Dislocations and Cracks

B. A. Bilby
Department of the Theory of Materials, University of Sheffield

We consider the advance of a crack tip in a direction continuing the line of the crack in terms of the crack extension force, a concept first introduced by Irwin.[1] Let unit length of a crack tip move from ξ to $\xi + \delta\xi$ and let $\delta\varepsilon^{EL}$ and $\delta\varepsilon^{POT}$ be the changes in the elastic energy of the body and the potential energy of the loading system. Then, if $\delta\varepsilon^{TOT} = \delta\varepsilon^{EL} + \delta\varepsilon^{POT}$, the crack extension force G is

$$G = -\partial\varepsilon^{TOT}/\partial\xi \qquad (1)$$

Draw a surface Σ round the crack tip and regard the surroundings of Σ as the loading system; the energy $-\delta\varepsilon^{POT}$ entering Σ from its surroundings is the sum of $\delta\varepsilon^{EL}$, which is stored, and $G\delta\xi$, which is available at the tip to drive the crack. In an elastic body we regard the crack tip as an elastic singularity or inhomogeneity, or we represent the crack itself as an array of crack dislocations.[2,3] Thus we need the force on an elastic singularity, the general theory of which was developed in 1951 by Eshelby, using the elastic energy-momentum tensor.[4] If all sources of internal stress and inhomogeneities within a surface Σ are displaced by $\delta\xi_\ell$ then $-\delta\varepsilon^{TOT} = F_\ell\delta\xi_\ell$ where

$$F_\ell = \int_\Sigma P_{\ell j}\, dS_j \qquad (2)$$

and

$$P_{\ell j} = W\,\delta_{\ell j} - u_{i,\ell}\, p_{ij} \qquad (3)$$

$P_{\ell j}$ is the spatial part of the elastic energy-momentum tensor, W is the elastic energy density, p_{ij} the stress, and u_i the displacement; the comma denotes differentiation with respect to x_ℓ. Since $P_{\ell j,j} = 0$ the integral is independent of the form of Σ, provided the singularities remain inside it. For a crack along the x_1 axis $G\delta\xi_1$

is the net energy available, $-\delta\varepsilon^{TOT}$, when we make a small cut at the tip to extend it $\delta\xi_1$. Thus (2) when $\ell = 1$ gives $F_1 = G$. The form (2) applied to a crack was also given in 1968 by Rice,[5] who denotes F_1 by J, and by Cherepanov.[6] By transforming (2) other path independent integrals for F_ℓ may be obtained,[4,7] one of which (with $\ell = 1$ and applied to a crack) was given by Sanders in 1960.[8] Eshelby[7,9] shows that (2) and these transforms are valid for non-linear materials undergoing finite deformations, with suitable definitions of the quantities involved.

In a linear elastic material, the field near the crack tip is completely characterised, for the appropriate mode of loading, by the stress intensity factors K_1, K_2 and K_3. If the loading modes are studied separately we can express the crack extension forces G_1, G_2 and G_3 in terms of the K_1, K_2 and K_3 values, obtaining $G_i = K_i^2/2M$; where for i = 1 and 2 M is $\mu/(1-\nu)$ for plane strain and $\mu(1+\nu)$ for plane stress and for i = 3, $M = \mu$. If $2\gamma_i$ is the energy necessary to form unit area of the two surfaces produced when the tip advances in any mode, a necessary condition for extension is that G_i should reach a critical value

$$G_{ic} = K_{ic}^2/2M = 2\gamma_i \qquad (4)$$

For $K_1 = (\pi c)^{\frac{1}{2}}\sigma$ we thus obtain the Griffith condition. Alternatively, we may following Barenblatt[10,11] use the moduli of cohesion $M_i^c = (\pi/2)^{\frac{1}{2}} K_{ic}$, for i = 1, 2, 3. These measures of fracture toughness are thus indirect ways of describing the effective surface energies of fracture G_{1c}, G_{2c} and G_{3c}. The necessary conditions for failure may also be expressed in terms of the critical densities of crack dislocations describing the crack shape.[12] For an ideal, sharp, brittle, crack we suppose that the necessary condition that G_1

reaches a critical value G_{1c} is also sufficient, since for any load there is always a bond near the tip on the point of rupture, in which therefore the stress is at the maximum of the stress-displacement curve for the bond. The stability of a reversible crack of this kind is like that of a pair of dislocations of opposite sign. Under a uniform load there is a critical separation of crack tips (or dislocations) for which there is unstable equilibrium. By adjusting the loading so that G diminishes or is constant with crack extension we can have stable or neutral equilibrium.

We can use this analysis to discuss a much wider class of fractures if, following Irwin[1] and Orowan[13] we interpret the surface energy of fracture to include the energy absorbed by all processes contributing directly to the tearing apart which occurs when the new surfaces are formed. However, although we can in this way discuss the important quasi-brittle and fast ductile fractures where the contribution to the effective surface energy of the work done in non-linear flows at the crack tip is dominant, it is not always easy to decide what part of the energy absorbed in the non-linear processes is to be included in γ. More understanding is possible if we use a model which takes explicit account of the non-linear processes. Then, however, we lose in general the simple characterisation of the crack tip field by the one parameter K. As many have emphasised[14,15,16] all we can then say is if two crack tips are loaded elastically to the same K value and the non-linearity is then switched on, then the non-linear forces will produce similar physical effects provided these are confined to a small enough region round the crack tip. It is in this limited sense only that we can still say that a critical K or G criterion means that extension occurs when the physical state of the two crack tips is the same.

Considerable effort is currently being devoted to the calculation of plastic-elastic fields near crack tips. There has been some progress using analytical methods[(17)] but the full treatment of any realistic problem is likely to involve extensive numerical computation, particularly if the effects produced by tip blunting and sliding off at sharp corners, important in relating the macroscopic plasticity to microscopic mechanisms are to be included.[(18-21)] Any simple model which can aid understanding is therefore valuable.

We may very generally represent the zone of non-linear deformation running from a crack tip by a continuous distribution of dislocations which have emanated from the tip itself or which have been created near it. The dislocations here are not necessarily those of crystal plasticity. They can give a formal description of the macroscopic deformation in this region which occurs by plastic, viscous or viscoelastic flow, and by brittle cracking and void formation. An account of a very simple theory based on this type of description, the Bilby-Cottrell-Swinden theory, was given by Cottrell in 1960[(22)] and developed to discuss a wide range of problems in fracture.[(23-33)] Though other arrangements have been discussed,[(29)] including the representation of the relaxation by large discrete dislocations,[(34,35)] in the original treatment the relaxation takes place by the formation of an array of dislocations collinear with the crack tip. Mathematically this picture is equivalent to the removal

of the singularity at the crack tip by the introduction of a region of constant stress at the end of the crack. Dugdale[(36)] used this procedure to estimate the length of the plastic zones round slits in thin sheets (but not to discuss fracture). Leonov and Onyshko[(37)] report that Vitvitski and Leonov[(38)] have used the method in a theory of brittle fracture; it is also closely related to Barenblatt's work.[(3,10,11)] Finally, it is salutary to note that a similar idea was used in 1933 by Prandtl.[(39)] The theory has been extended to include the effect of notches[(40-42)] and has been widely used[(3,31)] and elaborated[(43,44)] in discussions of fracture, particularly in post yield fracture mechanics.[(45-47)] It has also been applied to fatigue.[(48-53)] The model leads to the relations[(23)]

$$R/c = (a-c)/c = \sec(\pi\sigma/2\sigma_1) - 1 \qquad (5)$$

$$\phi(c)/c = (4\sigma_1/\pi M)\, \ell n\,(a/c) \qquad (6)$$

where R = a-c is the extent of the dislocation array near a crack of length 2c in an infinite medium and σ is the applied stress. In mode 3, these relations are valid also for a surface crack of length c, and Howard and Otter[(54)] have shown that these same functions, multiplied by numerical factors, represent R and $\phi(c)$ approximately for a similar relaxed surface crack in mode 1. Equation (7) predicts the observed extent of plastic zones surprisingly well.[(36,55,56)] R and $\phi(c)$ are also rough estimates of the maximum extent of the plastic zone (not directly ahead of the crack in plane strain), and the crack tip displacement calculated from plasticity theory.[(23,27)] For example, $\phi(c)$ in the small scale yielding approximation is $\alpha K^2/E\sigma_1$ with $\alpha = 0.89$; estimates based on plastic elastic calculations give $\alpha = 0.717$,[20] 0.613[5] and (from finite elements) 0.425.[19]

The DBCS model provides a system in neutral equilibrium under a uniform applied load[3,57,58] which represents relaxation due to non-linear forces at the crack tip. In applying it to discuss fracture however we use the dislocation array at the crack tip to provide also a two-parameter model of the effective surface energy γ of the fracture process. The expression for γ is $\gamma = \frac{1}{2}\sigma_1\phi_c$, where ϕ_c is a critical value of $\phi(c)$. It is assumed that when $\phi(c) = \phi_c$ the crack will run and failure ensue; thus a critical displacement criterion is used. We may set up an integral equation[27] to describe the deformation of a narrow layer of material in the plane of the crack, whose strength $\sigma_1(\phi)$ depends on the relative displacement ϕ which it undergoes. Taking $\phi(x) = 0$ for $|x| > a$, the dislocations of the BCS model may be regarded as the solution of this equation for an artificial rectangular law of force $\sigma_1(\phi)$, where σ_1 rises from zero to σ_1 at $\phi = 0$, and then remains constant until $\phi = \phi(c)$, when it drops to zero. When the relative displacement $\phi(c)$ at the crack tip reaches ϕ_c, we can regard the crack as about to extend. Moreover, the work to fracture unit area of the layer is

$$2\gamma = \int_0^\infty \sigma_1(\phi)\, d\phi = \sigma_1\phi_c \qquad (7)$$

In this way we obtain the critical displacement criterion[22,23] for failure ($\phi(c) = \phi_c$) and the expression $\sigma_1\phi_c$ for the work of fracture 2γ. When $\sigma \ll \sigma_1$, the relations (5) and (6) together with $2\gamma = \sigma_1\phi_c$ give the Griffith criterion; moreover, under these conditions the theory coincides with that of Barenblatt. Generally, (5) and (6) give the fracture stress σ_f[27,30] as

$$\sigma_f/\sigma_1 = (2/\pi)\cos^{-1}\{\exp(-c^*/\pi c)\} \qquad (8)$$

where $c^* = M\phi_c/4\sigma_1$. The condition $c \simeq c^*$ defines the crack length at which the material becomes notch sensitive.[27] When $c \gg c^*$, (8) reduces to the Griffith condition and we have a "low stress" failure with $\sigma_f \ll \sigma_1$. For $c < c^*$, σ_f approaches σ_1 the strength of the layer ahead of the crack.

By suitably interpreting the parameters σ_1 and ϕ_c a wide class of fractures can be discussed. We note first that we can identify two fundamental mechanisms of fracture; the separation of surfaces brittle on an atomic scale, as in crystalline cleavage, and failure by a ductile sliding off process, which does not involve any true cracking. Usually we meet some combination of these processes. Moreover, the scale on which they operate is of great importance. The engineer's greatest concern is with low stress catastrophic failures, and these we can understand if we note[24] that two modes of fracture can be distinguished. In a stable non-cumulative or non-localised mode, the dislocations spread rapidly throughout the whole net section of the material much faster than the crack, which can then only advance by continuing this process. Such a mode is typified by ductile fractures in which the material fails by macroscopic sliding off rather than by true cracking. The non-linear flow cannot become unconstrained without spreading throughout the whole net section. The stress σ_1 must be reached throughout, and as long as this mode persists we have $\sigma_f \sim \sigma_1$ and a condition which is not notch sensitive. An unstable cumulative or localised mode occurs when a similar localised set of dislocations moves ahead of the crack as it passes through the material. The crack can advance without dislocations spreading throughout the net section; we have a "low stress" fracture with $\sigma_f << \sigma_1$, $c >> R$ and $R \sim \pi c^* \sim E\phi_c/2\sigma_1 \sim E\gamma/\sigma_1^2$. Such fractures can occur whenever $c \gtrsim c^* \sim R/\pi$, and the condition established is a notch-sensitive one. The magnitudes of c^* and R may vary enormously, however, for they are governed by σ_1 and ϕ_c (with $\gamma = \frac{1}{2}\sigma_1\phi_c$). An example of a localised mode is ideal brittle fracture. Similar mechanically are the important <u>discontinuous</u> modes where small cracks or holes form ahead of the main crack and the bridges between them then fail in a ductile manner. Here we have a mixture of the two mechanisms of fracture. Ductile materials fail[27] not because their ductility is exhausted, as is sometimes said, but because they lose the capacity to harden, so that the non-linear flow becomes concentrated and large strains occur. Such large strains are possible whenever free surfaces allow large geometry changes, as at blunting tips, or, as here, in the internal necks between cracks and voids. These necks are like the atomic bonds in brittle fracture, though on a vastly different scale. We expect the heterogeneities in the material (grain size, inclusions) to generate the small cracks or holes ahead of the main fracture and ϕ_c to be of the order of their spacing. Other localised modes occur when non-linear deformation may spread right across the specimen because it is small in one dimension. For example[27,28] the mode 1 plane stress ductile necking of thin sheets, the mode 3 ductile tearing of thin sheets, and the 45^o shear mode in steel plates which combines a component of sliding off with plastic expansion of holes in the shear zones. Typical values of σ_1, ϕ_c and γ are[27]

	σ_1 dyne cm^{-2}	ϕ_c cm	$R = \pi c^*$ cm	γ erg cm^{-2}
Brittle	4×10^{11}	5×10^{-9}	10^{-8}	10^3
Discontinuous ductile-cleavage	2×10^9	10^{-2}	5	10^7

We see that the notch sensitivity of the discontinuous mode begins only for notches or cracks of macroscopic size, while the brittle solid is weakened by the smallest notch. In very large specimens, and in large structures in practice, failure may occur well below general yield. This is because ϕ_c can be accommodated by plastic flow at the crack tip before general yielding occurs, and the equation (8) predicts very well the reduction of failure stress which occurs with increasing size.[27,28] By using the formulae of the BCS theory for a plate of finite width,[30] Bilby and Swinden[29] and Smith[59] have shown how it may be used to predict the range of notch sizes which may cause catastrophic failure in a structure whose size, yield stress and loading are given; and also how to find the safe loading of the structure for a given notch size. Extensive numerical calculations for other geometries have been presented recently in a similar way by Hayes and Williams.[60]

This size effect appears again in tests on tough laboratory specimens, which do not fracture until well above general yield. Heald, Spink and Worthington[32] have recently applied the BCS theory to this situation, by setting in equation (8) $\sigma_1 = \sigma_u$ where σ_u is the ultimate tensile strength, and writing $2\gamma = \sigma_u\phi_c = K_{1c}^2(1-\nu)/2\mu$. They also define an apparent fracture toughness by

$$K_A = (\pi c)^{\frac{1}{2}} \sigma_f = 2\sigma_u \left(\frac{c}{\pi}\right)^{\frac{1}{2}} \cos^{-1} \quad \exp\left[-\frac{\pi K_{Ic}^2}{8\sigma_u^2 c Q^2}\right] \qquad (9)$$

where Q is a compliance factor introduced to allow for different geometries. This formula gives reasonable agreement with post yield fractures in a wide range of materials. The effect of a notch is also included by using a result of Smith.[40] It is suggested that by this analysis K_{1c} values may be derived from "invalid" ASTM tests and that the fracture behaviour of large structures may be predicted from small scale tests; also that the use of fatigue cracked specimens may not be necessary. In a paper presented to this congress[33] these authors show that these equations also agree well with a recent collection of data by Orange[61] and provide a more consistent method of correlating it. The effect of temperature on K_{1c} for an aluminium alloy is shown to be due to the variation of σ_u.

The critical displacement appropriate for the initiation of fracture from a notch will depend on the notch radius ρ and on the experimental conditions.[28] The initiation of microcracks is we believe due to the stresses set up by groups of dislocations representing twinning and slip. There are a number of specific mechanisms,[62] but the detailed correlation of their operation with specific notch geometries and experimental conditions and with macroscopic deformation fields is still incomplete. In steels at about room temperature the crack often begins in a ductile manner and then changes to brittle or semi-brittle cleavage as it spreads. The main effect here is probably the reduction of γ with crack velocity due to the high sensitivity of the yield stress to strain rate.[28] There is currently much interest in this period of initial

stable ductile growth, and the transition to fast fracture,[20,63-66] and here again the DBCS model through its extensions by Cherepanov[6,67] and Wnuk[68,69] provides some understanding.

It is instructive to evaluate the energy release rate $F_1 = J$ for the DBCS model by integrating the energy-momentum tensor round the non-linear layer ahead of the crack; we find generally[9,17] that

$$F_1 \equiv J = \int_0^{\phi(c)} \sigma_1(\phi)d\phi = \sigma_1\phi(c) \qquad (10)$$

This we can also see if we note that F_1 is the resultant force in the 1-direction on all the singularities inside. On each dislocation the elastic field must produce a force $+\sigma_1 b$ to balance that $-\sigma_1 b$ due to the resistance σ_1, so that for n dislocations $F_1 = n\sigma_1 b = \sigma_1\phi(c)$, since $nb = \phi(c)$. There is now no force on the crack tip itself but $\sigma_1\phi(c)\delta\xi$ is the energy released by the elastic surroundings if the whole system of crack and dislocations is displaced in the 1-direction by $\delta\xi$. As is obvious from the above argument, this energy is just equal to that absorbed in advancing all the dislocations by $\delta\xi$ against the resistance σ_1. Although the displacement $\delta\xi$ is not an equilibrium displacement we might perhaps have anticipated this result from the fact that the DBCS model gives a crack in neutral equilibrium.[3,57,58] However, as Swinden[57] shows, the term $\sigma_1\phi(c)$ is not the whole of the available elastic energy when the crack advances in an equilibrium manner, there is an additional term, which is again absorbed by work required to advance the dislocations. The energy released from the surroundings is also exactly absorbed by the plastic work[70] in the dynamic DBCS model treated by Atkinson.[71] In general we have $F_1 = (4\sigma_1^2 c/\pi M)\ \ell n\ \{\sec(\pi\sigma/2\sigma_1)\}$ and this when σ/σ_1 is small reduces to the linear elastic value $F_1^L = K^2/2M$ ($K^2 = \pi c\sigma^2$). F_1/F_1^L increases from unity with increasing σ/σ_1, so that the crack appears to the far elastic field as if it were a longer one with its end somewhere in the yielded zone. This is a general result, not confined to the DBCS model.[9,17,64] However, we must use this interpretation with some caution, for the energy released is used in rearranging the dislocations, and this rearrangement must contribute to the actual fracture process if the calculated energy release is actually to be used to advance the crack.

When we take a more realistic model of the crack tip plasticity, it seems obvious that the crack and its plastic zone again forms a system in neutral equilibrium in the sense that the energy released is absorbed by plastic work. We can evaluate again the ratio F_1/F_1^L for the exact antiplane solutions of power law hardening which are available, taking the path of integration in the elastic region or along the elastic-plastic boundary, as has been done by Rice,[17] and we can make similar evaluations numerically in plane strain. In deformation plasticity the F_1 integral is path independent and gives the energy release rate for a crack in the artificial non-linear material which reproduces the stresses and shape displacements of the plastic-elastic problem. Indeed, it is reported[47] that Hayes shows the integral path independent also for a Prandtl-Reuss material. We might expect the integral taken for an incremental theory in any case to be approximately path-independent for the type of centred fan anticipated about the (non-blunted) crack tip.[71] However, the significance of these integrals requires further study, as the following argument shows. Representing the crack tip plasticity by a continuous distribution of dislocations, we see that $F_1\delta\xi$ is the

energy released when the crack <u>and all the dislocations in the plastic zone are displaced in the 1-direction by $\delta\xi$</u>. Probably some at least of this energy is again used up in moving the dislocations; moreover, it is not the energy released (and absorbed by the dislocations) in an equilibrium displacement of the crack and the plastic field. It is thus not obvious what the calculated energy release has to do with the crack extension; we can only fall back, as before, on an appeal to a comparison of the behaviour of different cracks for which the calculations give the same F_1 value.

We can obtain further insight by using the theory of continuous distributions of dislocations. This leads to the following (path dependent) integral giving the resultant force on all dislocations within a surface Σ[73]

$$Q_\ell = \int_\Sigma (W\,\delta_{\ell j} - P_{ij}\,\beta^E_{\ell i})\,dS_j \qquad (11)$$

Here $\beta^E_{\ell i}$ is the elastic distortion tensor giving the incremental elastic displacement $du^E_i = dx_\ell \beta^E_{\ell i}$. Since $\beta^E_{\ell i} = u_{i,\ell}$ in the elastic region, Q_ℓ reduces to F_ℓ for a path outside the plastic region. It may however be shrunk to lie within the plastic region and can be evaluated when $\beta^E_{\ell i}$ is known there. For the small scale antiplane yielding from an edge slit discussed by Hult and McClintock[74] it gives, for a boundary defined by $r = \rho(\theta)$ within the plastic region

$$Q_1 = \frac{k^2}{\mu}\int_{-\pi/2}^{\pi/2} \cos\theta\,\rho(\theta)\,d\theta \qquad (12)$$

Taking $\rho(\theta) = \varepsilon\cos\theta$ we get $Q_1 = (\pi k^2\varepsilon/2\mu)$. This tends to zero as $\varepsilon \to 0$, so that, as in the BCS theory, we find no force on the crack tip itself. Note incidentally that $F_1 = \sigma_1\phi(c)$ is not a general relation; in the antiplane strain solutions the integrands in the integrals for F_1 and $\phi(c)$ differ by a factor $\cos\theta$, and in small scale yielding $F_1 = \frac{\pi}{4}\sigma_1\phi(c)$. In plane strain we have for symmetrical arrays of dislocations which have glided from the crack tip that $F_1 = \sigma_1\phi_2(c)$ only if $\theta = 45^\circ$; this relation also holds for the linear array of climbing dislocations.

With the Q_ℓ integral we can examine the assumptions used in the BCS fracture theory. We recall the important classification of modes of fracture into localised modes (of greatest practical interest) and non-localised modes. It is precisely in the localised modes that a similar distribution of dislocations moves along with the crack as the fracture proceeds. Moreover the forward movement of these dislocations actually represents the process of fracture, so that the work required to move them is the effective surface energy. If then we close the integral Q_ℓ down to enclose a narrow region ahead of the crack tip, we do surround a system of dislocations which moves forward with the crack itself, and Q_1 will give an energy release rate which may be set equal to 2γ. If $\sigma_1(\phi)$ is constant, then $Q_1 = \sigma_1\phi_c$, where ϕ_c is the critical displacement characterising the localised mode. This is the BCS theory.

It is clearly necessary for a complete model of the fracture process that we should set up the equation of motion of the crack. Although many important physical effects will be adequately described by a kinetic theory where the strain rate dependence of γ (and so the velocity dependence) is considered, a complete discussion must include inertia terms. We must use more care in discussing the energy balance about the crack tip when it is in motion.(9,75,76) If the instantaneous crack tip velocity is v,

the total inward rate of energy flow $I(\Sigma)$ across a surface Σ surrounding the tip and moving with it must equal the rate of increase of energy $\dot{\mathcal{E}}(\Sigma)$ stored within Σ plus the rate vG at which energy flows to the tip.

$$I(\Sigma) = vG + \dot{\mathcal{E}}(\Sigma) \qquad (13)$$

$I(\Sigma)$ is the rate at which the surroundings do work on the material inside Σ, $\int p_{ij}\dot{u}_j dS_j$, plus the net rate at which the interior of Σ acquires potential and kinetic energy by the convected flows at its leading and trailing boundaries. G may be expressed as a general path independent integral if the dynamic elastic field is a special one moving rigidly with the crack tip. For a general dynamic field, however, the integral is independent of the path only when $\Sigma \to 0$. Provided that the dynamic elastic field in the neighbourhood of the crack tip tends to a field which moves rigidly with the tip G may be written

$$G = \lim_{\Sigma\to 0}\int_\Sigma H_{1j}\, dS_j \qquad (14)$$

where $H_{1j} = (W + T)\,\delta_{1j} - p_{ij}\,\partial u_i/\partial x_1$ and T is the kinetic energy density. The integral (14) is path independent only when $\Sigma \to 0$. Atkinson and Eshelby(75) show that the uniformly expanding crack in plane strain(77,78) gives a G vanishing at the Rayleigh velocity and explain a different conclusion reached by Craggs.(79) For antiplane strain,(80) G vanishes at the shear wave velocity.

The energy balance about a moving crack tip has been discussed in terms of a dynamic cohesion modulus,(81,82) and in terms of the thermodynamic power balance.(6,83,84) To set up the equation of motion of a crack tip in an elastic solid it is necessary to find the elastic field which the tip moves arbitrarily and this was first done by Kostrov[81] and Eshelby.[85,86] Eshelby discusses in antiplane strain a finite stationary crack of length ℓ whose right hand tip (x_1) moves in an arbitrary way $x_1 = \xi(t)$ for $t > 0$. Freund[87] has extended the work by treating the semi infinite crack in plane strain. Eshelby shows that G depends on ξ and $\dot{\xi}$ but not on $\ddot{\xi}$, so that the crack has no inertia. Given the fracture energy as a function of ξ and $\dot{\xi}$ the equation of motion is

$$G(\xi,\dot{\xi}) = 2\gamma\ (\xi,\dot{\xi}) \qquad (15)$$

Again, G falls to zero at the shear wave velocity in antiplane strain and at the Rayleigh velocity in plane strain. In antiplane strain the velocity dependence of G arises from a factor $A^2(\dot{\xi}) = |(1 - \dot{\xi}/c)/(1 + \dot{\xi}/c)|^{\frac{1}{2}}$, where c is the shear wave velocity. $A^2(\dot{\xi})$ rises as $\dot{\xi}$ falls, and this behaviour may help to explain crack branching.[9] In a homogeneous medium the energy release rate must be 2γ just before branching and roughly 4γ immediately after. Any discussion of branching which neglects this point may violate the conservation of energy. The crack can however increase G as required by dropping its velocity. We can find the velocity which causes G just to double if the crack is suddenly halted; this gives a critical branching velocity of $\dot{\xi}_b = 0.6c$ in antiplane strain. Of course, the geometry and stress field at the tip will influence the details of the branching process, and branching may well often occur because the surface energies of the new paths are lower than that of the original. Nevertheless, the proper energy balance must always be considered. These ideas on the dynamic G factor have found an interesting application in the theory of flint knapping,[88] an ancient craft which has relevance to crushing, grinding, blasting, shattering and impact testing.

We have tried in this survey to show how a simple model can provide understanding of a wide class of fractures. Engineers will not need to be cautioned that the ideas of such a model must be critically assessed by experiment and testing before they can be applied to practice. Nevertheless it appears that certain aspects may be of some semi-empirical value. The full interpretation of fracture in particular materials and of the relation between the macroscopic continuum mechanics and the microscopic mechanics must await further experimental, mathematical and numerical analyses of the non-linear processes at the crack tip and of the energy balance involved there when the crack is in motion. Of these we hope to hear more in the specialist sessions of this meeting.

I am much indebted to Professor J. D. Eshelby, Dr. I. C. Howard and Dr. P. T. Heald for many valuable discussions, and to Mr. G. E. Cardew for assistance in computation.

References

(1) G. R. Irwin, 1948, In "Fracturing of Metals", ASM, Cleveland, p. 147.
(2) J. Friedel, 1956, "Les Dislocations", p. 214, Gauthier-Villars, Paris.
(3) B. A. Bilby and J. D. Eshelby, 1968, In "Fracture, Volume I", (H. Liebowitz, ed.), Academic Press, New York, p. 99.
(4) J. D. Eshelby, 1951, Phil. Trans. Roy. Soc., A244, 87.
(5) J. R. Rice, 1968, J. appl. Mech., 35, 379.
(6) G. P. Cherepanov, 1968, Int. J. Solids Structures, 4, 811.
(7) J. D. Eshelby, 1956, Solid St. Phys., 3, 79.
(8) J. L. Sanders, 1960, J. appl. Mech., 27, 352.
(9) J. D. Eshelby, 1970, In "Inelastic Behaviour of Solids", M. F. Kanninen (ed.), McGraw Hill, New York, p. 77.
(10) G. I. Barenblatt, 1959, PMM, 23, 434, 706, 893.
(11) G. I. Barenblatt, 1962, Advan. Appl. Mech., 7, 55.
G. I. Barenblatt and G. P. Cherepanov, 1961, PMM, 25, 46.
G. I. Barenblatt and G. P. Cherepanov, 1961, Prikl. Mat. Mech., 25, 752.
(12) E. Smith, 1965, Intern. J. Fracture Mech., 1, 204.
(13) E. Orowan, 1949, Rept. Progr. Phys., 12, 214.
(14) D. C. Drucker, 1971, In "Fracture, Volume II, (H. Liebowitz, ed.), Academic Press, New York, p. 473.

(15) J. R. Rice, 1967, In "Symposium on Fatigue Crack Propagation, 1966", STP 415, ASTM, Philadelphia.
(16) J. D. Eshelby, In "Fracture Toughness", 1968, ISI Publication 121, p. 30.
(17) J. R. Rice, 1971, In "Fracture, Volume II, (H. Liebowitz, ed.), Academic Press, New York, p. 192.
(18) N. Levy, P. V. Marcal and J. R. Rice, 1971, Nuclear Engineering and Design, 17, 64.
(19) N. Levy, P. V. Marcal, W. J. Ostergran, and J. R. Rice, 1971, Int. J. Frac. Mech., 7, 143.
(20) J. R. Rice and M. A. Johnson, 1970, In "Inelastic Behaviour of Solids" (M. F. Kanninen et al., ed.), McGraw-Hill, New York, p. 641.
(21) D. C. Drucker and J. R. Rice, 1970, Eng. Frac. Mech., 577.
(22) A. H. Cottrell, 1961, In "Symposium on Steels for Reactor Pressure Circuits", 1960, Special Report No. 69, Iron and Steel Institute, p. 281.
(23) B. A. Bilby, A. H. Cottrell and K. H. Swinden, 1963, Proc. Roy. Soc., A272, 304.
(24) A. H. Cottrell, 1962, In "Properties of Reactor Materials and the Effects of Radiation Damage", (D. J. Littler, ed.), Butterworths, London, p. 5.
(25) A. H. Cottrell, 1963, Proc. Roy. Soc., A276, 1.
(26) A. H. Cottrell, 1964, Proc. Roy. Soc., A282, 2.
(27) A. H. Cottrell, 1965, In "Fracture, Proceedings of First Tewksbury Symposium, 1963" (C. J. Osborn, ed.), Butterworths, London, p. 1.
(28) A. H. Cottrell, 1965, Proc. Roy. Soc., A285, 10.
(29) B. A. Bilby and K. H. Swinden, 1965, Proc. Roy. Soc., A285, 22.
(30) B. A. Bilby, A. H. Cottrell, E. Smith and K. H. Swinden, 1963, Proc. Roy. Soc., A272, 304.
(31) E. Smith, 1965, In "Proceedings of the 1st. International Conference on Fracture, Sendai, (T. Yokobori et al. eds.), Japanese Society for Strength and Fracture of Materials, Tokyo, Volume I, p. 133.
(32) P. T. Heald, G. M. Spink and P. J. Worthington, 1972, Mater. Sci. Eng., 10, 129.
(33) P. J. Worthington, G. M. Spink and P. T. Heald, 1973, In "Proceedings of 3rd International Congress on Fracture, Munich 1973, to be published.
(34) C. Atkinson and T. R. Kay, 1971, Acta Met., 19, 679.
(35) C. Atkinson and D. L. Clements, 1973, Acta Met., 21, 55.
(36) D. S. Dugdale, 1960, J. Mech. Phys. Solids, 8, 100.
(37) M. Ya. Leonov and L. V. Onyshko, 1961, Vopr. Mashinoved i Proch. v Mashinostr. Akad. Nauk Ukr. SSR, 7, 16.
(38) P. M. Vitvitskii and M. Y. Leonov, 1960, Vses. Inst. Nauchn. Tekhn. Inform. Akad. Nauk SSSR Pt. 1, p. 14.
(39) L. Prandtl, 1933, Z. f. angew. Math. und Mech., 13, 129.
(40) E. Smith, 1967, Proc. Roy. Soc., A299, 455.
(41) E. Smith, 1967, Int. J. Engng Sci., 5, 791.
(42) B. A. Bilby and P. T. Heald, 1968, Proc. Roy. Soc., A305, 429.
(43) B. V. Kostrov and L. V. Nikitin, 1967, Prikl. Math. Mekh, 31, 334.
(44) P. F. Arthur and W. S. Blackburn, 1968, In "Fracture Toughness of High Strength Materials", I.S.I. Special Publication 120, p.

(45) A. A. Wells, 1965, Brit. Welding Journ., 12, 1.
(46) F. M. Burdekin, D. E. W. Stone and A. A. Wells, 1965, In "Fracture Toughness Testing", ASTM, Philadelphia, p. 400.
(47) G. R. Egan, 1973, Eng. Fract. Mech., 5, 167.
(48) J. Weertman, 1966, Int. J. Frac. Mech., 2, 460.
(49) B. A. Bilby and P. T. Heald, 1968, Proc. Roy. Soc., A305, 429.
(50) P. T. Heald, 1968, Int. J. Frac. Mech., 4, 365.
(51) P. T. Heald and B. A. Bilby, 1968, In "Fracture Toughness of High Strength Materials", ISI Publication 120, p. 63.
(52) J. Weertman, 1969, Int. J. Frac. Mech., 5, 13.
(53) P. E. J. Flewitt and P. T. Heald, 1968, In "Fracture Toughness of High Strength Materials", ISI Publication 120, p. 66.
(54) I. C. Howard and N. R. Otter, private communication.
(55) G. T. Hahn and A. R. Rosenfield, 1965, Acta Met., 13, 293.
(56) A. S. Tetelman, 1964, Acta Met., 12, 993.
(57) K. H. Swinden, 1964, Ph.D. Thesis, University of Sheffield.
(58) T. Yokobori and M. Ichikawa, 1966, Rept. Res. Inst., 2, 21, Research Institute for Strength and Fracture of Materials, Tohoku University, Sendai.
(59) E. Smith, 1965, Proc. Roy. Soc., A285, 46.
(60) D. J. Hayes and J. G. Williams, 1972, Int. Journ. Frac. Mech., 8, 239.
(61) T. W. Orange, 1971, Eng. Fract. Mech., 3, 53.
(62) N. J. Petch, 1968, In "Fracture, Volume I", (H. Liebowitz, ed.), Academic Press, New York, p. 351.
(63) F. A. McClintock, 1968, J. Appl. Mech., 35, 363.
(64) F. A. McClintock and G. R. Irwin, 1965, In "Fracture Toughness Testing", ASTM STP 381, p. 84.
(65) F. A. McClintock, 1969, In "Physics of Strength and Plasticity", (A. S. Argon ed.), M.I.T. Press, p. 307.
(66) C. A. Berg, 1970, In "Inelastic behaviour of solids", (M. F. Kanninen et al. ed.), McGraw-Hill, New York, p. 171.
(67) G. P. Cherepanov, 1968, PMM, 33, No. 3.
(68) M. P. Wnuk, 1971, Int. Journ. Fract. Mech., 7, 217.
(69) W. G. Knauss, 1973, Appl. Mech. Rev., 26, 1.
(70) J. D. Eshelby, 1970, In "Inelastic behaviour of Solids", (M. F. Kanninen et al., ed.), McGraw-Hill, p. 635.
(71) C. Atkinson, 1967, Ark. Fys., 35, 469.
(72) I. C. Howard, private communication.
(73) B. A. Bilby and J. D. Eshelby, unpublished.
(74) J. A. H. Hult and F. A. McClintock, 1957, In "Proceedings of the 9th International Congress on Applied Mechanics, 8, 51.
(75) C. Atkinson and J. D. Eshelby, 1968, Int. Journ. Fract. Mech., 4, 3.
(76) B. A. Bilby, 1972, In "Amorphous Materials", (R. W. Douglas and B. Ellis ed.), Wiley, p. 489.
(77) K. B. Broberg, 1960, Ark. Fys., 18, 159.
(78) K. B. Broberg, 1964, J. Appl. Mech. 31, 546.
(79) J. W. Craggs, 1963, In "Fracture of Solids", Interscience, New York, p. 51.
(80) A. Austwick, 1968, Dissertation for M.Sc. (Tech.), University of Sheffield.
(81) B. V. Koshov, 1966, PMM, 30, 1042.
(82) G. I. Barenblatt, V. M. Entov and R. L. Saljanik, 1970, In "Inelastic behaviour of solids", (M. F. Kanninen et al., ed.), McGraw-Hill, p. 635.

(83) G. P. Cherepanov, 1967, PMM, 31, 476.
(84) C. Atkinson and M. L. Williams, 1973, Int. J. Solids Struct., 9, 237.
(85) J. D. Eshelby, 1969, J. Mech. Phys. Solids, 17, 177-
(86) J. D. Eshelby, 1969, In "Physics of Strength and Plasticity", (A. S. Argon ed.), M.I.T. Press, p. 263.
(87) L. B. Freund, 1972, J. Mech. Phys. Solids, 20, 129, 141.
(88) J. G. Fonseca, C. Atkinson and J. D. Eshelby, 1971, Int. Journ. Fract. Mech., 7, 421.