67] to the evaluation of the fracture energy, and also by Garmong and Thompson [68], who show that high local elongations are to be expected in the area of the neck due to the lateral support provided by the matrix. This phenomenon has often been observed experimentally (see, for example, [69-71]).

The complementary case, in which the fibres are ductile but the matrix is brittle, was discussed by Cooper and Kelly [72]. They showed that, in the case of a constant interfacial shear stress, the length of fibre on each side of the crack which is subject to plastic deformation is $\sigma_f d/4\tau$. This is half the fibre critical length, i.e. $\ell_c/2$. Thus the contribution of the fibres to the work of fracture is

$$G_f = V_f U_f \ell_c/2 \qquad (24)$$

where U_f is the work done in deforming unit volume of the fibre material to its U.T.S.

In view of the various types of fibre pull-out mechanisms discussed above in section 2, one can question the utility of equation (24), particularly as we have in mind fibres which are capable of some plastic deformation. The experiments of Morton and Groves [20], in which they observe very long pull-out lengths in metal wire-reinforced resins due to the plastic contraction of the wires away from the matrix are possibly more relevant in this case. Nevertheless, Gerberich [73,74] found that in the ductile matrix-ductile fibre system aluminium-stainless steel, the contribution of the fibres to the fracture energy of the composite could be satisfactorily described by an expression similar to equation (24), and that when this was added to the contributions of the matrix, obtained by the use of equation (27), he obtained a good correlation with the experimentally measured fracture toughness.

3.3.2 *Brittle fibres*

Brittle fibres and whiskers are generally characterized by a decrease in strength as length is increased. The strength of continuous fibres is usually measured on lengths of 25 mm or more, but the length required to transfer the stress from fibres to matrix is normally less than 1 mm. The fibre length having the appropriate strength lies somewhere between these values, so that measurements of fibre strengths often underestimate the strength. The expression for composite strength may be written

$$\sigma_{cu} = \alpha V_f \sigma_{fu} + V_m \sigma_{mu} \qquad (25)$$

where α is a constant, which for a well-made composite can be greater than 1.0 because of this effect. Rosen [75] used a statistical approach and showed how α varied with fibre length and the coefficient of variation of the fibre strength.

Parratt [76,77] suggested a graphical method for determining the appropriate fibre strength. This is illustrated in Figure 17, which shows a length vs. mean strength curve for glass fibres, taken from the results of Metcalf and Schmitz [78]. Superimposed on the curves are lines, given by the equation $\sigma = 4\tau\,\ell/d$ for different values of τ. The intersection of these lines with the strength curve gives an indication of the appropriate fibre strength for the composite. It can be seen that the error in using the 25 mm value for the strength is likely to be substantial, unless τ is 1MPa or less. However, one may argue that the strength used should be for a fibre length of several times the value used. (Parratt considered the strength of fibre bundles, as well as that of individual fibres. Fibre bundles have strengths of about 0.7 of the values shown in Figure 16, except for very short lengths).

Although neither approach has been widely used, (probably owing to the paucity of suitable data), they are nevertheless important since they can explain some unexpectedly high results for the strength of composites containing brittle fibres, or whiskers.

4. COHIBITIVE EFFECTS

This section describes contributions to toughness which result from one component of a composite reducing a property of the other component.

4.1 Matrix constraints

In a ductile matrix, in the absence of fibres, the plastically deformed zone at fracture surfaces can be as large as 1 cm or more. This can be reduced, and hence the work of fracture reduced by the constraining effect of the fibres. The contribution of the matrix to the work of fracture was studied by Cooper and Kelly (1967). In their simple treatment, they considered the behaviour of the matrix bridges which are left spanning the crack after the failure of the fibres (assumed brittle). The maximum load which can be carried across these bridges is $\sigma_{mu}V_m$, which is then transferred back into the fibres over a distance x on either side of the crack, where

$$x = \frac{V_m \sigma_{mu} d}{4\tau V_f} \tag{26}$$

The volume of matrix undergoing plastic deformation is thus $2xV_m$ per unit area of crack, and it was assumed that the contribution to the fracture energy of this deformation was approximately equivalent to that of deforming the same volume of matrix to its U.T.S., σ_{mu}. The matrix contribution to the work of fracture thus becomes:

$$G_m \sim \frac{V^2_m}{V_f} \cdot \frac{\sigma_{mu} d}{2\tau} \cdot U_m \tag{27}$$

where U_m is the work done in deforming unit volume of the matrix to its U.T.S.

Experimentally, the validity of this equation was studied by Cooper and Kelly [79], and R. E. Cooper [80]. The former studied the variation of fibre size, while the latter investigated both the effects of fibre size and volume fraction. In general, reasonable agreement was found, although Cooper [81] observed the dependence on fibre volume fraction to lie between $G_m \propto V_m$ and the relation $G_m \propto V_m^2/V_f$ predicted by equation (27). Subsequently, Thomason [82] has obtained a plane-strain solution for rigid fibres in a rigid-plastic matrix. He obtains a slip line field which is quite similar in its predictions to the deformation observed experimentally by Cooper and Kelly, and the work has been extended to the estimation of the fracture energy [83]. The estimated values, however, come to only one tenth of those obtained in the experiments. Thomason points out the value of having a strong, as well as a tough, matrix. His argument is essentially that since the matrix is constrained by the fibres and its ductility will be limited in any case by their presence, it is better to increase the yield stress at the expense of some ductility in order to maximise the total work of fracture. Another interesting observation is that under certain circumstances the matrix may be so constrained that the fibres will fracture at a distance from the crack plane before the matrix can yield: this is evidently another way of describing the transition between single and multiple fracture in metal-matrix composites.

4.2 Stress concentrations at fibre ends and breaks

The effects of fibre ends and broken fibres have been very widely studied, not only from the point of view of their influence on the adjacent fibres but also for their effects on the surrounding matrix. The work of Hedgepeth [84] on planar composites has been extended [85] to include fibrous rather than laminar structures and to allow for yield in the matrix. Not surprisingly, the stress concentrations found in the fibre case are lower than for the planar composite. Numerical methods were required to

obtain solutions, and the actual values of the stress concentrations look quite small, until it is realised that even in the case of the fibrous structure, a region of nine fibre breaks (three fibres broken across the crack diameter) can reduce the strength of an infinite composite to only two thirds of its undamaged strength.

The above analysis, and further extensions which have been made subsequently (e.g., [86-88]) were made using an elastic shear-lag analysis, which considers only shear stresses in the matrix. One of the failings of the theory is that it does not take account of the relative volume-fractions of fibre and matrix, nor does it give information about the distribution of stress in the matrix. To obtain further information, one is obliged to use techniques such as finite-element analysis, which is much more laborious (see, for example [89-93]).

Lockett [94,95] has compared the results obtained by shear lag analysis with the more accurate numerical techniques. As might be expected, the agreement is good for high fibre volume fractions or low matrix elastic properties, but otherwise the shear lag analysis tends to overestimate the stress concentrations in the adjacent fibres. Lockett estimates a maximum stress concentration in the case of the lamina, of 1.16, which is to be compared with Hedgepeth's value of 1.33. By analogy, he expects the true value in the case of the fibrous geometry to be less than Hedgepeth's value of 1.15, and even questions whether it will be significant at all in view of the other variables in real systems.

Experimental studies to measure the stress-concentration due to fibre ends or breaks have nearly always used photoelastic techniques. In many cases, the object was to measure the rate of load transfer from fibre to matrix [96-100]. Allison and Holloway [101] have investigated the effect of fibre end shape, and have shown that a hemispherical end produces a greater stress-concentration than a square one, and Durelli et al [102] have investigated the stress-concentrations due to fibres arranged in groups. A particularly attractive study has been made by MacLaughlin [103] in which both "fibre" and "matrix" were made of a photoelastic material, so that the stress concentrations in each could be observed simultaneously (see Figure 18).

A consistent conclusion to be drawn from all this work is that whereas the stress concentrations in the adjacent unbroken fibres will probably be low enough to be neglected, the stress-concentrations in the matrix will be very high. (MacLaughlin, for example, reports experimentally observed matrix stress concentration factors of up to thirteen times in some cases). These observations seem to be borne out in the case of real composites. Wadsworth and Spilling [104] and Mullin and Mazzio [105,106] have observed matrix cracking or shear failure associated with fibre breaks in carbon fibre-reinforced resins, and Mullin et al [107] have observed similar effects in boron-reinforced resins, often with the formation of interesting symmetrical crack patterns (Figure 19). The presence of these stress concentrations invalidates the simple elastic shear lag analyses, except at very low applied stresses (see, for example [51]).

4.3 Statistical aspects of failure

4.3.1 *Cumulative versus non-cumulative fracture*

Two views may be taken of the failure of composite material. The first is that damage accumulates progressively throughout the volume of the composite as the load is increased. This damage may take the form of fibre fractures, regions of failure of the interface, or the formation of voids and fractures in the matrix. As loading is continued, these points of damage will multiply until a point of instability is reached and the material fails. This fracture mode is termed *cumulative failure*. In the extreme, as for example when the fibre-matrix bond is poor, the individual points of failure will be independent of one another, and the failure may be treated statistically. Cumulative failures are typically observed in weakly-bonded composites reinforced by fibres with a wide scatter in strength. The fracture is normally very irregular, and may often be described as shaving-brush-like. At the opposite extreme, *non-cumulative failure* is obtained when the coupling between the elements of the composite is so good that the failure of any element is communicated to adjacent elements which fail in their turn and so precipitate failure of the whole composite. Failure thus occurs from the severest individual flaw at the moment when it reaches its own growth stress. Non-cumulative fractures are characteristic of well-bonded systems reinforced with uniform fibres; metal-metal systems normally show this type of behaviour. A typical non-cumulative failure in boron-aluminum from Herring et al [108] is shown in Figure 20. As we have seen with multiple fracture, a parallel can be drawn between composites and conventional metals: cumulative failure can be regarded as the analogue of the gradual exhaustion of the work hardening capacity of an un-notched specimen, whereas non-cumulative failure can be compared to failure arising from a crack or notch. Obviously, there is a whole range of intermediate conditions to be found in real composites - even in a rope the fibres are not totally independent, and there are very few composite systems in which the fibre-matrix coupling is so good that failure of one fibre immediately precipitates failure of the composite.

4.3.2. *Statistical theories for composite strength*

The statistical treatment of the strength of fibrous materials has a long history, starting with the early work of Pierce [109] and Weibull [110] on the strength of chain links loaded in series. The complementary problem of links taken in parallel, was treated by Daniles [111], and their use was reviewed and extended by Epstein [112]. Coleman [113] obtained numerical values for the strength of fibre bundles having a Weibull strength distribution, and a synthesis of all these appraoches was undertaken by Gucer and Gurland [114].

The approach of Gucer and Gurland was applied to composite materials by Rosen [75], who showed that the strength of the composite could be greater than indicated by the rule of mixtures (section 3.3.2).

This theory has subsequently been extended and enlarged [115-120]; the approach has been to extend the statistical theory so as to be able to treat non-cumulative as well as cumulative fracture. As a first step, the results of Hedgepeth [84] were used to estimate the influence of one or more pre-existing fibre breaks on the probability of one of the adjacent fibres failing. Figure 21, from Zweben and Rosen [117] shows the probability of the formation of a given number of fibre breaks in boron-reinforced aluminium as a function of applied stress. These are compared with the experimentally observed range of composite failure stresses, which were found to correlate with the predicted stress for the formation of fracture nuclei containing between two and four broken fibres. Qualitative agreement with the theory was obtained in the degradation in strength of boron-aluminium composites after exposure to high temperatures [121].

The statistical approach to composite failure is capable of considerable refinement, and it will be interesting to follow further developments. In view of the considerable difficulty in obtaining statistical information on fibre (and matrix) properties, however, one can question whether it will be very widely applied quantitatively in practice. In the authors' view, one of the most outstanding contributions of the theory at present is in the clarification it has brought to the distinction between the cumulative and non-cumulative failure modes.

5. THE RELEVANCE OF FRACTURE MECHANICS TO COMPOSITE MATERIALS

The question of the applicability of fracture mechanics to crack propagation and failure in composite materials is open to question [122]. In this section, we examine this question and attempt to establish the conditions under which a fracture mechanics approach will be justified.

5.1 The fracture criterion

Cooper [33] has discussed the general conditions which must be fulfilled for fracture to occur in a composite material. He proposed that fracture, like any other physical process, will not occur unless two separate criteria are fulfilled. These are in general terms the kinetic and thermodynamic criteria, by which it was understood that the former required that the process could be initiated (by achieving some activation energy or by passing a critical initiation point) while the latter required that in the further development of the process, there would be a net energy loss to the system. Applying these ideas to the fracture of a body from a crack or notch, the first criterion should be understood to require that it should be physically possible for the crack to advance; in the case of a metal, this means that the fracture stress must be exceeded at the root of the notch, while for the composite, it demands that there is a sufficient stress concentration for the next unbroken fibre (or element of matrix) to be fractured. The thermodynamic criterion for crack growth is of course the well-known requirement that there will be a net loss of mechanical energy to the system as the crack grows. In the case of conventional materials this leads directly to the Griffith-Orowan-Irwin fracture mechanics.

The energy criterion of fracture mechanics has achieved such dominance in the description of fracture that the "activation" or stress criterion as a separate entity is scarcely recognised. It is important to recognise, however, that both are necessary requirements to fracture. As an illustration, compare the behaviour of ductile and brittle metals.

A ductile metal having a sharp notch can tear, but does so under a rising load curve. If it is sufficiently notch insensitive, the tearing process can proceed until the cross section is so reduced that general yielding occurs. In this case there is sufficient stress concentration for the initiation of crack growth, but insufficient stress for the energy criterion for failure. On the other hand, a higher stress than that required for the energy criterion for failure will be needed to propagate a crack in a brittle metal having a blunt notch. In this case we have a situation where there is more than sufficient strain energy present in the specimen to be able to complete the fracture once initiated but because there is no local region which is sufficiently highly stressed for the material actually to be torn apart, a crack cannot grow.

The above simple arguments may seen trivial when applied to the failure of isotropic and homogeneous materials, but the authors believe that they are instructive when applied to composites. We have postulated above that a crack can be prevented from propagating in a material when *either* the stress *or* the energy criterion is not satisfied. Taking the stress criterion first, we believe that this amounts to searching for a mechanism which will in some way prevent or delay the propagation of a fracture at the root of a notch. Such a mechanism has been proposed by Cook and Gordon [3] who drew attention to the stress concentrations (both in tension and in shear) at the root of a notch which tend to cause failure on planes *normal* to the crack plane. They therefore suggested that by sufficiently weakening the fibre-matrix interface, cracks could be blunted by a process of splitting or delamination ahead of the crack. This mechanism thus fulfills our requirement for fracture prevention by interfering with the stress criterion, for if the material delaminates, the stress-concentration at the root of the crack disappears, and the next unbroken fibre cannot be fractured until the overall stress reaches the un-notched fracture stress. In general terms, we can extend this concept to any mechanism which prevents the propagation of the crack either by reducing the stress-concentration at the crack root or by placing a sufficiently strong barrier in the path of the crack. In either case, the condition for its further propagation must be evaluated in terms of the increase in load necessary to compensate for the drop in the stress-concentration at the crack tip, or the increase in load necessary to cause failure of the strong point. The condition for the material to fracture will not be related in any direct way to the strain energy content of the specimen.

The energy criterion deals essentially with the steady-state growth of the crack. For this to be the criterion which governs crack growth, there must be no irregularity in the crack tip stress needed to cause the failure of consecutive elements of the material, when considered on the same scale as the crack length. This is, of course, a question of macroscopic versus microscopic dimensions. If we take a composite consisting of, say, brittle fibres well bonded to a ductile matrix, and we observe the fracture behaviour of the composite with cracks or notches which are of the same order of size as the fibres (Figure 22a) it is clear that the stress necessary to cause the next small increment of crack extension will in no way be related to the fracture energy of the composite when measured in a large cross-section. The ease of propagation of the notch will depend upon whether its tip is in the fibre or the matrix, and whether it has just penetrated or has practically severed the element in which it is growing. The situation will be similar in the case where partial or complete delamination occurs (Figure 22b). In either of these cases, the only possible "fracture mechanics" approach would be to treat the crack propagation in each element separately, using the relevant fracture energy for each phase and crack propagation mode, and having regard for the constraint imposed on each phase by the other.

Contrast, however, the situation where we have deep cracks propagating in large pieces of the composite (Figure 23). In these cases, the influence of the failure of each fibre or element of matrix on the behaviour of the whole body is minimal, and from the exterior the crack will be observed to propagate in a uniform manner under a load which does not vary erratically between successive small increments of crack advance. Under these conditions there can be no reason why an energy balance cannot be applied to the growth of the crack, provided that due account is taken of all the contributions to the energy consumed during growth of the crack, and due allowance is made, in the calculation of the change in the strain energy

of the body, that it will be elastically anisotropic.

As we have seen elsewhere in this review, analogies can be drawn between the behaviour of the homogeneous and isotropic metals, and composites, and the same is true for the applicability of fracture mechanics. It is well recognised in the field of the fracture of metals that the propagation of a crack is irregular when seen on the same scale as the grain size. In steels, for example, near the ductile-brittle transition, some grains may fail by plastic flow, while others will fail by cleavage. Clearly, one would not hope for success in applying continuum fracture mechanics on this scale while using values of fracture energy obtained from a standard CHARPY specimen. Similarly, when testing larger pieces of ductile material, it is known that the behaviour of shallow but sharp notches is not to be explained in terms of the mechanics of steady-state crack propagation because of the non-equilibrium form of the crack. Finally, it is recognised that valid measurements of the fracture toughness will not be obtained if the size of the plastic zone is of comparable dimensions to those of the test piece.

Zweben [122] pointed out that the shear-lag analyses of Hedgepeth [84] and others predict that the relationship of composite notched failure stress σ_{cnu} to the crack length a (i.e. the number of consecutive fibres broken) is very close to the form $\sigma_{cnu} = Ka^{1/2}$, and that in view of the fact that this relation is based purely on the calculation of stresses, the experimental observation of relationships of the form $\sigma_{cnu} = Ka^{1/2}$ cannot be taken as a proof of the validity of fracture mechanics to composite materials.

Zweben is clearly dealing with the stress criterion: it is tacitly assumed that the thermodynamic criterion is already satisfied, since once the failure stress of the unbroken fibre is reached, it fails immediately, and the whole of the stress concentration is passed onto the following fibre. Contrast this behaviour with that which would be found if the fibres were ductile. In this case, yield of the first unbroken fibre would occur as predicted by the stress criterion, but because of the plasticity of the fibre, it would not fail immediately. Because of its extension, however, stress would be transferred to the following fibres, which again, would yield in their turn. At this point, where should we define the front of the crack, and what is the remote stress to cause final failure to the first unbroken fibre? Certainly, it is in principle possible to calculate the crack growth stress under these conditions, in the same way that it is in principle possible to calculate the whole elastic-plastic stress field at the root of a growing crack in a metal. However, it also becomes necessary to enquire where the strain energy is drawn from to accomplish these deformations. At this point, therefore, we must also introduce the energy balance, to determine whether energy must continue to be supplied from the external loading system to maintain growth (i.e. crack growth under increasing or constant stress at "general yield") or whether there is sufficient stored energy in the system to complete the failure in an unstable manner under falling load. This is the domain of fracture mechanics.

Experiments on the growth of cracks from sharp notches in a composite were reported by Cooper and Kelly [79] (and are discussed by Zweben [122]). The system chosen was copper reinforced by ductile tungsten wires. In these experiments, it proved possible to observe the three stages of crack growth discussed above: growth from a sharp notch under rising load conditions, steady growth under essentially constant load, and catastrophic final failure. Such observations are difficult to explain on the basis of a simple stress criterion. The above arguments could no doubt be repeated for the case of brittle fibres in a ductile matrix, for a brittle-brittle system showing fibre debonding and pull-out, etc. Without going into detail, however, it seems clear that there will always be some circumstances under which the failure will be governed by a stress criterion, and others under which it will be governed by a fracture mechanics approach. In addition, it is evident that, as with metallic specimens, not all experimental measurements of the fracture toughness parameters will be valid. In particular, it will be necessary to distinguish stages of initiation and early growth of the crack from those of steady-state propagation, when the damaged zone at the root of the crack has reached its equilibrium size, and to pay attention that the damaged zone does not become comparable in size with the dimensions of the test piece.

5.2 Some experimental measurements of fracture toughness parameters

There has been considerable interest in the applicability of fracture mechanics to the failure of composite materials. In general, these studies have concentrated upon two questions, the first being the applicability of the concepts of fracture mechanics, and the second being to relate the measured fracture energies to the micromechanical processes which occur during fracture.

In investigations of the applicability of fracture mechanics to composites, a considerable amount of work has been done on glass-resin systems. Owen and Bishop [123] investigated the behaviour of a reinforced polyester. They found that K_c was independent of crack length when measured parallel with the fibres in unidirectional material, but increased in specimens reinforced with chopped-strand mat or balanced weave material. To obtain a value of K_c which was independent of crack length, they used Irwin's crack-tip correction factor together with an equivalent yield stress, and were then able to predict the failure of specimens with a circular hole with reasonable accuracy. This work was then extended [124] to the testing of plate and box-beam specimens, again with good agreement being obtained between theory and experiments. Spencer and Barnby [125] have used a compliance method to obtain a critical stress intensity factor in glass-fibre-resin composites having a variety of notch and fibre angles; they draw attention to the fact that the compliance-crack length curve is very different from that of conventional materials because the composite is anisotropic and inhomogeneous [126]. The importance of anisotropy has also been emphasised in some preliminary work by Konish et al [127] who show that the concepts of linear elastic fracture mechanics (l.e.f.m.) are only applicable provided that due allowance is made for anisotropic effects.

The effects of notch geometry and specimen dimensions have been studied by a number of workers. Guess and Hoover [128] measured the critical stress intensity K_c and the work of fracture G for a series of carbon-carbon composites. They found that provided the specimens did not delaminate (which was, in fact, the "toughest" mode of failure) K_c and G were independent of the notch depth-to-width ratio and crack width. K_c and G did not correlate very well, but gave the specimens the same ranking. Ellis and Harris [129] have investigated the fracture behaviour of epoxy resins reinforced by carbon fibres of both high and low modulus. They found that in both cases, the work of fracture was independent of notch width and there was only a small dependence on notch root radius and strain rate. On the other hand, there was a strong dependence upon fibre orientation (as might be expected), on specimen length and on the notch depth. They used a l.e.f.m. approach on a double edge-notched plate and showed that K_c is related to the strain energy release rate G, obtained from compliance measurements, but not so

well connected with the work of fracture. High- and low-modulus carbon fibres and S-glass in resin matrices were studied by Beaumont and Philips [130] who found that the notch root radius has a significant effect on the apparent fracture toughness parameters. In particular, they found that as the relative sharpness of the notch was increased, there was a progressive improvement in the agreement of observation with the predictions of l.e.f.m. They found that the work of fracture of the carbon composites could best be explained by contributions from fibre pull-out, while that of the glass-reinforced material could be explained by the energy required to debond the fibres from the matrix. Fitz-Randolph et al [131] have measured the fracture energy of boron-epoxy composites. They found the fracture energy to vary both with the crack depth and also with the measurement method. The former variation appears to be linked to the form of the stress-distribution in the notched material, since they found that the work of fracture was attributable to the energy consumed in debonding the fibres from the matrix, and that post-fracture examination showed that the debond length varied with the depth of the original notch.

Phillips [132] has made a critical comparison of several methods for obtaining the fracture toughness parameters. Using carbon-fibre laminates, he measured the variation of compliance with crack length, the total work of fracture, and the tensile failure stress of centre-cracked plates. His analysis of the latter method depended upon the Griffith-Irwin failure criterion, and hence the fracture toughness parameters derived from this test could be compared with those obtained from the other two tests to give a critical test of the relevance of linear elastic fracture mechanics. In fact, he found that the measured K_c obtained by l.e.f.m. was largely independent of crack length, and that it compared favourably with values of the fracture toughness obtained by other methods.

There has been widespread recognition that the values of fracture energy deduced from the initiation stage of crack-growth, and which are normally deduced by use of l.e.f.m., are often considerably less than those derived from other measurements, such as the "work of fracture" method (see, for example, [133-137]).

This is so not only when the values are measured parallel with the fibres, but also in the transverse direction [134]. The reason for this appears not to be due to the non-applicability of the fracture mechanics approach but to the genuine measurement of two different parameters. The l.e.f.m. approach measures the difficulty of initiating the crack, while the fracture energy method measures its resistance to propagation. If in the former case, failure is taken to have occurred when the crack can first be detected, it is probable that the only energy-absorbing processes to have been called into play will have been those involved in the failure of the matrix and the fibre-matrix interfaces. Such energy as remains to be absorbed in completing the fracture by, for example, pulling out the broken fibres will only be completely consumed when the crack opening displacement has finally reached the maximum pull-out length, and thus it will only be measured in specimens which are large enough for these displacements to be included in the fracture test. This type of behaviour is precisely paralleled by that of ductile metals which are sharply notched; if one measures the loads at which the first local yielding occurs at the root of the notch, low values of the fracture toughness will be obtained. The full value of the fracture energy will only be measured when the crack has grown sufficiently for the equilibrium form of the plastic zone to be developed at the root of the crack, and this will only be obtained in a sufficiently large test specimen.

This explanation appears to be well supported by the experimental evidence. Both Beaumont and Philips [130] and Gershon and Marom [137] account for the experimentally obtained values of the initiation fracture energy by the energy required to debond the fibres from the matrix, or, in general, the formation of new surfaces, while the propagation-values are increased by the contribution of fibre pull-out. Beaumont and Harris [135] observe that the initiation value is similar to the "elastic strain energy release rate at the initiation of fracture of a brittle orthotropic solid", and that furthermore, when measured parallel to the fibres, the initiation value and the propagation value are similar, and not dissimilar to the fracture energy of the resin alone. Harris et al [138] consider that they can account for the propagation fracture energy of glass-reinforced resins in terms of the contributions from fibre pull-out.

In addition, the particular specimen dimension and loading system may have an influence. We cite, as one example among many, the work of Prewo [139] on the impact energy of the boron-aluminium system, in which he found a large amount of deformation and hence energy absorption on the compressive failure side of the specimen. In addition, it was shown that only twenty percent of the energy consumed was dissipated in the immediate vicinity of the fracture, while the rest was lost in interlaminar shear and other non-localised deformation processes. This casts considerable doubt on the usefulness of the impact method for testing the fracture toughness of composites.

Reviewing all these diverse experimental observations, it must be admitted that the direct applicability of fracture mechanics is not universally proven. The authors feel, however, that this is not through any basic failing of fracture mechanics, nor because it is inapplicable to composite materials. Rather the tests which are applied to the composite should be closely examined, as one does with metallic specimens, and it should be established precisely what parameter one is measuring (initiation or propagation values for example). Finally, it should be determined whether the parameter which is being measured is relevant to the subsequent use of the material, bearing in mind what has been called the "structural integrity" (section 2.5).

6. SUMMARY AND CONCLUSIONS

The experimental and theoretical work which has led to our present understanding of the fracture of composites has been critically reviewed, and the many processes which have an effect on the toughness of composites have been classified into three main groups.

The first group comprises processes that lead directly to energy dissipation during crack propagation. These processes generally arise from fibre movements relative to the matrix, and the many proposals for making these as large as possible have been critically reviewed to determine their potential usefulness for tough, load bearing structures. A criterion of structural integrity is introduced and it is shown that stress redistribution makes the greatest contribution to toughness when the crack opening has to be minimized.

The second group contains indirect processes that cooperate to enhance the toughness of a material, i.e. syngertic effects. The combination of brittle fibres with a brittle or ductile matrix can bring about interactions which result in non-linear stress-strain relationships, analogous to those of ductile metals. In addition, some matrix properties can be improved by

the presence of fibres, and, also, there are situations where the matrix can have a beneficial effect on the fibre strength and ductility.

The third group consists of those interactions between fibres and matrix which lead to a reduction in a property of the fibres or matrix. For example, constraints due to fibres at the crack face can reduce the depth of the deformed layer in the matrix in this region, and thus reduce the apparent toughness of the matrix. In addition, stress concentrations, at fibre ends and breaks, can contribute to early failure of the matrix. Cumulative vs. non-cumulative failure are also discussed in this context, and the importance of this distinction is emphasized.

The question of the relevance of fracture mechanics to composite materials is also discussed. The size of the fibres relative to the test specimen size is a very important factor to be considered when tests for toughness are to be made. Also important is the relevance of the parameter being measured, to the subsequent use of the material, bearing in mind what has been called the structural integrity.

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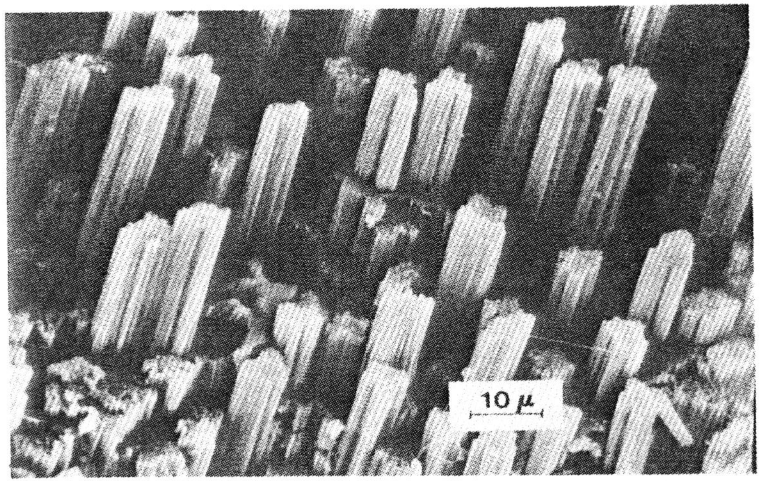


Figure 1 Fracture surface showing pulled-out fibres. In many cases G_{fp} can be calculated from the pulled-out lengths using plausible values of τ. Carbon fibres in epoxy resin (after Pinchin and Woodhams [5]).

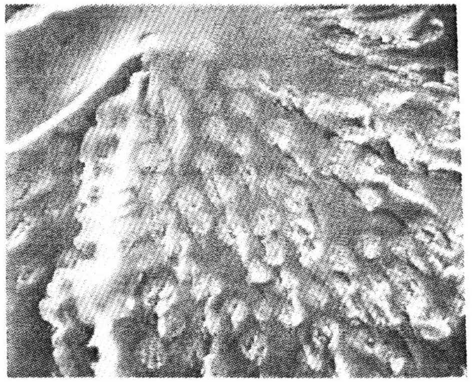


Figure 2 Brittle fracture of a carbon-carbon composite due to having too good a bond.

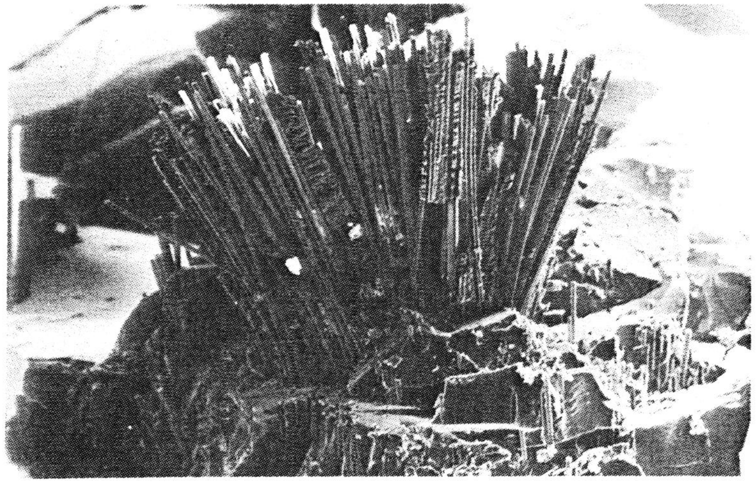


Figure 3 Fracture surface with fibre bundle. At least half of the work of fracture comes from stress relaxation and redistribution.

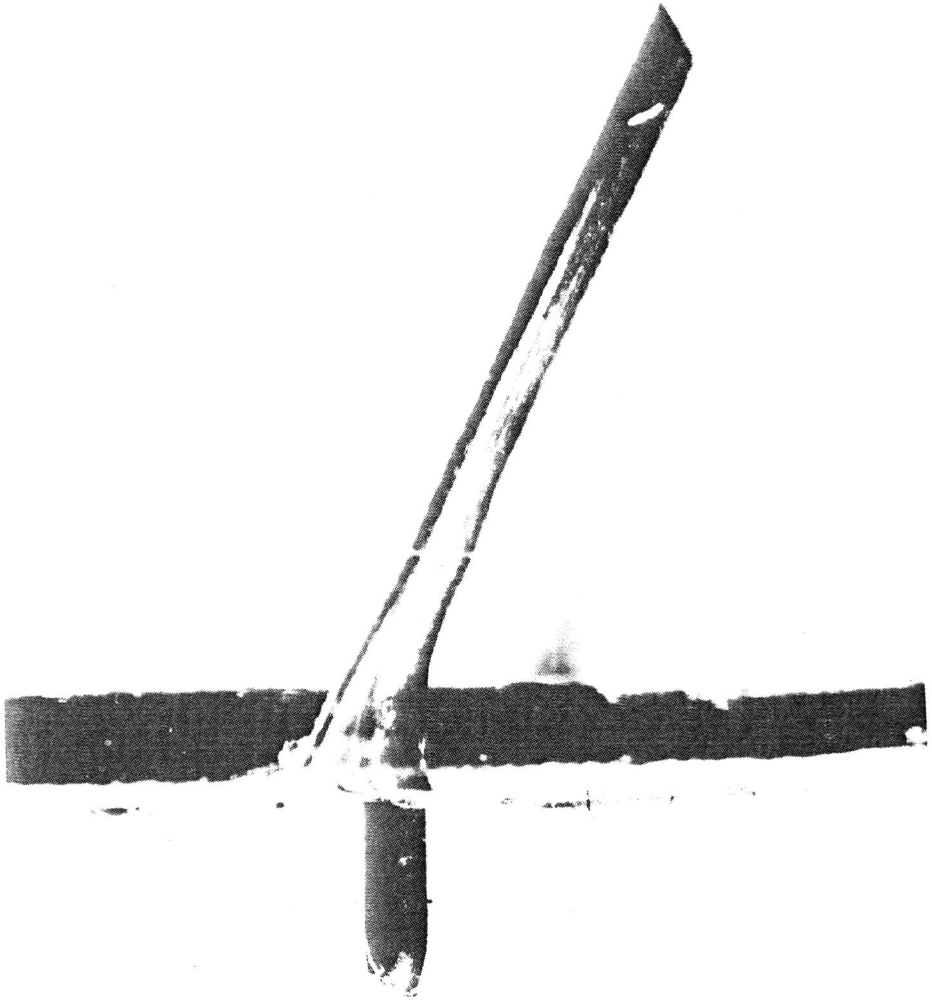


Figure 4 Flexure of oblique fibres at fracture surface. Plastic work in flexing the fibres, and elongation of the holes in the matrix at the fracture surface contribute substantially to the work of fracture.

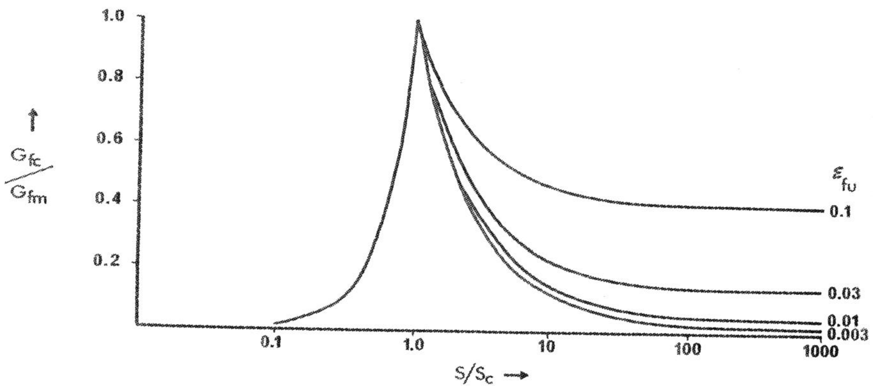


Figure 5 Effect of aspect ratio on work of fracture, for fibres with different ultimate tensile strains. Curves dimensionless. Any fibre-matrix combination can be fitted.

$$G_{max} = \frac{1}{6} V_f d\tau s_c^2 \ , \quad s_c = \sigma_{fu}/2\tau$$

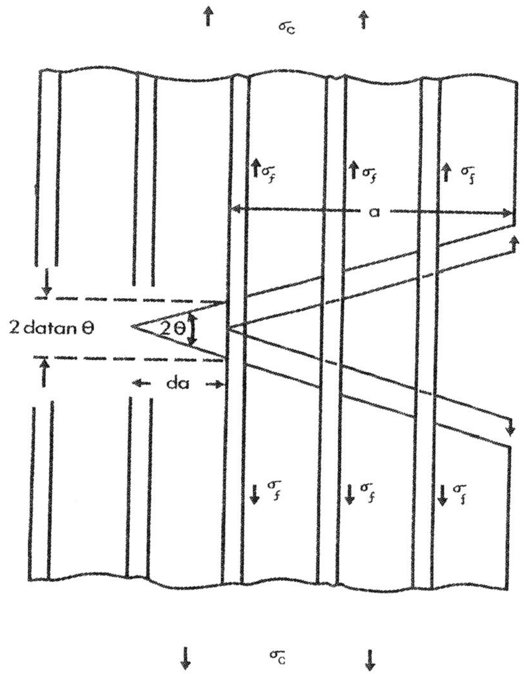


Figure 6 Crack opening with continuous pull-out

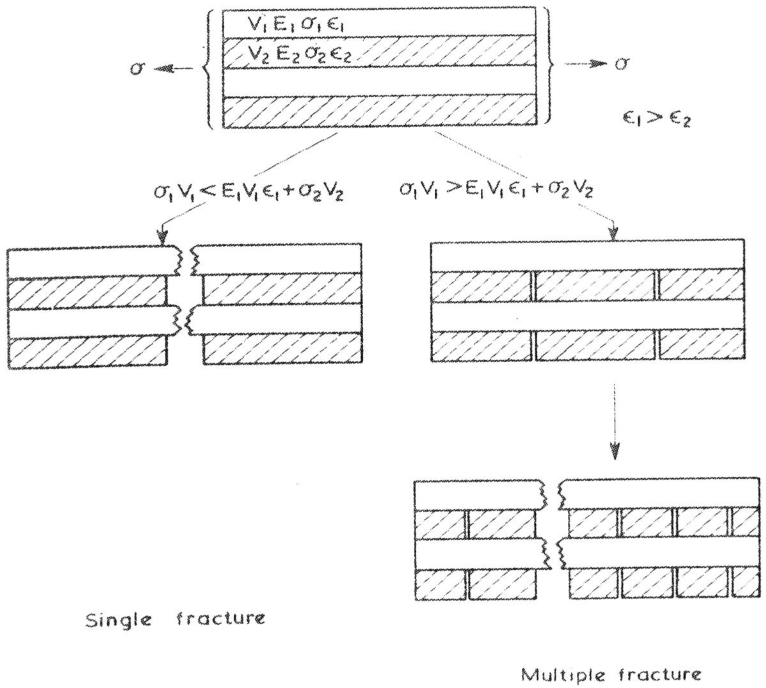


Figure 7 Single and multiple fracture: the more brittle phase is shaded. ε_1 and ε_2 are ultimate tensile strains, and σ_1 and σ_2 are ultimate tensile stresses.

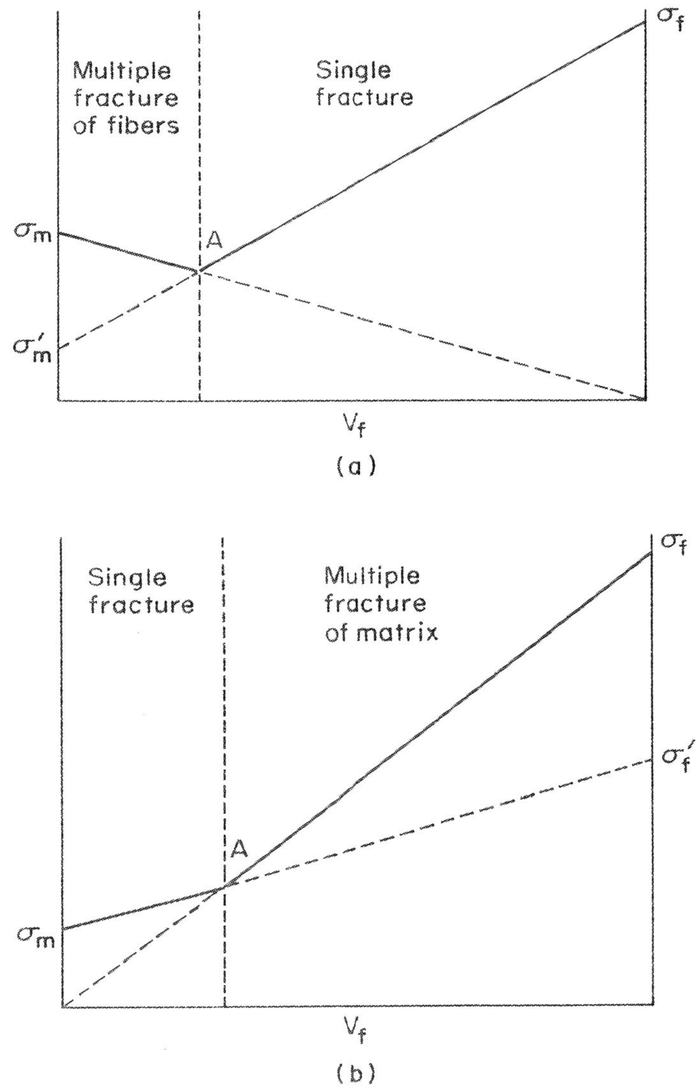


Figure 8 Illustrating the conditions under which single and multiple fracture occur.

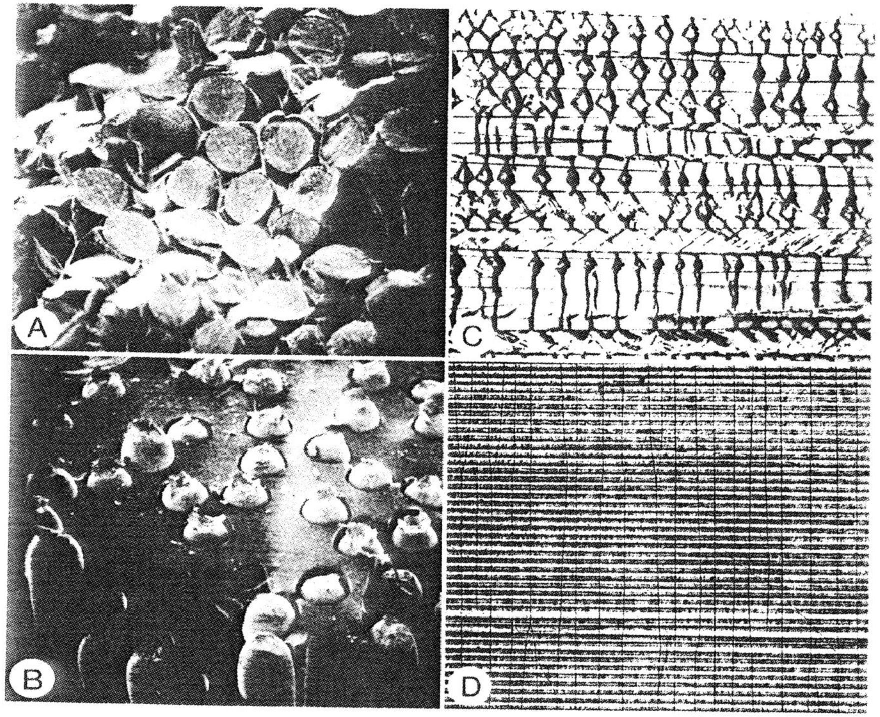


Figure 9 Examples of single and multiple fracture:

(A) Single fracture, ductile matrix: tungsten in copper,
(B) Single fracture, brittle matrix: phosphor bronze in an epoxy resin,
(C) Multiple fracture, ductile matrix: the Co-CoAl eutectic after Cline [38],
(D) Multiple fracture, brittle matrix: steel wire reinforced epoxy, after testing at 77K.

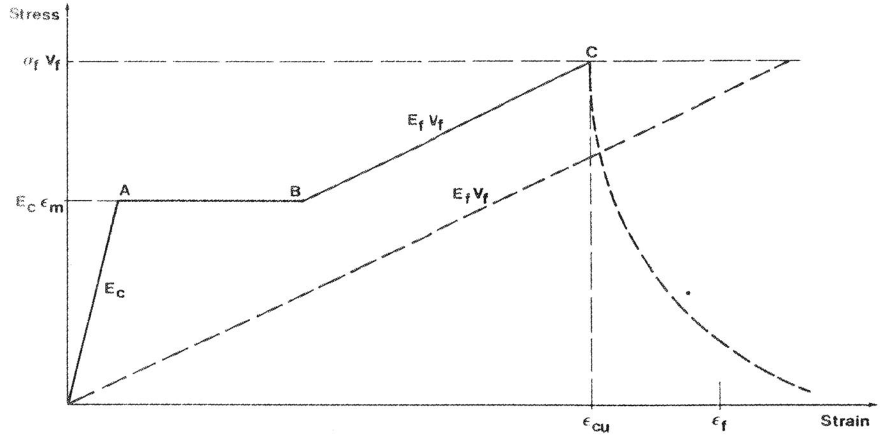


Figure 10 Theoretical stress-strain curve with a brittle matrix.

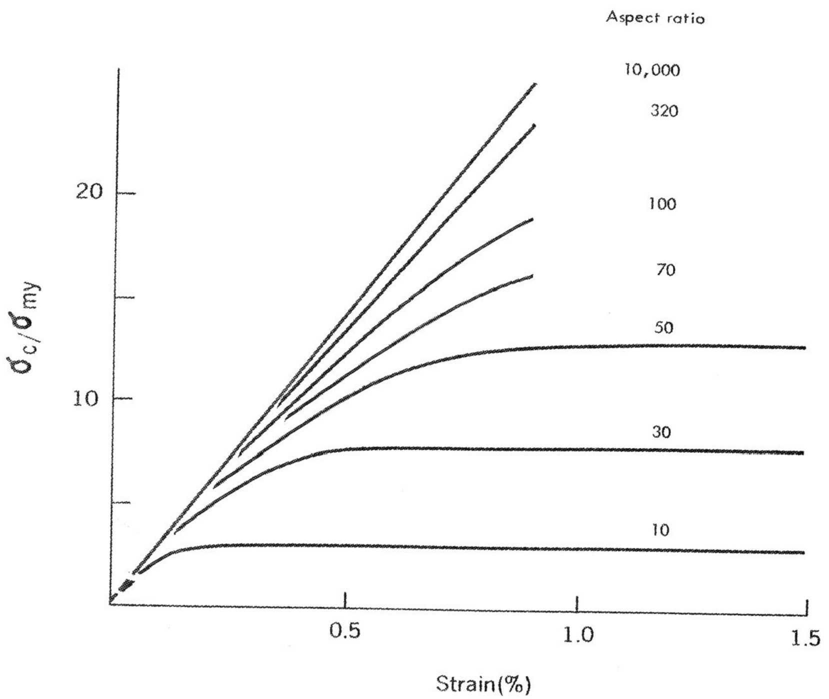


Figure 11   Theoretical stress-strain curves with a metal matrix for fibres with a number of aspect ratios. (Boron-Aluminium; σ_{my} is the matrix yield stress).

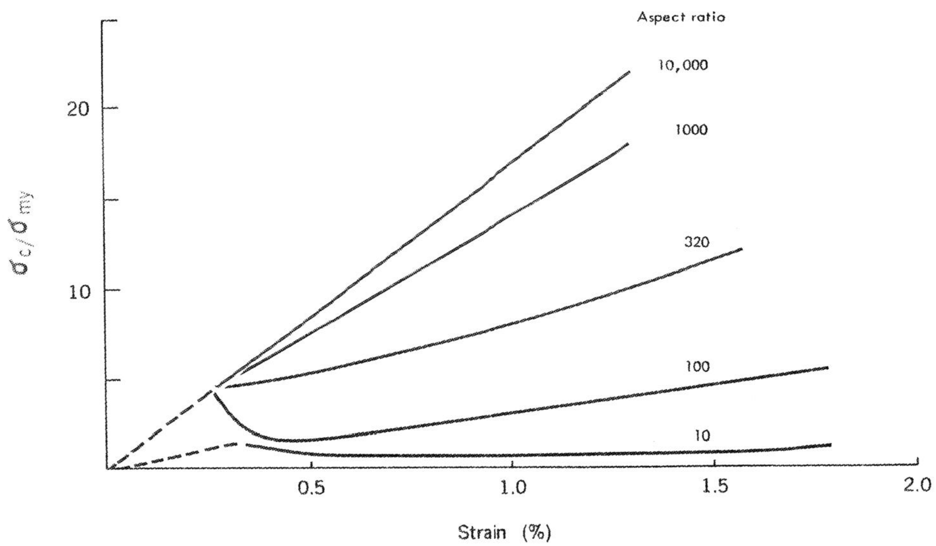


Figure 12   Theoretical stress-strain curves with a polymer for fibres with a number of aspect ratios (steel-polycarbonate; σ_{my} is the matrix yield stress).

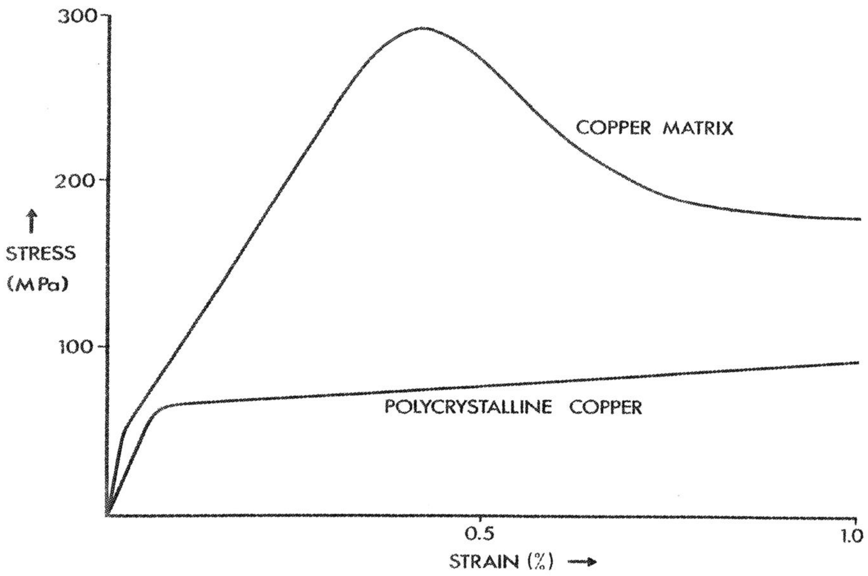


Figure 13 Derived stress-strain curve of the matrix in tungsten fibre reinforced copper, compared with stress-strain curve for copper with 12 micron grain diameter. (V_f = 0.38, d = 10μm). (After Kelly and Lilholt [54]).

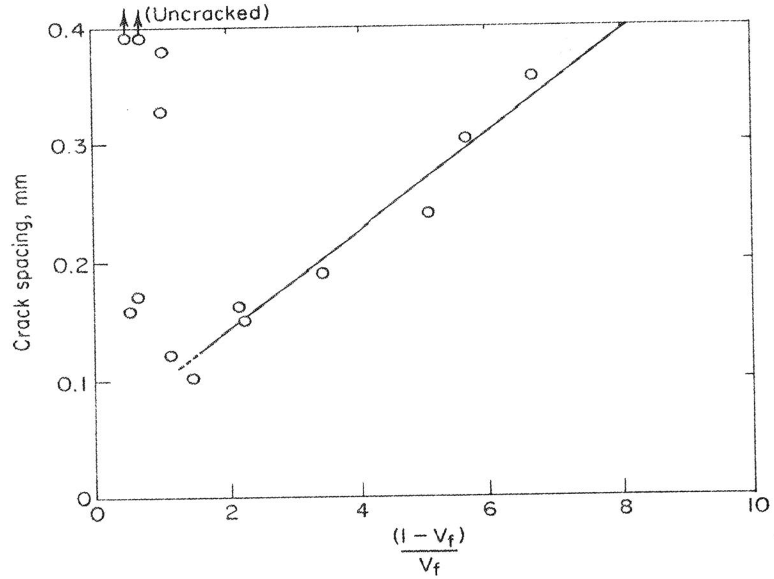


Figure 14 Suppression of multiple fracture at high volume fractions of fine wire in steel-reinforced epoxy resin tested at 77K (after Cooper and Sillwood [47]).

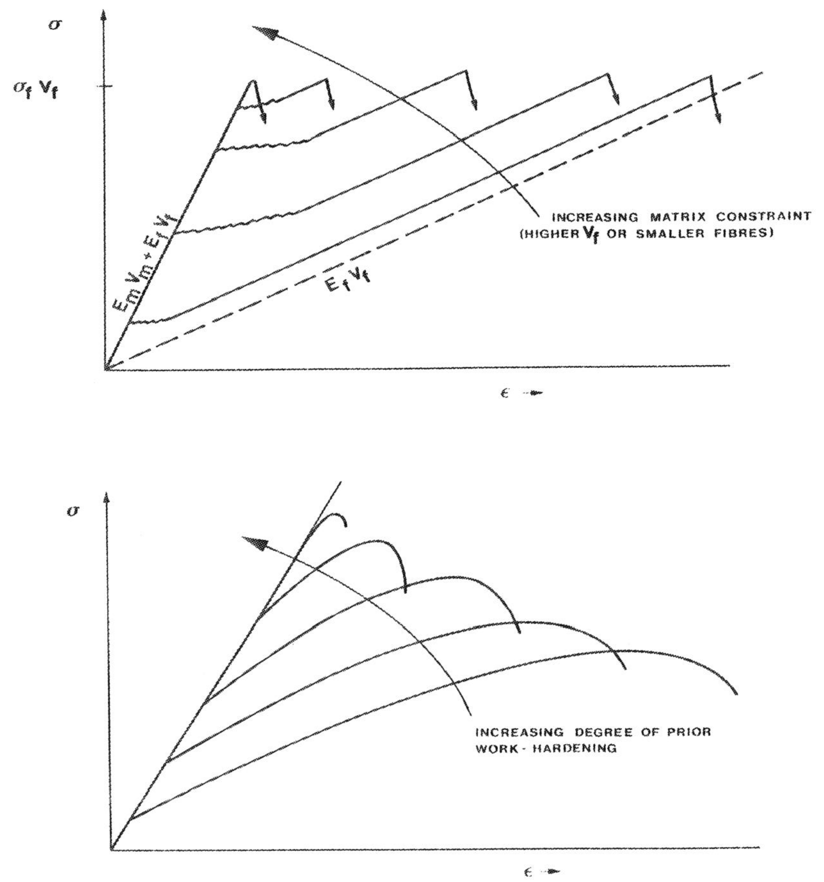


Figure 15 Illustrating the similarity between increasing the failure strain of the matrix in a composite which shows multiple matrix fracture, and of applying increasing degrees of prior work-hardening to a homogeneous ductile metal.

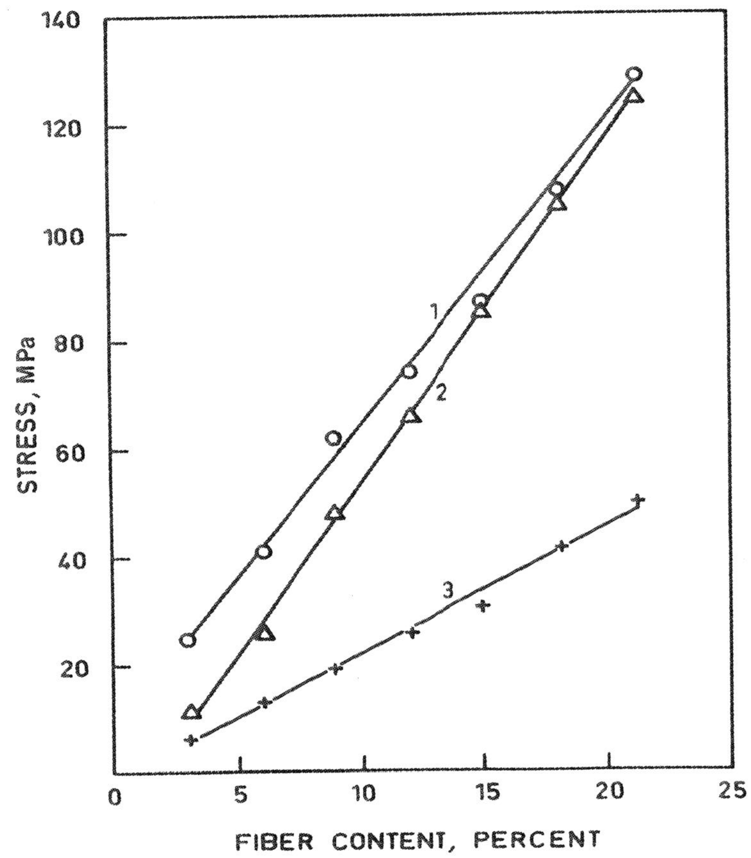


Figure 16 Suppression of multiple fracture in glass reinforced cement: change in tensile strength of composite as a function of glass content. (1) ultimate strength, (2) stress causing cracks to form in hardened paste, (3) limit of proportionality. Adapted from Biryukovitch, et al [64].

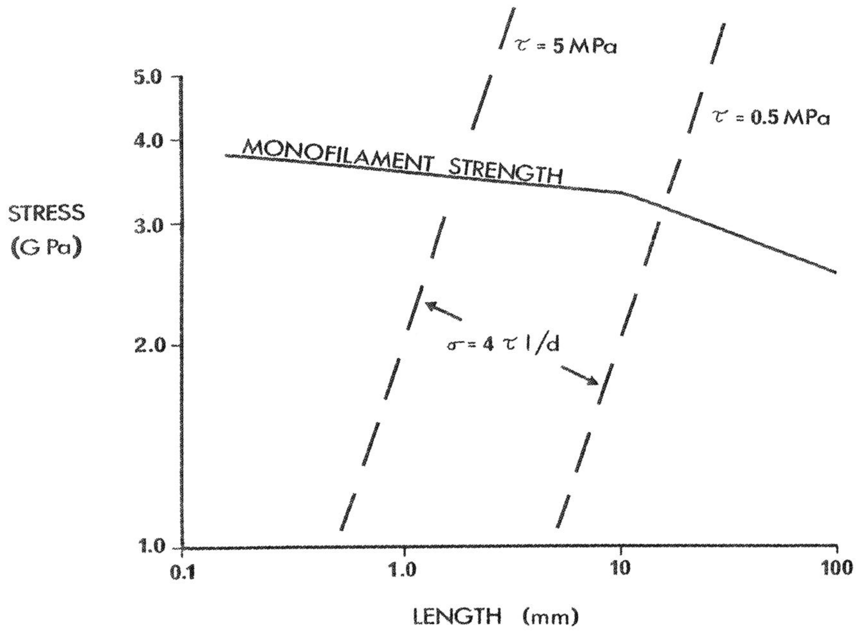


Figure 17 Graphical method illustrating appropriate fibre strength for composite, after Parratt [76,77]. Intersection of line $\sigma = 4\tau\ell/d$ with line for fibre strength gives fibre pull-out length and fibre strength at this length. Fibre strength curves taken from Metcalf and Schmitz [78]. Fibre diameter is about 10 microns.

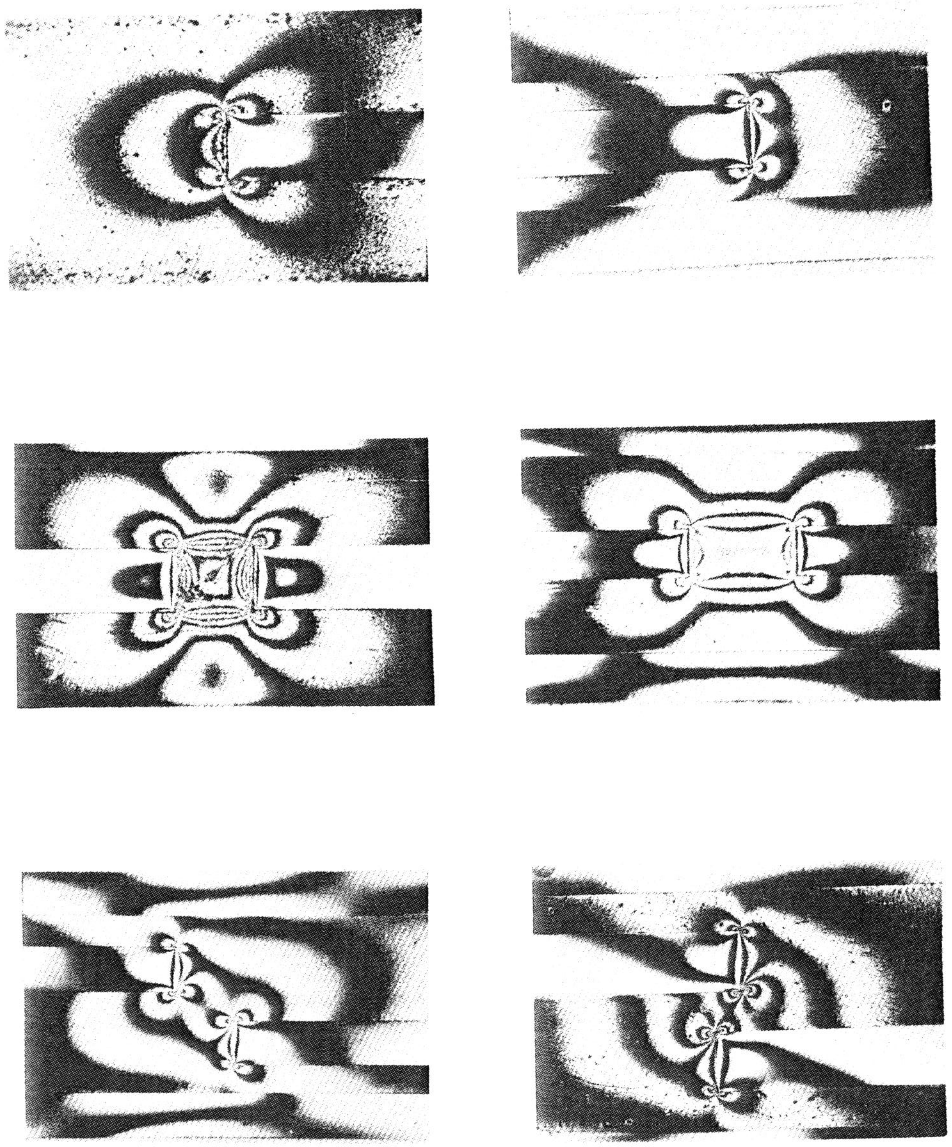


Figure 18 Photoelastic models showing the stress distribution near various types of discontinuity (after MacLaughlin [103], U.S. Department of the Army photograph).

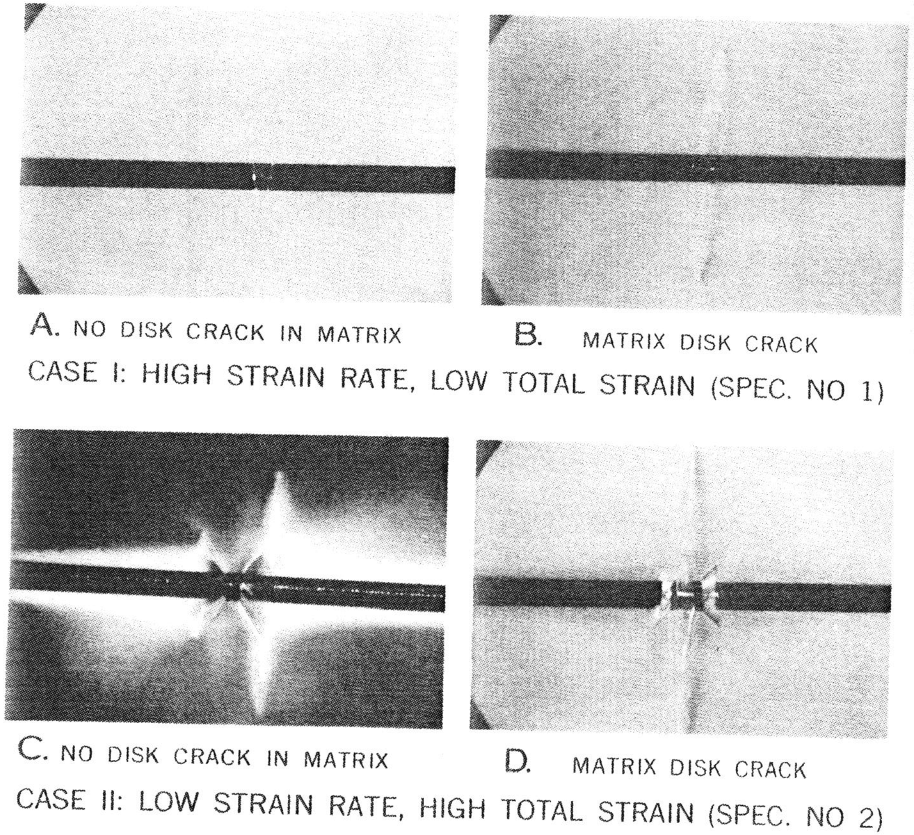


Figure 19 Various types of matrix crack formed as a consequence of filament failure in a boron-epoxy composite (after Mullin, et al [107]).

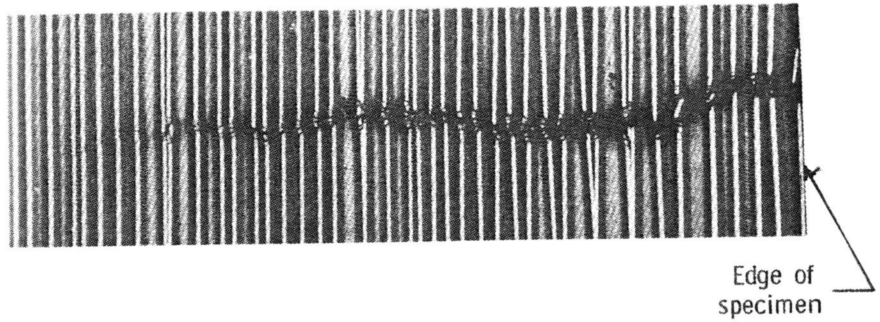


Figure 20 Non-cumulative failure in boron-aluminium (after Herring, et al [106]).

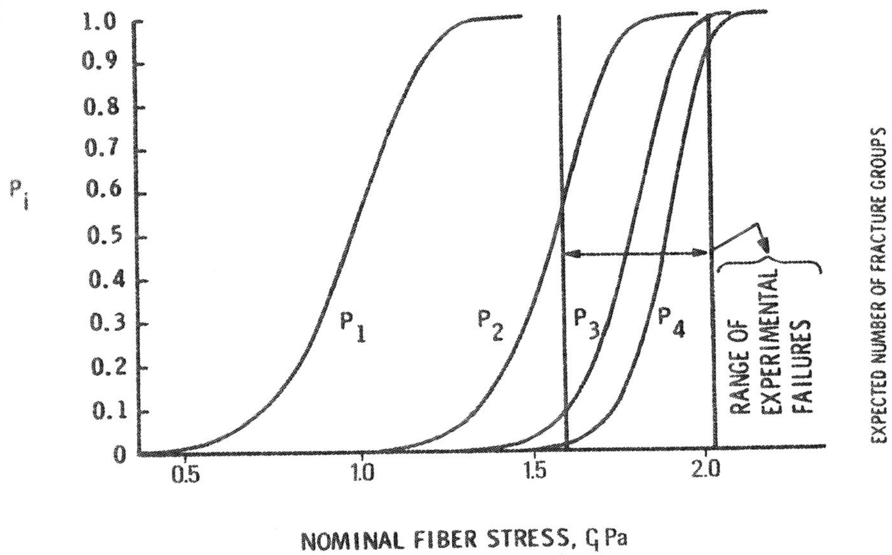
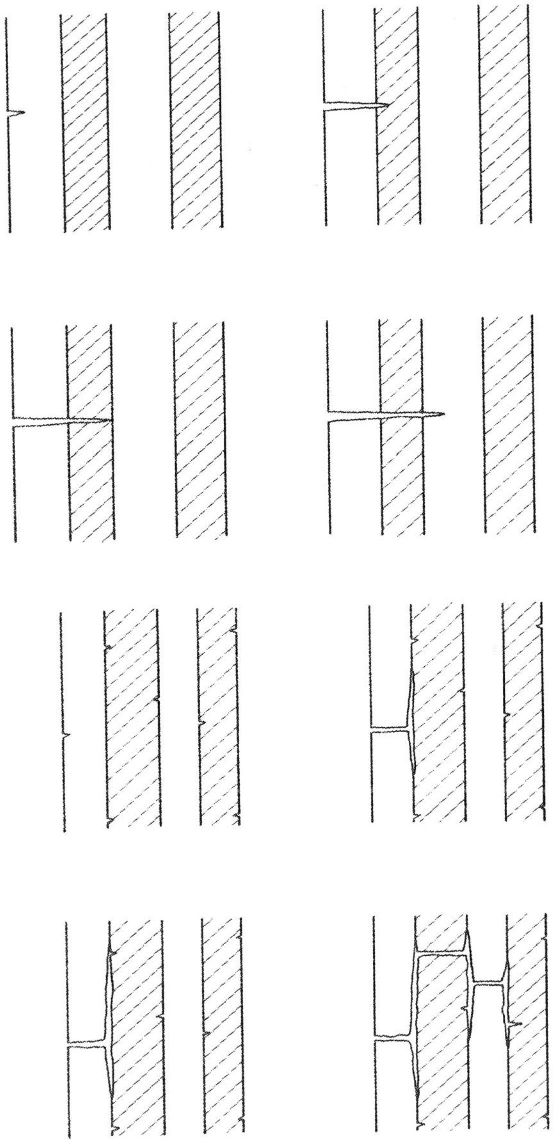


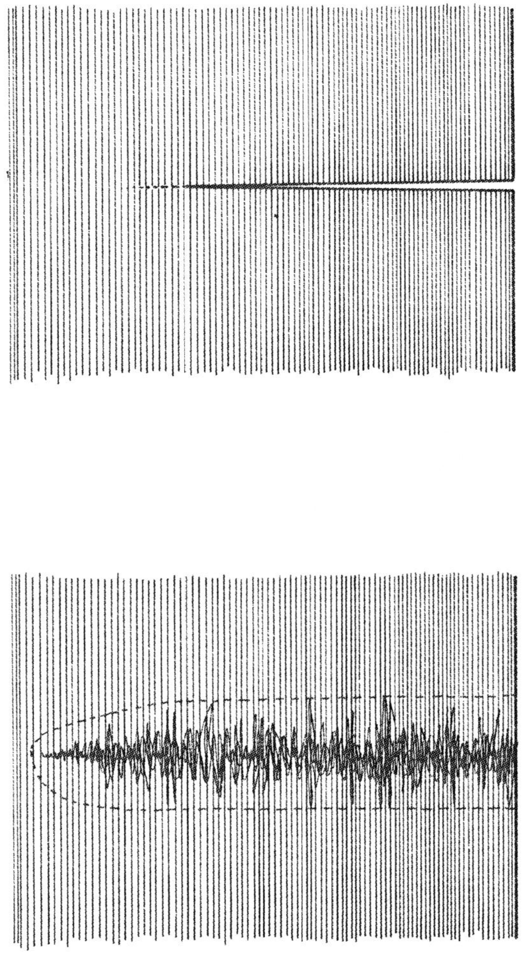
Figure 21 The probability (P_i) of a given number of fibre breaks in boron-aluminium as a function of applied stress.



(A)

(B)

Figure 22 Cracks or notches the same order of size as the fibre diameter: (A) good fibre-matrix bond, and no significant delamination; (B) with delamination.



(A)

(B)

Figure 23 Cracks or notches which are very much larger than the fibre diameter: (A) good fibre-matrix bond and no significant delmaination; (B) with delamination.