Fracture Toughness of Fiber-Reinforced Composites

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The effects of reinforcing fibers on the fracture toughness and critical stress intensity factor of an epoxy resin are investigated. By varying the volume percentage of glass fibers oriented parallel to a crack, it is found that there exists an optimum fracture toughness of the composite dependent upon the fiber volume percentage and the constituents of the composite. The simple relationship between k_c, the critical stress intensity factor, and G_c, the crack toughness, for a homogenous material in the linear elastic theory of fracture mechanics is found to hold with reasonable accuracy for the case of a crack extending parallel to the fibers when the orthotropic elastic constants for the composite system are used in the calculation.

From classical fracture mechanics it is known that two parameters used to measure fracture resistance, the critical stress intensity factor k_c and the fracture toughness G_c are related for homogenous anisotropic materials [1] such that they represent a single fracture criterion. An experimental program was carried out to study the conditions for which this type of relationship can be expected to apply to fibrous composites. Both compliance tests, which enable a direct determination of G_c, and fracture tests, from which k_c can be calculated, were performed on compact edge-notched specimens (see [2] for a detailed description of the test specimen). Compact edge-notched specimens supplied by AFML were used. The composite materials tested include samples of the resin matrix ERL-2256/ZZL0820 in which uni-directional E-glass fibers are embedded. All specimens were tested with an initial crack inserted in the direction parallel to the fiber. The E-glass composite specimens consisting of 10, 20, 50 and 60 percent fiber volume fraction with the following elastic properties:

TABLE 1 - Elastic Constants

Volume Fraction %	E_1 psi x 10^6	E_2 psi x 10^6	ν_{12}	μ_{12} psi x 10^6
0	0.50	0.50	0.35	0.35
10	1.51	0.60	0.33	0.33
20	2.52	0.66	0.32	0.32
50	5.55	1.33	0.28	0.30
60	6.56	1.70	0.26	0.29

The compliance test is designed to determine the G_c-value of a cracked specimen by direct measurement of the displacement of a certain gage length L for varying lengths of cracks and at different critical loads P. For a compact tension specimen, L corresponds to the distance between the applied loads P and the displacement of the load points is therefore the changed length ΔL. This gives the rate of change of $\Delta L/P$ with the crack length a, and hence from [3]

$$G_c = (P^2/2B)[\partial(\Delta L/P)/\partial a] \quad (1)$$

can be computed at fracture giving G_c. The quantity $\Delta L/P$ is called the "compliance" of the specimen, and B stands for the thickness of the specimen.

Fracture tests were run on the same experimental set up that was described above for compliance measurement. The idea here is to measure critical values of the load P and crack length a at the point of unstable crack extension so that the stress-intensity factor as derived from the linear theory of elasticity [3]

$$k = [P\sqrt{a}Y(a/W)]/[BW\sqrt{\pi}] \quad (2)$$

can be calculated. In the above equation, $Y(a/W)^*$ is a function depending upon the ratio of the crack length to the width of the specimen W.

Since the precise relationship between the strain energy release rate G and the stress-intensity factor k of a heterogenous system such as the fiber reinforced composite is not known and cannot be easily obtained analytically, it would be informative to investigate the possibility of using an orthotropic model for the composite specimen. Sih and Liebowitz [1] obtained a relationship between G_1 and k_1 for a generally anisotropic material in which the direction of crack propagation is collinear with the original crack and the crack is opened by a constant surface pressure. For this case the system is said to be orthotropic and the

*Bowie and Freese [4] have shown that, for a wide range of geometries, the anisotropic stress intensity factor is closely approximated by its isotropic counterpart.

resultant relationship for plane strain is

$$G_1 = \pi k_1^2 (b_{11}b_{22}/2)^{1/2}\{(b_{22}/b_{11})^{1/2} + [(2b_{12} + b_{66})/2b_{11}]\}^{1/2} \quad (3)$$

where the elastic coefficients, b_{ij}, are related to the principal elastic constants as follows

$$b_{11} = a_{11} - (a_{12}^2/a_{22}),\quad b_{22} = a_{22} - (a_{23}^2/a_{22})$$

$$b_{12} = a_{12} - (a_{12}a_{23}/a_{33}),\quad b_{66} = a_{66} = 1/\mu_{12}$$

and

$$a_{11} = 1/E_1,\quad a_{22} = 1/E_2,\quad a_{12} = -(\nu_{12}/E_1),\quad a_{23} = -(\nu_{23}/E_2)$$

Values for the principal elastic constants are shown in Table 1.

Values of $G_c^{(2)}$ with the superscript (2) signify the fracture toughness value obtained directly from the compliance measurement technique and hence $k_c^{(2)}$ is calculated indirectly using Eq. (3). On the other hand $k_c^{(1)}$ with the superscript (1) is found from a fracture test and $G_c^{(1)}$ is computed form Eq. (3). All these are average values for their respective fiber volume fractions. The percentage of deviation compares the experimentally measured values of G_c and k_c using Eqs. (1) and (2). The idea is to check whether Eq. (3) would hold for a fiber reinforced composite. The deviation is based on the value which was found more directly, i.e.,

$$G_c - \%\ \text{deviation} = [(G_c^{(2)} - G_c^{(1)})/G_c^{(2)}] \times 100 \quad (4)$$

and

$$k_c - \%\ \text{deviation} = [(k_c^{(1)} - k_c^{(2)})/k_c^{(1)}] \times 100 \quad (5)$$

The unit for G is in-lb/in^2 and k is measured in lb/in$^{3/2}$.

TABLE 2 - Orthotropic Model

Volume Fraction	Fracture		Compliance		% Deviation	
%	$G_c^{(1)}$	$k_c^{(1)}$	$G_c^{(2)}$	$k_c^{(2)}$	G_c	k_c
0	0.75	373	0.77	380	+2.5	-2
10	4.05	1,020	2.98	865	-36	+15
20	3.63	965	2.47	800	-47	+17
50	2.32	1,090	2.31	1,090	-0.5	0
60	1.97	1,100	1.81	1,060	-9	+4

Note, from the data in Table 2, that the fracture toughness G_c of the glass fiber composite first increases with the fiber volume fraction and then decreases. The peak value of G_c which is approximately three times higher than the fracture toughness of the matrix material occurs at approximately 10 percent volume fraction. This means that in the low fiber volume fraction load transfer to the crack tip is relatively low while the load transfer in the high fiber volume fraction range is high, thus lowering the fracture strength of the composite.

The qualitative features of the theoretical prediction are in agreement with the experimental data indicating that there exists an optimum fiber volume fraction for which the composite achieves maximum fracture toughness.

REFERENCES

[1] Sih, G.C. and Liebowitz, H., "Mathematical Theories of Brittle Fracture", Mathematical Fundamentals of Fracture, H. Liebowitz(ed.), Academic Press, New York (1968), pp. 67-190.

[2] Sih, G.C., Hilton, P.D. and Wei, R.P., "Exploratory Development of Fracture Mechanics of Composite Systems", Air Force Report AFML-TR-70-112 (1970).

[3] Sih, G.C., et al, "Fracture Mechanics for Fibrous Composites", ASTM Publication, (1972), (in Press).