A Comparison of the Prediction of the Stress Concentration Theory of Fracture Mechanics with a Number of Actual Failures of Large Plant Components

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By way of introduction, the equilibrium of the simple system shown in Fig. 1 will be considered. This figure shows a through thickness crack of length $2l$ located in a flat plate loaded with a uniform stress along the two boundaries parallel to the crack. The magnitude of this stress, which will cause the crack to propagate and fracture the plate is denoted by f_g. The presence of the crack causes a stress concentration in the crack tip region, and, considering unit plate thickness, then at the failure stress the load shed by the crack is $2l\ f_g$, half of which is carried in the material at each end of the crack. The distribution of this load is shown schematically in Fig. 1. Here the mean value of the stress in the region in which fracture occurs is F, and S is the equivalent width of the region over which it may be considered to act. That is $(F - f_g)S$ = cross hatched area = $f_g l$

This equation will only be of use if both F and S are independent of crack length. That this is so can be shown by re-arranging to $f_g = F - f_g l/S$, and plotting experimental data in the form f_g against $f_g l$; straight line plots are obtained, Refs. 1, 2 and 3. The value of F is given from the ordinate intercept and for all materials for which test data is available, is within experimental error equal to the UTS of the material as it should be to satisfy the limit condition that $l \rightarrow 0$, $f_g \rightarrow$ UTS. Although pressure components frequently experience a bi-axial or sometimes tri-axial stress system the unflawed bursting pressure is normally that corresponding to the uni-axial UTS as measured in an ordinary tensile test. The behaviour of Zircaloy 2 is an exception since its UTS is raised above the uni-axial value by bi-axial stress conditions and the plot in this case is a straight line with the ordinate intercept equal to the bi-axial UTS obtained from the unflawed bursting pressure. The value of S for the particular material can obviously be determined from large flat plate tests as in Fig. 1. However, it is desirable to be able to determine the fracture characteristics of a material from small specimens. In Ref. 3 it was shown that $S = \alpha(\phi/f_y)^{1/3}$ where ϕ is the Charpy V energy at the test temperature, f_y is the yield strength, and are measured in ft.lbs and T.s.i. respectively. The parameter α is proportional to the elongation and this dependency is shown in Fig. 2 which is derived from published test data on various steels. (f_g, F, l, ϕ, f_y are given, hence α can be determined). Here elongation is measured on a gauge length of $4\sqrt{A}$.

The application of this theory to more usual practical defects is described in Ref. 4. The same principle is used; thus if an embedded penny shaped crack is considered, Fig. 3, the load shed by the crack, considering a segment is f_g.(area ABC). If f_g varies, it is $\int_{ABC} f_g\, dA$. This is carried by the area CBED where CD = S.

In the ensuing pages the actual sizes of defects which caused failure in various large plant items are compared with the sizes calculated by this theory.

Example 1 Extrusion Press Hydraulic Cylinder, Material ASTM A 27 Grade 70–36 Cast Steel, Ref. 5.

Failure occurred from a large fatigue crack, shown in Fig. 4, originating on the inner diameter.

Material properties: UTS = 69 K.s.i., f_y = 30.7 K.s.i. (13.7 T.s.i.) Charpy V = 8 ft.lbs., Elongation 27.5%, Maximum Lame hoop stress = 13 K.s.i.

From Fig. 2, $\alpha = 1.8$ $\therefore$ $S = 1.8\,(8/_{13.7})^{1/3} = 1.5$ in.

The load originally carried by area ABCD in the absence of the crack is given by:–

$$\int_{R_i}^{r_c} f_g \, dr$$

here f_g = Lamé tangential stress

$$= p.\, R_i^2 \left(1 + R_o^2/r^2\right) \big/ \left(R_o^2 - R_i^2\right)$$

With the crack present, this load is carried at crack tip by cross hatched area; this is $\left(F - f_{g_{r=c}}\right) S$

These equations are then equated giving:–

$$\left\{F - p \frac{R_i^2}{R_o^2 - R_i^2}\left(1 + \frac{R_o^2}{r_c^2}\right)\right\} S = p \frac{R_i^2}{R_o^2 - R_i^2}\left\{r_c - R_i - \left(\frac{R_o^2}{r_c} - \frac{R_o^2}{R_i}\right)\right\}$$

which can be solved for r_c and hence crack depth, which is 8.65". The actual crack depth, as scaled from photograph is 8.75".

Example 2 Seamless Boiler Drum, Ref. 6, Material Th31R

Failure occurred from a fatigue crack emanating at an un-reinforced penetration and propagating in a longitudinal plane. Fig. 5 refers. Failure pressure 290 ats. (Hoop stress f_h = 13.8 T.s.i.)

Material properties: UTS = 45 T.s.i., f_y = 32 T.s.i., Charpy 8.6 ft.lbs at 20°C, 4 ft.lbs at 0°C., Elongation δ_5 = 9.5%. Note that this elongation is not measured on a G.L. of 4 $\sqrt{A}$. To find the equivalent elongation on a G.L. of 4 $\sqrt{A}$ the method of Fenton (Ref. 7) is used. This gives an elongation of 11% and an α value of 0.3. In the absence of the crack, the hoop stress in the vicinity of the hole is given by $f_g/f_h = 1 + 0.75\, r_o^2/r^2 + 0.75\, r_o^4/r^4$ (2.1 stress field). This leads to a very complex integrand and this relation can be approximated by $f_g/f_h = 2.4 - 0.292\, r_o/r$

Load carried by area ADE before cracking $\doteqdot \int_{r_o}^{R+S} f_g\, y \, dr$

Load carried by area BCED after cracking = area DBCE x F where DB = S.

These loads are then equated and solved for R and hence crack length giving:–

if Charpy = 4 – crack length = 0.38"
if Charpy = 8.6 – crack length = 0.485
Actual length from scaled photograph = 0.47"

Example 3 Seamless Boiler Drum (Ref. 8) Pressure cycled to failure Longitudinal through thickness defect of total length 312 mm caused failure. Vessel 43.4 I.Dia., 4.34" thick, hoop stress at failure 8.25 T.s.i., UTS = 34.7 T.s.i., f_y = 19.5 T.s.i., Charpy V 16.4 ft.lbs, Elongation δ_5 = 28%, equivalent to 32% on 4 $\sqrt{A}$, α = 2.2.

Because this is a through thickness defect a bending stress acts on the material at the crack tip in addition to the stress due to membrane stresses. Failure equation, Ref. 2, is:–

$$f_g\,(1/S + 1) + k f_g\, l^2/D^2 = F \qquad (D = \text{I. Dia.})$$

and f_g = 8.25 T.s.i., $S = \alpha\,(\Phi/f_y)^{1/3}$ = .482", k = 12, F = 34.7 T.s.i. This equation solves to give l = 6.25", hence total crack length = 12.5" (318 mm) which may be compared with the actual overall crack length of 312 mm.

Example 4 PVRC No. 1 vessel (Ref. 9) A105 nozzle, Fig. 6 refers. UTS = 66 K.s.i., f_y = 35.1 K.s.i., Elongation 34.4%, Charpy V = 40 ft. lbs. The hoop stress at failure in the vessel shell was 39 K.s.i. Since this is above the nozzle material yield stress it will be assumed that the nozzle material is also at this stress level.

Load shed by crack:– $f_g\, r^2 \theta/2$

Load taken by area ABCD when crack formed $= (F - f_g)\left\{(r+S)^2 \theta/2 - r^2 \theta/2\right\}$

Here F = 66.6 K.s.i., f_g = 39 K.s.i., S = 3.44" giving r = 3.66". The actual crack radius as scaled from photograph is approximately 4".

Example 5 Heavy Section Steel pressure vessel forging. Ref. 5, Fig. 7 refers. Failure occurred from a ⅛" deep fatigue crack which developed in 1,550 cycles, at the base thread. The stress at the thread root was of yield magnitude. So here $f_g = f_y$. UTS = 116 K.s.i., f_y = 96 K.s.i., Elongation 18%, Charpy V = 18 ft.lbs. Hence S = .525", f_g = 96 K.s.i., and using basic equation $f_g(1/S + 1) = F$, gives l = .11". Actual size approximately ⅛".

Example 6 Ref. 10. Turbine Disc failure originating from 1/16" deep x ⅛" long defect located at bottom of keyway machined in bore of disc. Without a defect stress was at yield level. So take $f_g = f_y$ and UTS = 55 T.s.i., f_y = 49 T.s.i., Elongation 22.5%, Charpy V at failure temperature, 84°C, not given but at 20°C is 13 ft.lbs and upper shelf is 54 ft.lbs. A value of 30 ft.lbs is used here. (S is not very sensitive to error in Φ in this range). This gives S = .74" and equation is:– 49(1/.74 + 1) = 55. Hence l = .086". Actual approximately .063".

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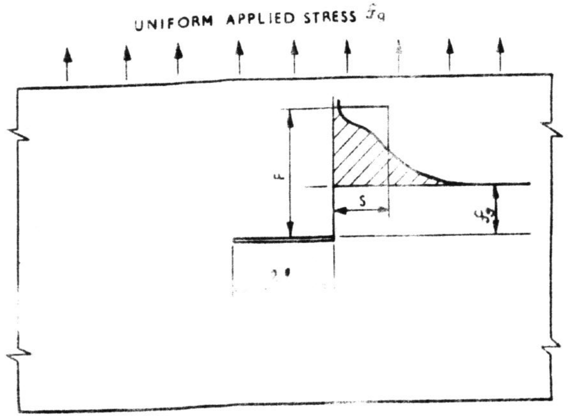

FIG. 1

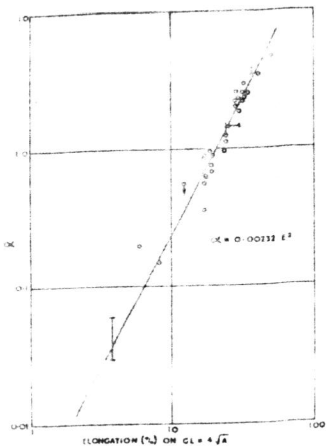

FIG. 2

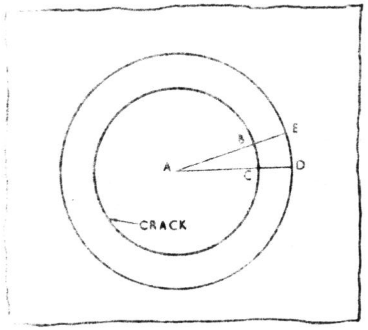

FIG. 3

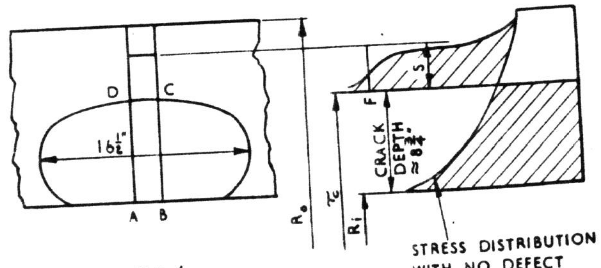

FIG. 4

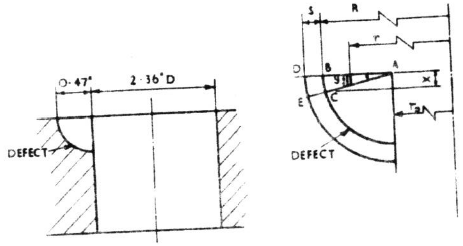

FIG. 5

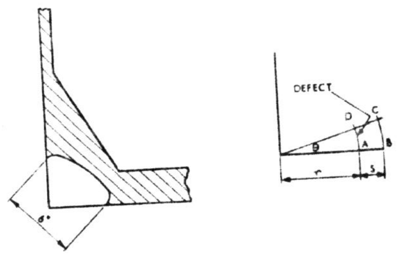

FIG. 6

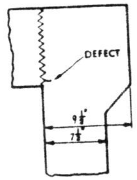

FIG. 7