The Fracture of Notched Specimens of Layered Composite Materials

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This paper considers a composite material consisting of alternate layers of fibre and matrix. Glass Reinforced Plastic made of glass mat approximates closely to such a simple model and a uni-directional or bi-directional lay-up of fibres can also be regarded as of this type, particularly if the loading is essentially in one of the fibre directions.

For uni-directional loading, it has been demonstrated (Ref. 1) that the displacement in the direction of loading in the nth fibre layer is governed by the equation (see Figure 1)

$$\frac{d^2\phi_n(x)}{dx^2} = \frac{G_m}{h_m h_f E_f}\left\{2\phi_n(x) - \phi_{n-1}(x) - \phi_{n+1}(x)\right\} \qquad (1)$$

where ϕ is the displacement of a fibre layer, E_f is the Young's modulus of the fibre material, G_m is the shear modulus of the matrix material, h_f, h_m are the thicknesses of fibre and matrix layers.

In the limiting case as h_f, $h_m \to 0$, we can replace $\phi_n(x)$ by $\phi(x,y)$ which then satisfies the equation

$$\frac{\partial^2\phi}{\partial x^2} + \beta^2\frac{\partial^2\phi}{\partial y^2} = 0 \qquad (2)$$

where $\beta^2 = \frac{(h_f + h_m)^2}{h_f h_m} \cdot \frac{G_m}{E_f}$, and is a constant for the material.

This is Laplace's equation and many methods exist for its solution. Analytical solutions are readily obtainable for many geometries but have the disadvantage that they lead to infinite stresses at the tips of cracks and notches. We show how, by reverting to Equation (1), one can estimate from an analytical solution finite values of the tensile and shear stresses at the tip of a crack or notch. This offers considerable advantages in fracture mechanics, giving a simple stress criterion for fracture instead of an energy criterion.

Analytical Solutions

Analytical solutions for various crack geometries will be published elsewhere. They have all been obtained for a uniform tensile load and the geometries concerned are shown in Figure 2, all relating to cracks in an infinite half-plane of material. For geometry (a), discussed in Ref. 1, the solution has also been obtained for a sheet of finite thickness, H, and it is found that the stresses near the tip are multiplied by the factor $\left\{\frac{2H}{\pi a} \tan \frac{\pi a}{2H}\right\}^{\frac{1}{2}}$.

Derivation of Finite Stresses at Crack Tip

At the crack tip, $\phi = 0$. Using an analytical value $\bar{\phi}$ of ϕ on the crack surface at a distance $(h_f + h_m)$ from the crack tip, the maximum shear strain, η_m, in the first matrix layer is immediately estimated as

$$\eta_m = \bar{\phi}\ (h_f + h_m)\ /\ h_m. \qquad (3)$$

For the tensile stress at the crack tip, we rewrite equation (1) in the form $\frac{d^2\phi}{dx^2} = \gamma^2(\phi - f)$

where ϕ is the displacement in the fibre layer at the root of the crack (the nth layer), $f = \frac{1}{2}(\phi_{n-1} + \phi_{n+1})$, obtained from the analytical solution, and $\gamma^2 = 2\beta^2/\ (h_f + h_m)^2$.

It can be deduced from this that the maximum tensile strain is

$$\left(\frac{d\phi}{dx}\right)_o = \gamma^2\int_o^\infty f(u)\ e^{-\gamma u}\ du, \qquad (4)$$

and the tensile stress can be calculated from this.

Mechanisms of Fracture

A systematic presentation of tip stresses for various geometries, using Equations (3) and (4), will be published elsewhere. In the meantime, we present a simplified discussion based on the analytical solution for geometry (a), given in Ref. 1. The tensile and shear stresses, σ_f

and τ_m, in the neighbourhood of the crack tip are given by

$$\sigma_f = \frac{\sigma_o}{\alpha}\left(\frac{a}{2r}\right)^{\frac{1}{2}} \cos\frac{\theta}{2}, \qquad \tau_m = \beta\sigma_o\left(\frac{a}{2r}\right)^{\frac{1}{2}} \sin\frac{\theta}{2},$$

in the notation of Figure 3, where $\alpha = h_f\ /\ (h_f + h_m)$, the volume fraction. Thus the ratio of maximum tensile stress to maximum shear stress is given by $\sigma_f/\tau_m = 1/\alpha\beta$.

Substituting values of fracture tensile and shear stress for σ_f and τ_m in this, we obtain an expression for the critical volume fraction which separates tensile failure (small α) from shear failure (large α) in the form

$$\alpha = \left(1 + \frac{\sigma_f^{\,2}}{E_f} \cdot \frac{G_m}{\tau_m^{\,2}}\right)^{-1}. \qquad (5)$$

It may be noted that $\sigma_f^{\,2}/E_f$ is related to the tensile energy density in the fibres at tensile failure while $\tau_m^{\,2}/G_m$ is similarly related to the matrix shear energy density.

Numerical Results

The quantities given in the theory are values for the simplified layered structure idealising the real material. For example, the shear stress in the matrix material will differ from that of the idealised layer assumed here and this will affect the assumed value of shear stress for fracture; indeed, this particular quantity may be a debonding stress rather than a fracture stress. It may well be that this model is more suited for qualitative than for quantitative discussion and the results given here are intended to be indicative only.

Some experimental stress distributions round a crack in CFRP sheet have been obtained at the Royal Aircraft Establishment, Farnborough, (Ref. 2) using a laser beam Moiré fringe technique. Tensile and shear strain contours are compared in Figure 4 with theoretical contours derived from Ref. 1, modified by the correction term for finite width given above (about 5%). These have been calculated assuming a value of $\beta = 0.18$ which is a not unreasonable value for the material used. A typical value of β for GRP would be around 0.3.

From Equation (5), critical values of volume fraction of 0.12 - 0.2 for CFRP and 0.04 for GRP are estimated. Thus GRP will virtually always fail in shear in the matrix. Tensile failure in the fibres of CFRP is more possible; a cross-ply lay up would have a volume fraction of 0.25 - 0.3.

Conclusions

This paper shows that it is possible to estimate the stresses in the neighbourhood of a crack or notch tip. These can be used either in conjunction with an energy criterion e.g. for fibre pull out or with a strength criterion for continuous fibre layers to estimate failure loads.

It is known that debonding or cracking of the matrix in shear is the usual cause of failure in GRP and in CFRP. It is shown here that tensile failure is more possible under some circumstances for CFRP and a criterion is indicated for determining the critical volume fraction. Whether tensile or shear failure is the more desirable is outside the scope of this paper.

References

1. King, J.L. Jnl. Strain Analysis 7, 2, 146-150 (1972).
2. Bishop, S.M. R.A.E. Technical Report (to be published).

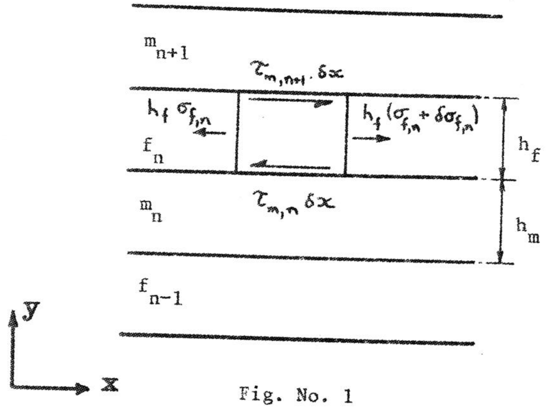


Fig. No. 1

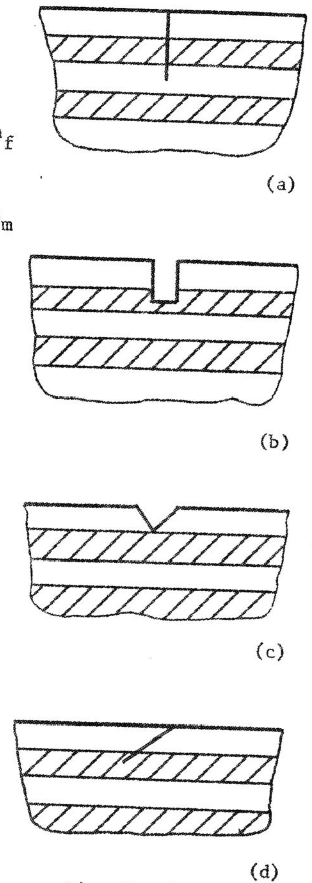


Fig. No. 2.

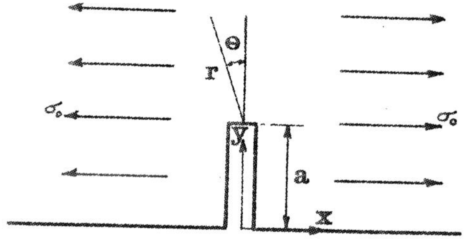
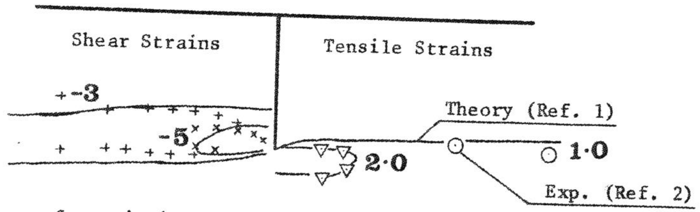


Fig. No. 3



All values of strain in terms of unit tensile strain at infinity.

Fig. No. 4