Fracture of Porous Viscoelastic Materials

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1. Introduction

Modern techniques and the increasing use of computers in the building industry allow more sophisticated constructions. It is, therefore, necessary to know more about the behaviour of building materials under normal and extreme conditions. Many attempts to describe the basic laws of deformation and fracture are to be found in the literature. Most contributions, however, are mainly based on phenomenological observations and consequently do not cover mechanical properties in general. In an effort to fill this gap we have studied the theory of crack propagation in a porous and viscoelastic material. At the same time, we carried out experiments on hardened cement paste, mortar and concrete. Special attention has been paid to the behaviour of hardened cement paste and concrete under a high sustained load. Uniaxial and biaxial states of stress have been studied. Though complicated, the characteristic properties of the investigated materials can be described with the help of the newly developed theory. The main ideas and the experimental results are described in detail in Ref. 1. Here we would like to give a brief survey of our theoretical approach and mention some experimental results.

2. Theory

2.1 Uniaxial State of Stress

The simplest element to be investigated in an idealized structure of a porous material is a circular hole in a homogenious and isotropic plate. In Fig. 1 the well known stress distribution around a circular hole is shown. As soon as the tensile strength of the material at the lines of maximum tensile stress is reached a crack will propagate into the material. It can be shown that the related crack length λ is dependent on the applied load q by the following expression /1/:

$$q = \sqrt{\frac{\pi E \gamma}{\lambda r}} \sqrt{\frac{(1+\lambda)^7}{(1+\lambda)^2 - 1}} \qquad (1)$$

Here λ is equal to the crack length l divided by the radius of the pore r. All other symbols in equation (1) have their usual meaning.

Equation (1) can be applied as long as the crack length l is small in comparison to the distance of individual pores, that means, as long as the interaction between two mutual approaching cracks is negligible. This is, of course, a severe limitation of the mathematical description. We, therefore, studied the crack propagation of two interfering cracks and finally the crack propagation in a material with pores distributed at random. In this model which is fairly close to a realistic porous structure the sum of the length of all individual microcracks has been calculated. Each time two pores become connected by the merging of two increasing cracks there is a sudden increase in the total crack length until a critical value is reached. Then the cracks propagate without any increase in load.

If the material is viscoelastic cracks can propagate due to the fact that creep occurs in the immediate vicinity of the crack tips. Čerepanov /2/ has shown that formulas which are derived under the assumption that a material reacts in an ideally elastic way can be extended so that they can be applied to describe the behaviour of a viscoelastic material as well. In this case the elastic modulus E has to be replaced by a time dependent function $\tilde{E}$ where:

$$\frac{1}{\tilde{E}} = \frac{1}{E} + \int_{\tau_1}^{t} K(t,\tau) \frac{\sigma(\tau)}{\sigma(t)} \frac{1}{E(\tau)} \, d\tau \qquad (2)$$

In equation (2) K (t,τ) has the following meaning:

$$K(t,\tau) = -E(\tau) \frac{\partial}{\partial \tau} \left[\frac{1}{E(\tau)} + C(t,\tau) \right] \qquad (3)$$

Now it is possible to calculate the time t when a specimen fails under a high sustained load, the load being applied at time τ. The basic assumption here is that the specimen will fail as soon as the total crack length increases due to creep of the material in the crack tips and reaches a value that equals the critical crack length of the short time experiment. The related strength under sustained load is then given by the following expression:

$$\eta(t,\tau) = \frac{m(t,\tau)\beta_K(t)}{\beta_K(\tau)} \sqrt{\frac{E(\tau)}{E(t)} \frac{1}{1+\varphi(t,\tau)}} \qquad (4)$$

$\beta_K(t)$ and $\beta_K(\tau)$ being the short time strength of a companion specimen at time t when the sample fails and at time τ when the load has been applied respectively. E(t) and $E(\tau)$ represent the elastic modulus at the indicated age.

Creep in the material near the crack tips not only increases the crack length but reduces the stress concentration at the same time which leads to an increase in strength. $m(t,\tau)$ takes this effect into consideration.

2.2 Biaxial State of Stress

To study the stress distribution around a pore and the crack propagation under biaxial stress we used a different model. In Fig. 2 a spherical hole is shown with a penny-shaped crack starting from the circular line of maximum tensile stress. By introducing polar coordinates it is possible to calculate the crack propagation under biaxial stress in a similar way as shown for the uniaxial state of stress and the result is, in fact, identical with equation (4). That means that one can expect the strength to decrease under high biaxial stress in the same way as detected in experiments using uniaxial state of stress.

3. Experimental Results

For the experiments, we used a concrete with a water cement ratio of 0.73. We added 1913 kg/m^3 river gravel aggregate. All specimens were tested at the age of 7 days when they had reached a strength of approximately 200 kp/cm^2. To prevent shrinkage stress the concrete prisms were covered with a layer of paraffin wax.

In Fig. 3 hollow circles indicate the results of tests carried out under uniaxial compressive stress. In the same diagram full circles represent the experimental results of the biaxial compression test. The drawn line has been calculated with the aid of equation (4). Within the usual range of scattering of this type of experiment fair agreement between theoretical prediction and experimental results has been found. The necessary data for the calculation, such as, the time dependence of the elastic modulus and the short time strength, has been determined with companion specimens. The statement that the sustained load strength is the same under uniaxial and biaxial state of stress has been confirmed.

This work was performed when J. Zaitsev was a research fellow at the Technical University in Munich.

References

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2. Čerepanov, G.P., Cracks in Solids, Int. Journ. Solids and Structures 4, 811 (1968)

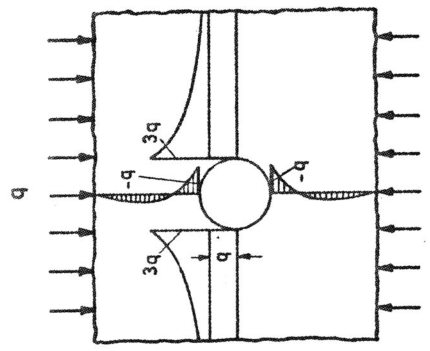

Fig. 1: Stress distribution in a homogenious material around a circular hole under uniaxial compression.

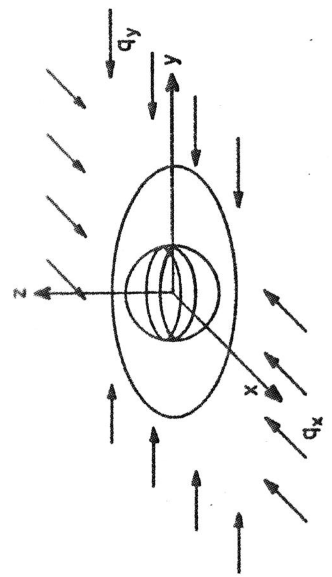

Fig. 2: Crack propagation into a homogenious material starting from a spherical hole under biaxial state of stress.

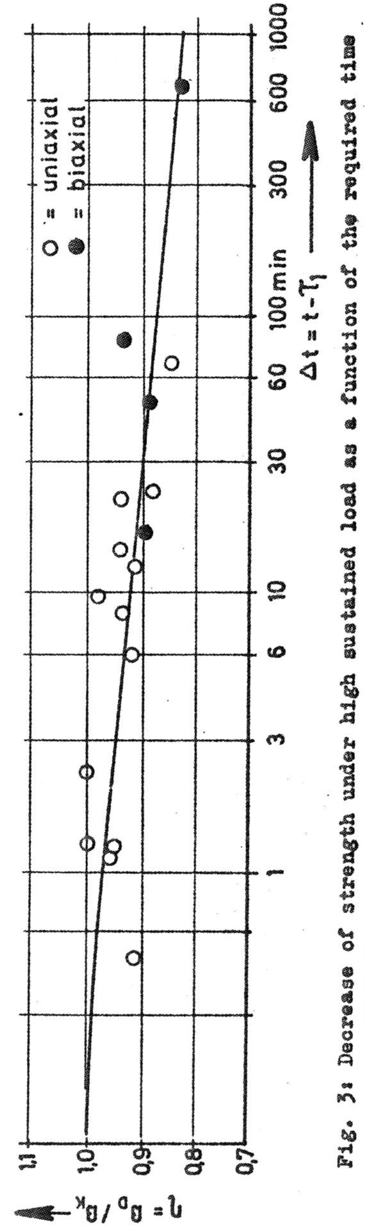

Fig. 3: Decrease of strength under high sustained load as a function of the required time until failure.