Post Yield Fracture Mechanics

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ABSTRACT

The use of fracture mechanics in the post yield regime is reviewed and compared with published fracture toughness tests on the Aluminium Alloy 2014-T6 at 20°K and 300°K. It is shown that the difference in the measured fracture toughness at the two temperatures arises from the change in tensile properties in that temperature range.

INTRODUCTION

It is now well established that, provided certain requirements concerning crack length and thickness are satisfied, linear elastic fracture mechanics can be used to specify critical defect sizes. In this note we consider the situation when the crack length requirement is violated: that is where the crack length is less than the characteristic length $(K_{1c}/\sigma_y)^2$, where σ_y is the material yield stress. We shall assume throughout that the thickness is sufficient to maintain plane strain conditions.

Orange (1971) has recently collected together data from fracture toughness tests in the post yield region and has attempted to interpret these results by a modification of Irwins (1962) result

$$K_{1c}^2 = \pi Q^2 \sigma_f^2 \left[c + {}^s/2\right] \qquad \dots \quad (1)$$

where σ_f is the applied stress at fracture, 2c the crack length and s is the plastic zone size which in L.E.F.M. is proportional to the square of the stress intensity factor. Q is a geometric factor. Orange's (1971) expression for the fracture stress may be written in the form

$$\sigma_f = \frac{K_{1u}}{\{\pi Q^2 [c + \alpha (K_{1u}/\sigma_u)^2]\}^{\frac{1}{2}}} \qquad \dots \quad (2)$$

where α is a constant, $1/\pi Q^2$, which depends only on the geometry, σ_u is the ultimate strength and K_{1u} is Orange's 'modified fracture toughness parameter'. Clearly if $c >> (K_{1u}/\sigma_u)^2$ this reduces to the familiar result $\sigma_f \propto c^{-\frac{1}{2}}$, whilst if $c << (K_{1u}/\sigma_u)^2$ $\sigma_f \rightarrow \sigma_u$ and equation (2) has the correct behaviour for short and long crack lengths. However, it is usual to present the results of fracture toughness tests in the form of a plot of the apparent fracture toughness against crack length and, provided the thickness requirement is satisfied, the apparent fracture toughness increases as the crack length increases reaching a 'constant' value when

$c > (K_{1c}/\sigma_y)^2$. Orange's modification of the Irwin theory is unable to predict this behaviour. In this note we show that if the effects of plastic relaxation at the crack tip are treated self-consistently then the form of the fracture stress against crack length curves and the apparent fracture toughness against crack length curves agree with the experimental data.

THE APPARENT FRACTURE TOUGHNESS AS A FUNCTION OF CRACK LENGTH

We adopt, for simplicity, the Vitvitskii-Leonov (1960), Bilby, Cottrell, Swinden (1963) model of fracture. In this model there is a relative displacement at the crack tip, $\delta(c)$, by virtue of the plastic relaxation there:

$$\delta(c) = \frac{4(1-\nu)\sigma_1 c}{\pi\mu} \ln\left\{\sec\left(\frac{\pi\sigma}{2\sigma_1}\right)\right\} \qquad \dots \quad (3)$$

where σ_1 is a stress which is characteristic of the strength of the material, μ and ν are the shear modulus and Poisson's ratio respectively. Fracture occurs when this displacement exceeds some critical value δ_c so that

$$\sigma = \sigma_f \text{ when } \delta(c) = \delta_c \qquad \dots \quad (4)$$

Cottrell (1963) defines the surface energy as the work under the force displacement curve so we have

$$2\gamma = \sigma_1 \delta_c = (1-\nu) K_{1c}^2 / 2\mu \qquad \dots \quad (5)$$

Combining equations (3), (4) and (5) gives the fracture conditions

$$\sigma_f = \frac{2}{\pi} \sigma_1 \cos^{-1}\left\{\exp-\left(\frac{\pi K_{1c}^2}{8\sigma_1^2 c}\right)\right\} \qquad \dots \quad (6)$$

If $c >> (K_{1c}/\sigma_1)^2$ then equation (6) reduces to the familiar Griffith-Irwin result whilst if $c \rightarrow 0$, $\sigma_f \rightarrow \sigma_1$. Thus we set σ_1 equal to the ultimate tensile stress of the material (see Heald, Spink and Worthington, 1972). The apparent fracture toughness is defined in the usual way

$$K_A = \sqrt{\pi c}\,\sigma_f = 2\sigma_u \left(\frac{c}{\pi}\right)^{\frac{1}{2}} \cos^{-1}\left\{ \exp -\left(\frac{\pi K_{1c}^{\ 2}}{8\sigma_u^{\ 2} c} \right) \right\} \quad \ldots \quad (7)$$

It is clear from this expression that $K_A \to 0$ as $c \to 0$ and that $K_A \to K_{1c}$ as c becomes large. Both equations (6) and (7) have the same behaviour as all the data collected by Orange (1971). In Figure 1 equation (7) is compared with the experimental data of Pendleberry (1962) on 2014-Tb Aluminium sheet at 20^oK and 300^oK. We have replaced c in equation (7) by Q^2c to account for the crack geometry.

DISCUSSION

It was found when comparing Pendleberry's data with equation (7) that the value of K_{1c} calculated using this expression was 27 $MNm^{-3/2}$ (30 k.s.i. $\sqrt{in}$) at both 20^oK and 300^oK despite the larger apparent toughness at 20^oK. The difference in the two curves arises from the different tensile strengths of the materials at the two temperatures, i.e. the U.T.S. is 687 MNm^{-2} (99.7 k.s.i.) at 20^oK and 497 MNm^{-2} (72 k.s.i.) at 300^oK. The agreement between equation (7) and the experimental results is good.

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