Fracture Toughness in Randomly Oriented Fibre Reinforced Materials

by M. R. Piggott

Abstract

For some time it has been possible to estimate the work of fracture for orientated fibre reinforced materials from the mechanical properties of fibres and matrix. Cases considered were: poorly bonded fibres, and coated fibres in brittle matrices, and well bonded fibres in ductile matrices. The crack was assumed to propagate in a plane normal to the fibres. Recently, however, a theory has been developed for the case of cracks propagating at other angles to a parallel array of fibres in a ductile matrix. This theory is extended to the case of materials reinforced with randomly oriented fibres, and the work of fracture is shown to be governed by the same equation as for the crack propagating normal to a parallel array of fibres, except for a numerical factor, and a factor depending on the ratio matrix flow stress to fibre strength. The optimum conditions for toughness are shown to be large fibre diameter, great fibre strength, small fibre modulus, and low matrix flow stress.

Introduction

Although the work of fracture, or fracture surface energy (= 1/2 work of fracture) has been measured for a number of different fibre-matrix combinations[1,2,3], and the theoretical understanding of the processes contributing to work of fracture is now quite advanced[4,5,6] the fibres have almost always been normal to the plane of fracture. This author has recently extended his theoretical treatment[5] to the case of oblique fibres. This theory considered a parallel array of fibres of uniform strength in a matrix which can exert a shear stress τ_y at the fibre surface, either by flow or friction, or by yielding of a special coating on the fibres. The extension for the oblique case is mainly concerned with fibres having a uniform maximum tensile strain for failure, in a yielding matrix[7]. This paper further extends the theory to include the case of fibres randomly orientated in a plane, and randomly oriented in space. In addition experiments are described to test the theory's predictions for the planar-random case.

Theory

In the previous paper[7] it was assuming that the force between oblique fibres and matrix in the region of a crack had components both normal to the fibre and parallel to it, the normal component decreasing with distance from the crack face. The fibre thus had maximum curvature close to the crack face, when stressed by the crack opening, in keeping with visual observations on steel wires embedded in clear polycarbonate.

The work contributed by each fibre was approximately

$$u = \frac{\pi d^3 \sigma_u^3}{48 \tau_y E_f} (1 - C \tan \phi)^3$$

and is given with sufficient accuracy by

$$u = u_0(1 - 3 C \tan \phi) \qquad (1)$$

where d = fibre diameter, σ_u = fibre ultimate strength, E_f = fibre modulus, C = constant determined by geometrical factors and the ratio τ_y / σ_u, ϕ is the angle between the fibre and the crack plane normal

and u_0 is the work for $\phi = 0$. For values of ϕ such that $3C \tan \phi \geq 1$ the approximations break down, and it is assumed that the work becomes vanishingly small.

We will examine the planar case first. If n is the total number of fibres crossing area A of crack face, then so long as A is sufficiently large, the number of fibres in the angular interval $d\phi$, initially at angle ϕ to the crack plane normal, will be $n d\phi/\pi$.

From Equation 1, the increment of fracture surface energy due to these fibres is $A d\gamma = u_0(1 - 3C \tan \phi)\frac{n d\phi}{\pi}$ so that integrating over the interval $-\phi_1$ to ϕ_1 where the work is not vanishingly small, i.e. where $\tan \phi_1 = C$, we obtain

$$\gamma_p = \frac{2\gamma_0}{\pi}\left\{\phi_1 + 3C \log \cos \phi_1\right\} \qquad (2)$$

where $\gamma_0 = n u_0/A$.

For the case of fibres oriented randomly in space we notice that all fibres at an angle between ϕ and $\phi + d\phi$ to the crack plane normal contribute the same amount of work. Again taking a large enough area A so that the fibres are smoothly distributed between all possible angles, we find that for a total of n fibres crossing A, $A d\gamma = u_0(1 - 3C \tan \phi).\ n \sin \phi d\phi$. This time we include all fibres if we integrate from $\phi = 0$ to $\phi = \phi_1$. Thus

$$\gamma_s = \gamma_0\left\{1 - \cos \phi_1 - 3C\left(\ln\left(\frac{1 + \sin \phi_1}{\cos \phi_1}\right) - \sin \phi_1\right)\right\} \qquad (3)$$

Experimental Method and Results

In order to simulate the fracture of specimens having continuous fibres, a soft polyester was cast into discs, having a slit completely crossing a diameter, except for a small amount of polymer at the centre. Through this central piece of polymer went 11 glass fibre bundles, impregnated with hard polyester so that each bundle behaved as a rigid rod, and oriented approximately at angles of 0°, ±15°, ±30°C, ±45°, ±60°, ±75° to the normal of the diametral slit. These specimens were broken in 3 point bending, and the work done was compared with that for similar specimens having the fibres all normal to the slit.

From prior tests[7], the value of C was found to be 0.196 for glass fibres in this polyester. Thus $\phi_1 = 60°$ so that $\gamma_p = 0.41\gamma_0$. However, the value of γ_p in these tests came to $(0.30 \pm 0.02)\ _0$.

Conclusion

The experimental results indicate that randomly oriented fibres contribute about 30% less fracture surface energy than would be anticipated if the fibres all behaved independently, and the work of fracture was merely the sum of the contributions of each fibre. Thus it is concluded that for planar random fibres the work of fracture appears to be reduced to about 1/3 that for the aligned fibre case, for glass fibres in soft polyester. For fibres randomly oriented in space, the work should be reduced still further (in theory to about 1/7th of the aligned case). For harder matrices the value of C will be greater (since τ_y is greater) so that the toughness of the

material should be reduced still further. However, as the fracture surface energy for the both random cases is proportional to the fracture energy for cracks propagating in a plane normal to aligned fibres, the optimum conditions for toughness of randomly oriented fibre composites should still be large fibre diameter, great fibre strength, small fibre modulus and low matrix flow stress.

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