Structural Reliability Characterization of Advanced Composites

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A kinetic model is postulated which considers the interaction of cumulative fatigue damage and chance overload on a component or structure under typical probabilistic service load histories. This model recognizes that: (1) materials fail from preexisting flaws; (2) that flaws develop in a characteristic manner which is determined by the material properties, state and magnitude of the stresses at the flaw perimeter, the history of the imposed tractions, the thermal and environmental histories; and (3) the critical load for structural failure is a decreasing function of the crack length.

Since the growing crack in a laminated composite or adhesive joint may not be detected by inspection procedures before reaching critical proportions, the reliability of a component or structure must include the chance of survival with an undetected crack. Fundamental to this operation is the definition of a "critical load". Failure occurs when the resistance F(t) of an element or structure is decreased through flaw development to the level such that the next load at that level, "the critical load", will produce rapid fracture. The fatigue lifetime, an induced stochastic variable, is simply the time required to produce a sufficient strength degradation to permit failure.

Let us assume that the damage rate accumulation may be approximated by a power-law growth equation

$$\frac{\partial C}{\partial t} = M \cdot C^r \tag{1}$$

where $\underline{r}$ is a positive exponent characteristic of material behavior and independent of test or service variables, and $\underline{C}$ is the dimension of the critical damage zone. The premultiplier $\underline{M}$ is proportional to the far field work inputed into the element [1] by the service environment

$$M = \text{CONST.}\ W^r \simeq AD^r F_{MAX}^{2r}$$

and $\underline{F}$ is the stress, $\underline{D}$ is the effective compliance, and $\underline{A}$ is a constant. In addition, assume that a "critical load" is correlated with

$$F\sqrt{C} = K \tag{2}$$

where $\underline{K}$ is an apparent toughness or work parameter. Integrating (1) and employing (2) gives a statement for the time dependent residual strength

$$F_i(t)^{2(r-1)} = F_i(t_o)^{2(r-1)} - (r-1)A_4F_{MAX}^{2r}(t-t_o) \tag{3}$$

of the i^{th} element in a population of N elements, where $A_4 = AD^rK^{2(r-1)}$. $F(t_o)$ is the initial static strength of the component possessing a Weibull (extreme value) distribution

$$P[F > F_i(t_o)] = \exp - [F/\hat{F}(t_o)]^{\alpha_o} \quad (4)$$

where α_o and $F(t_o)$ are the shape and scale parameters, respectively, for the Weibull distribution. Using equation (3) it is easily seen that the probability that $F(t) > F$ is just the probability that equation (3') holds.

$$F(t_o) > \left[F(t)^{2(r-1)} + (r-1)A_4F_{MAX}^{2r}(t-t_o)\right]^{1/2(r-1)} \quad (3')$$

Equation (4) gives this probability as

$$P[F(t) > F] = \exp - \left\{\frac{F(t)^{2(r-1)} + (r-1)A_4F_{MAX}^{2r}(t-t_o)}{\hat{F}(t_o)^{2(r-1)}}\right\}^{\alpha_f} \quad (5)$$

where $\alpha_f = \alpha_o/2(r-1)$. Note that when $t = t_o$, (5) reduced to (4), that when $t > t_o$ the result that $P[F(t) > 0] < 1$ is a positive probability that $F(t) = 0$: and that when $F(t) > 0$ the random variable $F(t)^{2(r-1)}$ is distributed as a translated three parameter Weibull distribution.

Note that the cumulative probability statement predicts a truncated residual strength determined by the peak fatigue loads and reduces to the extreme value statement for total fatigue failure:

$$P[F(t) > F_{MAX}] = \exp - \left[\frac{t}{\hat{F}(t_o)^{2(r-1)}/(r-1)A_4F_{MAX}^{2r}} + \frac{F(t)^{2(r-1)} - (r-1)A_4F_{MAX}^{2r}t_o}{\hat{F}(t_o)^{2(r-1)}}\right]^{\alpha_f}$$

which under the conditions of $F_{MAX} < F(t_o)$, $t_o = 0$ and $t \to \infty$ (t_b, which necessitates $F(t)^{2(r-1)} \to 0$)

yields

$$P\left[F(t) > F_{MAX}\right] = \exp - [t/\hat{t}_b]^{\alpha_f}, \quad (6)$$

where

$$\hat{t}_bF_{MAX}^{2r} = \frac{\hat{F}(t_o)^{2(r-1)}}{(r-1)A_4} = B. \quad (7)$$

Equation (7) is simply the power law representation of the classical S-N curve [2, 3].

One import of this work, as illustrated in the first assumption, is that damage development is analogous to chemical reaction rate theory. For example, eq. (1) may be interpreted as stating that the premultiplier term M is a function of the major test variables.

$$M = G(\text{Temp, Humidity}, F(t), F_{MAX}, \cdots)$$

For a thermal variation; if

$$M = A \cdot \text{EXP} - (\Delta H/RT) \quad (8)$$

then,

$$A_4 \equiv A_5 \cdot \text{EXP} - (\Delta H/RT) \quad (9)$$

where ΔH is a classical activation energy, R is the gas constant and A is a material parameter, A_4 is specified in Eq. (3-7). A change of temperature changes the rate of a da/dn law but not the slope on a da/dn plot:

$$\log\left[\frac{MT_i}{MT_o}\right] = \log\left[\frac{A_4(T_i)}{A_4(T_o)}\right] = \log \alpha_T = -\frac{\Delta H}{2.3R}\left[\frac{1}{T} - \frac{1}{T_o}\right] \quad (10)$$

Similar statements may be written for the adsorption of water or humidity in a composite [4, 5]. These effects are well known in viscoelastic analysis and produce a change in characteristic lifetime without changing the dispersion of the data

$$\hat{t}_{f(T_i)} \equiv \hat{t}_{f(T_o)}/\alpha_T \quad (11)$$

$$\alpha_o \text{ and } \alpha_f = \alpha_o/2(R-1) \neq G(\alpha_T, \cdots)$$

It is important to observe that equation (6) implies, for a constant mode of fracture, an approximately fixed fatigue life shape parameter independent of history load or environmental and scale effects. The fatigue shape parameter, for truncated random or constant amplitude histories is causually related to the static shape parameter via equation (5) although the failure times of the individual specimens are still stochastic quantities. These results imply that failure in quasi-brittle systems can be described by extreme value statistics and that the shapes of the static and fatigue distributions are material variables independent of structural geometry and scale.

The kinetic model presented here is consistent with some of the earlier results of Coleman [2] in that quasi-brittle solids, which correlate with Equations (6) and (7) also exhibit delayed creep failure data [5, 6] of the form

$$P[t_b] = \exp - [t_b/\hat{t}_b]^{1.0} \quad (12)$$

and lifetime function

$$\hat{t}_b \hat{F}_b^{2r} = \frac{\hat{F}(0)^{2(r-1)}}{(r-1)A'_4} \equiv B' \quad (13)$$

In this format the term A'_4 is history sensitive; it is not identical with A_4 of equations (3-7). Thermal and other environmental effects will still be contained in A'_4 as illustrated by equations (8-11). However, if a constant loading rate history, $F(t) = V \cdot t$, is applied, it can be shown [2] that the strength properties will be given as

$$P[F_b] = \exp - [F_b/\hat{F}_b]^{a_o} \quad (4')$$

where

$$a_o = 2r+1; \quad \hat{F}_b \equiv [B'(2r+1)V]^{1/2r+1} \quad (14)$$

and

$$t_b \hat{F}_b^{2r} = B'(2r+1) \quad (15)$$

The quantity B' is identical with the statement in equation (12). It is interesting to note: (1) that equations (7), (13) and (15) yield parallel "life curves" displaced by a history function; (2) that the shape parameters for creep, tensile fracture, and fatigue, equations (12), (14) and (5) are only a function of the flaw size exponent in equation (1)

$$a_o = 2r+1; \quad a_f = \frac{2r+1}{2(r-1)}$$

and are not a function of history, time scale or environment. It is important to note that the effects of a fatigue history is the acceleration of the average damage rate, A'_4, and a sharp alteration of the shape of the lifetime distribution function when compared against the creep experiment [5]. These results were achieved under the implicited assumption that the physical character of fracture is invariant with respect to the history of the external stimuli. Under this assumption a change in failure statistics, for a fixed history, must reflect a change in failure mode.

This assumption implies that there is, at least, one primitive form of fracture from which all other conditions associated with fracture can be derived as specific boundary value problems. While we are able to quantitatively interrelate creep and tensile fracture, only the qualitative character of the fatigue response can be evaluated. This condition exists because fatigue-flaw development processes are inherently non-linear history functions which will always be difficult if not impossible to estimate from cumulative damage theories. Servo hydraulic equipment, when interfaced with a computer controlled system can produce a real or accelerated load-time-power history $F(t)$ in the laboratory. Controlled environment may also be enforced upon the test. Consequently, the experimental laboratory may be employed to obtain an "analog" solution to lifetime and statistical estimates under fatigue

conditions. Thus, point estimates of A'_4 may be achieved for either laboratory coupons or full scale structures. The quality of this estimate is only contingent upon our knowledge of actual engineering histories and environments. These capabilities, supported by the structure reliability (extreme value) format presented in [7, 8 and 9] for either the computation of safety factors (static strength and/or life), constitute the basis for probabilistic structural design technology.

An extensive data base developed for fracture and fatigue of a composite system shall be presented in support of the results outlined herein.

The data base presented here will be typical of time-dependent quasi-brittle solids such as elastomers, composites, and adhesive joints. The extension of these results to metallic systems require the recognition that fracture in metals is complicated by additional fracture modes not specifically developed herein.

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