Unstable Crack Length, Effective Specific Surface Energy and COD Concept in the Brittle Fracture of Structural Steels

Albert Kochendörfer and Horst Dieter Schulze
Max - Planck - Institut für Eisenforschung, Düsseldorf, Germany
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ABSTRACT

The theory of brittle fracture of Stroh and Cottrell gives an unstable crack length of about 1/10 of the grain size, whilst stable crack lengths of at least one grain size are observed. Furthermore the calculated surface energy is about 1/50 of that evaluated from K_{Ic} values. By an improvement of the theory the first inconsistency becomes removed and both surface energies become equal. These results, allowing for the evaluation of K_{Ic} from measurements in the macroscopic brittle fracture range, are in accordance with the COD concept and enable a physical interpretation of this concept and of the fracture toughness.

1. Introduction

The dislocation theory of the brittle fracture [1,2] at first enabled to describe quantitatively correct the influence of the grain size d on the embrittlement transition temperature T_t. Qualitatively T_t increases with increasing d. The "classical" description has predicted the opposed dependence of $T_t(d_t)$ [3].

The dislocation theory has, however, some consequences not in accordance with experimental results or to be considered hitherto so. This concerns the fracture stress $\sigma_f=\sigma_{ft}$ at $T = T_t$ in dependence of $d = d_t$ [4]. But the results of recent investigations [5] indicate that at an accurate measurement of the test temperature and therefore at an accurate destination of Tt the experimental results agree with the theoretical conclusions, and that the results obtained in [4] are not in contradiction to these conclusions if the scattering of the points (σ_{ft}, d_t) is taken into account. But the fact that the theoretically predicted stable crack lengths are smaller by one order of magnitude than the observed ones, covering one or several grain sizes [6], is an inconsequence of the theory.

In this paper shall be shown that this inconsistency can be removed by an unconstrained improvement of the theory. At the same time results an (effective specific) surface energy γ of the same magnitude as obtained from the measured values of the fracture toughness K_{Ic}. These results are shown to be in accordance with the COD concept.

2. Fundamentals and results of the existing theory

The characteristic features of the theory and the conditions of applying them are discussed in [7c]. It is assumed that the dislocations coming from a source pile-up at a grain boundary and that the high stress field in the neighborhood of the pile-up either activates a source in this neighborhood by its shear stress component or overcomes the cohesive forces and generates a crack by its normal stress component. In the first case gliding continues and an equation for the yield stress σ_y results, in the second case a brittle fracture is initiated if the crack is unstable and an equation for the fracture stress σ_f results. Both possibilities are at first considered independently and then it is verified at which temperature $T = T_t$ the transition from brittle fracturing to plastic yielding, determined by $\sigma_f = \sigma_y$, occurs for a given grain size $d = d_t$. If σ_y is considered to be the macroscopic yield stress, at which full scale yielding has just taken place, T_t is the macroscopic brittle to ductile fracture transition temperature. In this paper all quantities and numerical values relate to this temperature, but for simplicity of denotation the index t is omitted. As far as it concerns the problems to be treated the conditions are the same for unnotched and notched specimens. Therefore the equations are only given for unnotched tensile specimens and the numerical values for a mild structural steel. As the equations are obtained for distinct models which not necessary must be true exactly the numerical values obtained by them are uncertain within a factor of the order of 1. For this reason the calculated numerical values are rounded.

According to Stroh [1] the energy U^n of a crack with length 2 l generated by a piled-up group of n dislocations is besides an additive constant

$$U = - \frac{[illegible]}{4\pi(1-\nu)} [illegible] \qquad (1a)$$

with b the Burgers vektor of the dislocations (= $2.5 \cdot 10^{-7}$ mm), G the shear modulus (= [illegible]$\cdot 10^4$ N/mm^2 = [illegible]$\cdot 10^{11}$ dyn/cm^2), ν Poisson's ratio (= 1/3). Cottrell [2] has furthermore taken into account the work U^σ done by the applied stress and the surface energy U^γ . For the first term he has used the Griffith formula for a slit type crack. A crack generated by a pile-up [illegible], however, a wedge type crack for which U^σ is four times larger if its length is denoted by 2 l, as revealed by the crack dislocation theory [7b, [illegible]]. It is

$$U^\sigma = - \frac{2\pi(1-\nu)\,\sigma^2 l^2}{[illegible]} \quad , \quad U^\gamma = [illegible] \qquad (1b,c)$$

From the condition $\partial U / \partial$ [illegible] = 0 with U = [illegible] + [illegible] + [illegible] follows for the critical unstable crack length 2 l^c and for the fracture stress σ_f [illegible] :

$$2\, l^c = \frac{[illegible]}{\pi(1-\nu)\,\sigma_f^2} \quad , \quad [illegible] \qquad (2a,b)$$

If no tensile stress is applied but only a shear stress in the glide plane of the dislocations the crack length 2 l^n is

$$[illegible] = \frac{[illegible]}{[illegible](1-\nu)} \qquad (3a)$$

If no dislocations are [illegible] in the glide plane the crack becomes unstable at the [illegible] length

$$[illegible] = \frac{[illegible]}{\pi(1-\nu)\,\sigma_f^2} \qquad (3b)$$

Therefore under the combined action of dislocations and tensile stress:

$$2\, l^c = \frac{2\, l^\sigma}{2} = 2\,(2\, l^n) \qquad (4)$$

Additionally the shear stress τ_f in the glide plane must fulfill the condition

$$\tau_f = \tau_{fi} + n\, b\, G / L \qquad (5a)$$

that the piled-up group with n dislocations and of the length L can be formed. As this happens in a favorable orientated grain for which $\tau \sim \sigma/2$ equation (5a) may be written as

$$\sigma_f = \sigma_{fi} + 2\, n\, b\, G / L \qquad (5b)$$

According to the discussions in [7c]:

$$L = 4\, d/3, \qquad (6)$$

therefore:

$$\sigma_f = \sigma_{fi} + 3\, n\, b\, G / 2\, d \qquad (7)$$

By elimination of n in the equations (2b) and (7):

$$\sigma_f(\sigma_f - \sigma_{fi}) = k_f^2 / d \quad , \quad k_f = \sqrt{3\, [illegible]} \qquad (8, 8a)$$

According to the measurements in [9] and [10] for a mild steel:

$d_t = 5 \cdot 10^{-2}$ mm, T_t = 90 K, $\sigma_f = 7.5 \cdot 10^2$ N/mm^2 = $7.5 \cdot 10^9$ dyn/cm^2 (9a,bc)

$\sigma_f - \sigma_{fi} = 2.5 \cdot 10^2$ N/mm^2 = $2.5 \cdot 10^9$ dyn/cm^2 (9d)

and independent of temperature

$$k_f = 90 \text{ N mm}^{-3/2} = 3 \cdot 10^{8} \text{ dyn cm}^{-3/2} \qquad (9e)$$

The following numerical values are obtained: From equation (2b):

$$\gamma / n = 0.94 \cdot 10^{-4} \text{ N/mm} = 94 \text{ erg/cm}^2 \qquad (10a)$$

from equation (7):

$$n = 420 \qquad (10b)$$

from equation (8a)

$$\gamma = 4 \cdot 10^{-2} \text{ N/mm} = 4 \cdot 10^{4} \text{ erg/cm}^{2} \qquad (10c)$$

from equations (4) and (3a) or (3b):

$$2\, l^{c} = 3 \cdot 10^{-3} \text{ mm} = d/16 \qquad (10d)$$

The value (10c) of γ follows also from the equations (10a,b).

The value of n depends somewhat on the grain size, approximatively as $\sqrt{d}$, but k_f is independent of d and therefore γ and $2\, l^c/d$. According to (10d) each crack longer than about d/10 should be unstable. Tetelman [11] assumes that such a crack propagates to the next grain boundary at which it is arrested because the surface energy becomes higher. Aurich [12] points out that such a crack could grow by dislocations coming from glide bands in the grain flowing into the crack as proposed by Orowan. But the equation of Griffith type for the fracture stress resulting in both cases is not in accordance with the experimental results [5]. Furthermore it has to be taken into account that the condition $\partial U / \partial l = 0$ has also stable solutions for stresses lower than the fracture stress and of lengths smaller than $2\, l^c$. Such cracks should be observed at least in some grains. This is, however, not the case, also if a specimen is loaded for some time just below the fracture stress and then unloaded [5]. For this reason the observation of cracks only with lengths of the grain size is not consistent with the consequences of the theory. It shall now be shown that the theory can be improved in such a way that it accounts for the observed crack lengths.

3. Improvement of the theory

3.1. Generation of a crack

One viewpoint is already regarded in the theory of Stroh [1] who has shown that a crack can only be generated by a piled-up group of dislocations if the shear stress has at least a value given by the equation

$$\tau_g = \tau_{gi} + 12\, \gamma_g / n_g\, b \qquad (11a)$$

which by introducing the tensile stress may be written as

$$\sigma_g = \sigma_{gi} + 24\, \gamma_g / n_g\, b \qquad (11b)$$

The index g shall refer to the generation process. Equations (3a) and (7) are likewise valid for this process:

$$2\, l_g^n = \frac{n_g^2\, b^2\, G}{8\pi(1-\nu)\,\gamma_g} \quad , \quad \sigma_g = \sigma_{gi} + \frac{3\, n_g\, b\, G}{2\, d} \qquad (11c,d)$$

From equations (11b,d,c):

$$n_g^2\, b^2\, G / \gamma_g = 16\, d \quad , \quad 2\, l_g = d \qquad (12a,b)$$

If therefore with increasing stress this has reached the value at which a crack can be generated, the crack has a length of the grain size, smaller cracks are not possible. This result, hitherto not taken into consideration, accounts for the fact that smaller cracks have not been observed and removes the inconsistency (10d).

If a crack has become generated the pile-up disappears and its back stress on the dislocation source relaxes. Therefore at a stress lower than σ_f further dislocations can be injected in the crack leading to a larger width than $n_g b$ and a larger length than $2\, l_g$ of it until it becomes unstable under the action of the tensile stress σ. The actual unstable length depends on the surface energy γ_p for the propagation and this itself on the plastic zone forming at the crack tip. For a sufficiently small γ_p the crack is unstable immediately with its generation, for a sufficiently high γ_p further dislocations must be injected in the crack until it reaches the unstable length. Before discussing this problem the values of γ_g and n_g shall be estimated. It may be assumed that the fracture stress σ_f is not essentially higher than the generation stress σ_g. Putting $\sigma_g = \sigma_f$ in equation (11d) with the values (9a,d):

$$n_g = 420 = n \tag{13a}$$

and from equation (12a):

$$\gamma_g = 1.1 \cdot 10^{-3}\ N/mm^2 = 1.\ 1 \cdot 10^{3}\ erg/cm^2 \tag{13b}$$

Therefore γ_g just agrees with the intrinsic surface energy. This is a reasonable result then the cracking of the crystal lattice can occur only by overcoming of the cohesive forces and needs no plastic deformations. These set in not before the crack is generated at its tip. With this result the assumption $\sigma_g = \sigma_f$ is justified then for a smaller σ_g equation (11d) would give a smaller value of n_g and then equation (12a) a smaller value of γ_g which is not possible. Therefore equation (11d) may be written as

$$\sigma_f = \sigma_{fi} + 3\ n\ b\ G\ /\ 2\ d \tag{14}$$

3.2. Propagation of the crack

Under the influence of the tension stress σ the crack become unstable. At this moment it shall contain n_p dislocations and the surface energy shall be γ_p. The course of the calculation is at first the same as in section 2, in the equations (1a-c) γ is to replace by γ_p, however. According to equations (2a,b) is now:

$$2\ l_p^c = \frac{2\ l_p^\sigma}{2} = \frac{\gamma_p\ G}{\pi(1-\nu)\ \sigma_f^2}\ , \quad \sigma_f = \frac{2\ \gamma_p}{n_p\ b} \tag{15a,b}$$

As σ_f has a fixed experimental value, from equations (2b) and (15b):

$$\gamma_p/n_p = \gamma/n \quad \text{or} \quad \gamma_p/\gamma = n_p/n = z \tag{16a,b}$$

with the numerical value (10a) of γ/n. Equation (7) cannot be used for n_p then it is valid only as long as the pile-up exists, i.e. for $n = n_g$ according to equation (14). From the equations (14) to (16b) result the equations (8) and (8b). The latter may be written as

$$k_f = \sqrt{3\gamma_p G/z} = \sqrt{3\gamma_p G\ n/n_p} \tag{17}$$

This result indicates also that $\sigma_g = \sigma_f$ must be put within the accuracy of the equations then otherwise a contradiction would arise. As no longer exists an independent equation for the separate evaluation of n_p and γ_p, the observation of the stable cracks in fractured specimens must be used for this evaluation. Assuming that the most frequently observed cracks with lengths of d and 2 d are stable ones but that cracks with lengths $\gtrsim$ 3 d are already unstable and in a few cases only observed because of the unloading after the fracture, it may be put

$$2\,l_p^c = 3d = 0.15 \text{ mm} \tag{18a}$$

i.e.

$$z = 48 \tag{18b}$$

From equations (10b,c) then follows

$$n_p = 2\cdot 10^4 \quad , \quad \gamma_p = 2 \text{ N/mm} = 2\cdot 10^6 \text{erg/cm}^2 \tag{18c,d}$$

This value of γ_p is equal to the value γ_{Ic} evaluated from K_{Ic}. For a steel of the kind considered here is $K_{Ic} = 0.9\cdot 10^3 \text{Nmm}^{-3/2}$ for the grain size (9a) and at temperatures in the neighborhood of the transition temperature (9b) [13] . It follows

$$\gamma_{Ic} = \frac{K_{Ic}^2}{2\ E} = 2\text{N/mm} = 2\cdot 10^6 \text{ erg/cm}^2 \tag{19}$$

Therefore within the accuracy of the numerical values:

$$\gamma_p = \gamma_{Ic} \tag{20}$$

4. Discussion

No doubt, by the present improvement of the theory the inconsistency between the predicted and observed stable crack length is removed. The reason for this inconsistency is that the crack has been considered as one with an arbitrary length whilst the shortest length to be generated is one grain size. A further satisfactory result is the equality of γ_p and γ_{Ic}. With regard to this an objection could be made: As the length $2l_p^c$ of the generated crack is small compared with the length 2a of the slit + fatigue crack of fracture toughness specimens, e.g. 2a=25mm for 1 CT specimens, it might be thought that the length Δl_p^P of the plastic zone at the tip of the former crack should be small compared with that length Δl_{Ic}^P of the latter. This assumption is, however, not correct. The linear elastic fracture mechanics appraoch (LEFMA) is only applicable if $\Delta l_{Ic}^P \ll 2a$ and then is $\sigma_f < \sigma_y$. But

for the fracture at the macroscopic brittle frature transition temperature for which $\sigma_f = \sigma_y$ it may be $\Delta l_p^P \sim 2\, l_p^c$. Indeed, by use of the COD concept this can be verified. The crack opening displacement δ^c is according to equation (15b) if σ_f is replaced by σ_y:

$$\delta_p^c = n_p\, b = 2\,\gamma_p/\,\sigma_y \tag{21a}$$

The LEFMA gives [7b]

$$\delta_{Ic} = K_{Ic}^2/E\,\sigma_y = 2\,\gamma_{Ic}/\,\sigma_y \tag{21b}$$

If according to the COD concept the initiation of the fracture by the instability of the crack shall be determined by a fixed value of δ, it follows

$$\gamma_p = \gamma_{Ic} \quad , \quad \delta_p^c = \delta_{Ic} = 5\ \mu\text{m} \tag{22a,b}$$

Equation (22a) expresses just the result (20). If the COD concept would have been put at the beginning, the values (18a,b) would have resulted. Applying the LEFMA the length of the plastic zone becomes

$$\Delta l_p^P = \pi(1 + \nu)\ G/\ 2\,\sigma_y^2 \simeq 2l_p^c = 3d \tag{23}$$

Under the macroscopic brittle fracture conditions the length of the plastic zone is therefore approximately equal to the unstable crack length.

Finally it can be said that the present condiderations are in accordance with the COD concept and that they enable to evaluate with a high accuracy the fracture toughness of a material from measurements in the macroscopic brittle fracture range. This is because the propagation of an existing crack in K_{Ic} specimens does not occur by a contineous fracturing of the crystal lattice immediately at the the tip of the crack but by a subsequent generation of new cracks by glide bands originated at the tip. Then the meaning of the COD concept is clear: it states that at the considered opening mode I and for a given grain size the generated crack becomes unstable at a certain fixed opening $n_p b$ and length $2 l_p^c$, independent of the specimen and notch geometry, i.e. of the local stress state. The geometry determines the critical value of the applied stress, i.e. the fracture stress. This varies from a value equal or higher than the uniaxial yield stress σ_y for a brittle fracture with preceeding full scale yielding to a value lower than σ_y for a brittle fracture with preceeding small scale yielding, i.e. under K_{Ic} conditions. Generally n_p and l_p^c depend on the surface energy and therefore on the temperature. For the steel considerd here, however, γ_p is independent of the temperature within the brittle fracture temperature range [9, 10, 13]. The plastic zone combined with the propagation of a crack, determing γ_p, is then represented by the slide band by which the following microcrack is generated. Now, $b\tau$ is the force acting on the unit depth of a dislocation and n_p dislocations move over distance ~d. As at the same time the length of the crack increases by d, the work done during the propagation per unit of new crack surface is given by

$$\gamma = n_p b \sigma_f/2 \qquad (24)$$

According to equation (15b) is

$$\gamma = \gamma_p = \gamma_{Ic} = K_{Ic}^2/E \qquad (25)$$

as should be according to the above considerations. These represent therefore a physical interpretation of the COD concept and of the fracture toughness.

These ideas have been used to evaluate the grain size dependence of the fracture toughness [14]. A more detailed investigation of the fracture behavior in the range from the macroscopic brittle fracture to the fracture under K_{Ic} conditions is presented in [10]. These investigations shall be continued with CT specimens.

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ZUSAMMENFASSUNG

Die Theorie des Sprödbruchs von Stroh und Cottrell ergibt eine instabile Rißlänge von etwa 1/16 der Korngröße, während stabile Rißlängen von mindestens einer Korngröße beobachtet werden. Weiterhin beträgt die berechnete Oberflächenenergie nur etwa 1/50 der aus den K_{Ic}-Werten erhaltenen. Durch eine Verbesserung der Theorie wird die erste Unstimmigkeit beseitigt und beide Oberflächenenergien ergeben sich gleich groß. Diese Ergebnisse, die eine Bestimmung von K_{Ic} aus Messungen im Bereich des makroskopisch spröden Bruchs ermöglichen, sind im Einklang mit dem COD-Konzept und ergeben eine physikalische Interpretation dieses Konzepts und der Bruchzahigkeit.

RÉSUMÉ

La théorie de la rupture fragile de Stroh et de Cotrell donne des longueurs des fissures instables de 1/16 de la grandeur des grains tandis que des fissures stables au moins d'une grandeur des grains sont observées. En outre l'energie de surface calculée n'est plus que 1/50 d'elle obtenue de K_{Ic}. Par un amendement de la théorie la première contradiction devient liquidée et les deux energies de surface obtient la même valeur. Ces résultats, permettant de déterminer K_{Ic} à des mesures dans la domaine de la rupture fragile macroscopique, sont en accord avec le concept COD et conduisent à une interprétation physique de ce concept et de la tenacité de la rupture.

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