The Hugoniotelastic Limit (HEL) and Influencing Factors

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One of the results of experimental high-pressure shock loading experiments of the last years is to have realized, that the earlier assumption of hydrodynamic response of materials at pressures higher than their yield strength isn't correct. S. MINSHALL[1] confirmed by experimental shocking of steel, that a two-wave-configuration may exist. He found an almost pressure independent fast 'elastic' wave of constant peakpressure and a slower pressure dependent 'plastic' wave. Fig. 1 represents this in general. The elastic wave shows nearly constant, very slightly increasing velocity. But from the original state p_o it is by reasons of stability, discussed from M.H. RICE, R.G. MCQUEEN, and J.M. WALSH[2], impossible to reach p_{II} by a single Rayleigh-line. State p_{II} is only reached by an elastic wave from p_o to HEL and a second plastic wave from HEL to p_{II}, latter wave-velocity depending on pressure. In state p_{III} the elastic and plastic wave velocities coincide and a stable hydrodynamic pressure dependent single shock velocity is reached.

Fig. 1 may be interpreted as a force-compression-diagramm, the hugoniotelastic limit is to be understood as the limit of (shock-)elasticity and also as a limit between the stability of one- and two-wave shock-configuration. But the HEL doesn't coincide with the static yield strength. A.C. HOLT[3] and others pointed this out in a macroscopic model. A quite different explanation may be obtained assuming a brittle fracture model according to A.A. GRIFFITH[4] for brittle materials.

Ceramics in general are polycrystalline brittle materials consisting of statistically oriented well-bonded single crystals displaying in general an elastic anisotropy in respect to the crystal-directions. In fig. 2 the Young's modulus-body of sapphire is shown.

One may assume, that only a two-crystal-configuration with maximum and minimum Young's modulus respectively in critical configuration to the shock-direction in an isotropic medium is present. Furthermore it is assumed, that fracture occurs, if the differences of the compression-strains of the various oriented two single-crystals equal the ultimate strain of the polycrystal when compression strength $\sigma_{comp.pc.}$ is reached and failure occurs.

$$\varepsilon_{max.sc.} - \varepsilon_{min.sc.} = \varepsilon_{comp.pc.} \qquad (1)$$

Considering this, one may calculate the HEL for various models on the nature of the shock-pressure. For dense alumina the following solutions are obtained:

a) Material is shock compressed hydrostatically

$$HEL > 23\ \sigma_{comp.pc.} \qquad (2)$$

b) Material is shock compressed uniaxially, whereas Poissondilatation is allowed

$$HEL > 8\ \sigma_{comp.pc.} \qquad (3)$$

c) Material is shock compressed strictly uniaxially without any dilatation

$$HEL = \frac{E_{max.sc.}\ E_{min.sc.}}{(E_{max.sc.} - E_{min.sc.})E_{pc.}}\sigma_{comp.pc.} = 3{,}1\sigma_{comp.pc.} \qquad (4)$$

According to the experimental HEL-measurements of T.J. AHRENS, W.H. GUST and E.B. ROYCE[5)] for dense alumina equation (4) holds. Assuming almost spherical grains in the material this result may be unterstood as the compression-wave-velocity always exceeding the shear-wave velocity. It could be shown[6,7)], that equ. (4) holds for refractory materials, and also for metals, like W, Al, Fe, Cu, and Pb, this relation holds in spite of not fulfilling the suppositions of brittle fracture.

Recalculating $\sigma_{comp.pc.}$ from experimental HEL-measurements[8)] the dynamic value is obtained. This value in comparison to the static one is always higher for dense materials. But for materials with some porosity the

dynamic compression strength is diminished and lower in respect to the static one. The first case is readily understood owing to dislocation dynamics. The second case ($\sigma_{comp.dyn.} < \sigma_{comp.stat.}$) however may be understood by introducing an acoustic-rheological model, which is briefly outlined:

Assuming that a single discontinuity with diameter 2 R is small in comparison to the depth of the shock-rise (the assumption of an indefinitely steep shock is an oversimplification because a material with losses is a low-pass filter), two cases are to be considered

a) Discontinuity is fixed in the matrix by viscous forces. In this case the material shows a homogeneous behaviour by dynamic attack.
b) Shock interacts with the discontinuity and the interaction overcomes the fixing viscous forces and the discontinuity gets a movability in respect to the matrix.

The limit between a) and b) is given by Roth's number Ω, which corresponds to Reynolds number and is in principle the ratio of the energy of acceleration to viscous dissipation. Introducing a loss-tangent for Young's modulus $tg\delta = E''/E'$, for dimension reasons the unity of area-density $\bar{\varrho}$, pressure (HEL) p, and shock-impedance $I = \varrho_\infty u_s$, whereby ϱ_∞ is the density, and u_s the shock-velocity, for Roth's number[9)]

$$\Omega = \pi^2 R^2 I^2 / 3 Z \bar{\varrho}^2 p^2 tg\delta \qquad (5)$$

holds and Z is a factor for the type of the acoustic oscillator:

Z = 3/8 for pulsating sphere, Z = 1 for acoustic dipole.

In equ. (5) a strictly limiting frequency f_1 and Küpfmüller's step-response function is assumed.

E. SKUDRZYK[10)] calculated the motion of a discontinuity in respect to the surrounding medium as function of the density ratio of discontinuity and surrounding medium ϱ'/ϱ_∞ and Ω, see fig. 3 and[10, 11, 12)].

W. JUST, U. DWORAK and C.O. LEIBER[9)] have shown, that in alumina of different grade and porosity losses of Young's modulus arise, resulting from discontinuities at a frequency range of 50 to 100 Mc. This loss determines dynamic fracture and is strictly related to the grade and porosity of the material. In brittle materials it is assumed that fracture occurs, if discontinuities start to move ($\Omega \approx 0{,}1 - 1$).

The discontinuity dynamics can best be determined by experiments on explosives and glasses. Also the mechanism of antiknocking by decomposition of organic lead compounds to small sphere lead oxide in combustion engines can be unterstood by this model.

Moreover a purely elastic loss is to be expected in general, resulting from the elastic anisotropy of the single-crystals in the matrix. The shock-directions diverge and converge and interfere. This elastic loss is found in acoustic absorption by wave length similar to grain size[13]. As alumina single-crystals show only relatively little anisotropy, this effect isn't taken into account.

Since their are some experimental facts, that a material damage occurs by oscillations and thermo-shock resulting in pores, which are growing, it should be discussed, whether fatigue could be pointed out also by an acoustic-rheological model of the outlined manner.

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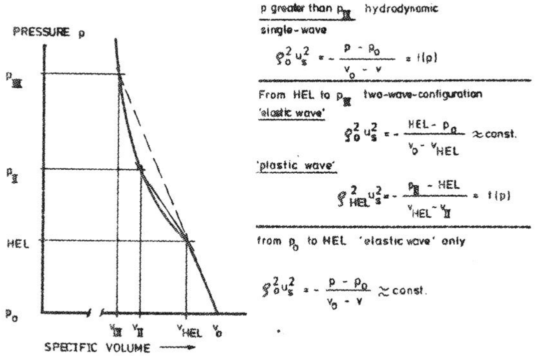

Fig. 1

Hugoniot-relation and HEL

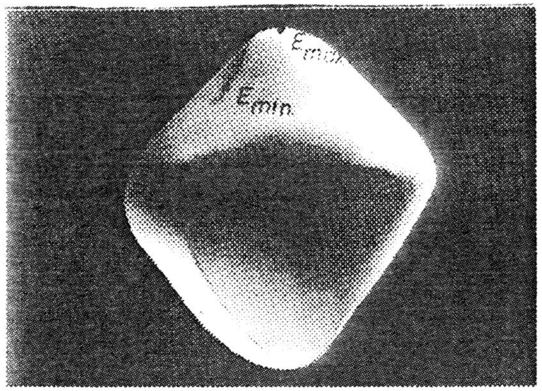

Fig. 2

Young's modulus body of sapphire

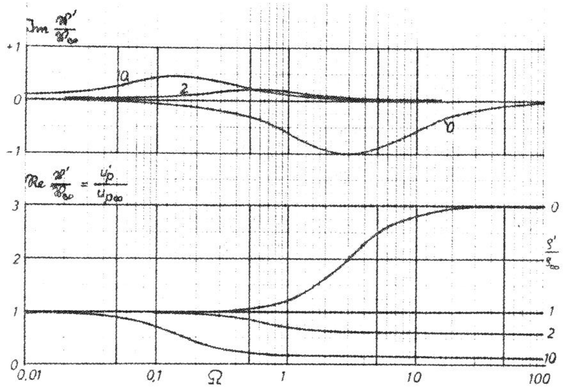

Fig. 3

Dynamic response of discontinuities according to E. SKUDRZYK

Re for motion, Im for interaction