Failure of Brittle Materials with Tougher Surface Coating

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It has safely been established in extensive tests using diverse experimental techniques that extensibility and bending and tensile strengths of brittle building materials with coarser structure grow considerably greater if such materials are coated with a surface layer of a tougher material /1,2/. This finding is in agreement with results of tests made on composite concrete members and on concrete with closely-spaced reinforcement.

All the cases mentioned involved strengthening of materials whose failure is preceded by a gradual development of microcracks. As a hypothesis proposed to explain the observed favourable phenomenon suggests, the energy balance of a critical crack in a brittle core with relatively low fracture energy is considerably improved by energy absorption in the tougher coating in the course of the latter´s larger straining in the region above the crack /1,2/; this proposition was also verified by photoelastic investigations /3/.

The improved energy balance and the corresponding enhancement of mechanical properties of the core can be determined explicitly. If the coating is adequately tough, its bond with the core fails near an opening-up crack over a short segment, so that an equilibrium is established between the appertaining tensile force in the coating and the forces transmitted to it by bond from the core on both sides of the crack. For the purposes of practical engineering it is wholly sufficient to introduce the following simplifying assumptions (see Fig. 1):

a) The crack is of an elliptical cross-section and completely unloads a region likewise being of an elliptical cross-section circumscribed around the crack;

b) Length l of the segment with failed core-coating bond is larger by far than the opening s of the crack but small compared with the length of the unloaded region of the core;

c) Local shear stress τ_b in the bond between the coating and the unloaded region of the core falls from its maximum (equal to the bond strength R_b) near the crack linearly to zero on the edge of the unloaded region (this corresponds to the linear course of the displacements of the core surface in this region);

d) Contribution A_1 of the coating to energy absorption is considered only in segment l ; in its calculation as well as in the determination of strain energy U released from the unloaded region, only the work of normal stresses and strains is taken into account.

The first case to be solved is that of a plane surface crack of depth c in a brittle core under simple tension and plane state of stress, spanned by a hitherto non-strained coating of relatively small stiffness which is uncapable of substantially affecting the core forces. In this case the length of the unloaded core region is $4c$. The critical depth c of a crack which would grow unhindered in the absece of the coating action, is given by the familiar formula

$$\frac{dU}{dc} - \frac{dA}{dc} = G_c \quad ; \qquad (1)$$

if depth c is increased by Δc , the crack opening near the core surface will increase by

$$\Delta s = 4\sqrt{\frac{G_c}{\pi E}\,\frac{\Delta c}{c}} . \qquad (2)$$

energy U released by the unloading of the adjoining core region will grow by

$$\Delta U = G_c \Delta c\left(1 + \frac{\Delta c}{2c}\right) \qquad (3)$$

and energy A absorbed in the growth of the core crack will increase by

$$\Delta A = G_c \Delta c \quad . \qquad (4)$$

In the formulae, E denotes Young's modulus of the core and G_c the critical energy release rate of the core defined on the case of a plane crack. For an elastic coating, length l of failed bond is obtained from formula (2) on the assumptions stated above, viz.

$$l = 4\sqrt{\frac{G_c}{\pi E}}\,\frac{\Delta c}{\sqrt{c}}\;\frac{E_1 t_1}{R_b(c+\Delta c)} \quad ; \qquad (5)$$

the energy absorbed in the process of straining of the coating in this segment amounts to

$$\Delta A_1 = \frac{1}{2}\,\frac{(\Delta s)^2 E_1}{l^2}\,t_1 l = \frac{2R_b\sqrt{G_c}\,\Delta c(c+\Delta c)}{\sqrt{\pi E c}} \qquad (6)$$

(E_1 - Young's modulus of the coating, t_1 - the thickness of the coating). After some handling the ratio of the total increment of energy absorbed in the core and in the coating, to the increment of energy released turns out to be

(7)

where the second term on the right-hand side represents the energy contribution of the coating. Released energy U is alone not sufficient for a crack spanned by the coating to begin to propagate unhindered; such propagation requires that the (average) stress σ in the core be increased, and since strain energy U is a quadratic function of stress, a running crack does not come into existence until after the stress

$$\sigma^+ = \sigma\sqrt{\frac{\Delta A + \Delta A_1}{\Delta U}} \qquad (8)$$

equal to the increased tensile strength of the core, has been reached. The corresponding increase of the extensibility is given by the course of the stress-strain diagram, the continuation of which beyond the limit of the original strength must be determined by experiment. For materials not deviating overly from Hooke's law the increase defined by formula (8) may be used for rough approximation.

For a crack which first grew restricted to the critical depth c while the coating above it was undergoing straining, the energy contribution of the coating obtained by a similar solution, turns out to be double. If the crack depth becomes larger than $0.5m$ and the major semi-axis of the unloaded region thus exceeds the maximum bond-stress transfer length m of the coating, the stress and strain in the coating cease to increase and only length l with failed bond continues to grow. The formula for the energy balance, which then starts to apply is

$$\lim_{\Delta c \to 0} \frac{\Delta A + \Delta A_1}{\Delta U} = 1 + \frac{R_b\sqrt{m}}{\sqrt{2EG_c}} \quad ; \qquad (9)$$

compared with the previous case, this represents considerable impairment and as a rule, immediate formation of a running crack. When the stress in the coating reaches the yield limit, length l no longer grows with the opening-up of the crack; only the coating continues to strain and then absorbs merely half the amount of energy than before. If a plane crack advances on the whole width b of a rectangular cross-section coated also on both side walls, the second term on the right-hand side of formula (7) and similarly so in other cases, must be multiplied by

$$\frac{\pi^2}{4}\left(\frac{E_1 t_1}{E_1' t_1'}\right)^2 \frac{c}{b} , \qquad (10)$$

Fig. 1: Bond stress and unloaded region for a spanned plane crack

Fig. 2: Bond stress for a spanned penny-shaped crack

where the values with prims refer to the coating on the side walls.

The region unloaded by one half of a penny-shaped crack centered on the core surface and with radius c has the form of a semi-ellipsoid of revolution with length $\frac{8}{\pi}c$ under simple tension. On assumptions same as above, the stresses in the core-coating bond follow the surface of a conoid (Fig. 2). Following integrations and some handling the formula for the energy balance improved by the originally non-strained coating becomes

$$\lim_{\Delta c \to 0} \frac{\Delta A + \Delta A_1}{\Delta U} = 1 + \frac{8 R_b}{\pi^2} \sqrt{\frac{c(1-\nu^2)}{\pi E G_c}} \quad , \quad (11)$$

where: ν - Poisson's ratio of the core; this formula is of the same type as formula (7). If in the course of the preceding opening-up of the crack the local stress and strain of the spanning-over coating grow from the very beginning, the energy contribution of the coating is again double.

The stress gradient in the core cross-section (under bending, etc.) bears a considerable effect on the released energy U and, in this way, also on further quantities. We shall give here only the result of the solution for a plane crack, plane stress-state, validity of Hooke's law and a stress-less coating, where the improved energy balance is

approximately defined by the formula

$$\lim_{\Delta c \to 0} \frac{\Delta A + \Delta A_1}{\Delta U} = 1 + \frac{2 R_b \sqrt{x}}{\sqrt{\pi E G_c}} \; \frac{\sqrt{\frac{c}{x}}\left(1 - \frac{c}{x}\right)}{\sqrt{\beta}} \qquad (12)$$

and in case of a coating already strained, by

$$\lim_{\Delta c \to 0} \frac{\Delta A + \Delta A_1}{\Delta U} = 1 + \frac{4 R_b \sqrt{x}}{\sqrt{\pi E G_c}} \; \frac{\sqrt{\frac{c}{x}}\left(1 - \frac{3c}{4x}\right)}{\sqrt{\beta}} \quad . \quad (13)$$

In the formulae, x denotes the distance of the neutral axis from the tension face of the core, and β is given by

$$\beta = 1 - \frac{3c}{2x} + \frac{2c^2}{3x^2} \quad . \quad (14)$$

If dangerous cracks occur at spacings lesser than the overall length of the adjoining unloaded regions, there come into play also the effect of appropriately smaller released strain energy of the core and the controlling action of the coating on the opening-up of neighbouring cracks, which circumstances improve the energy balance still further.

In all the cases mentioned above the increased strength of the core is determined by help of formula (8) but with a restriction concerning the relative stiffness of the coating. Thus, e.g. the increased tensile strength cannot exceed the value

$$\sigma^+ = \sigma\sqrt{2} \qquad (15)$$

if a very rigid coating completely prevents a crack at the core surface from opening up and thus turns a surface crack into an internal crack with half the dimension c.

It follows from the solutions presented in the foregoing that the effect of a tougher coating on the improvement of mechanical properties of a brittle core is indirectly dependent on the Young's modulus and on the G_c - value of the core material and directly dependent on the bond strength between the two materials and on the depth of the dangerous crack so that it practically fails to manifest itself in materials with fine structure and critical cracks of minute size (e.g. glass).

References:

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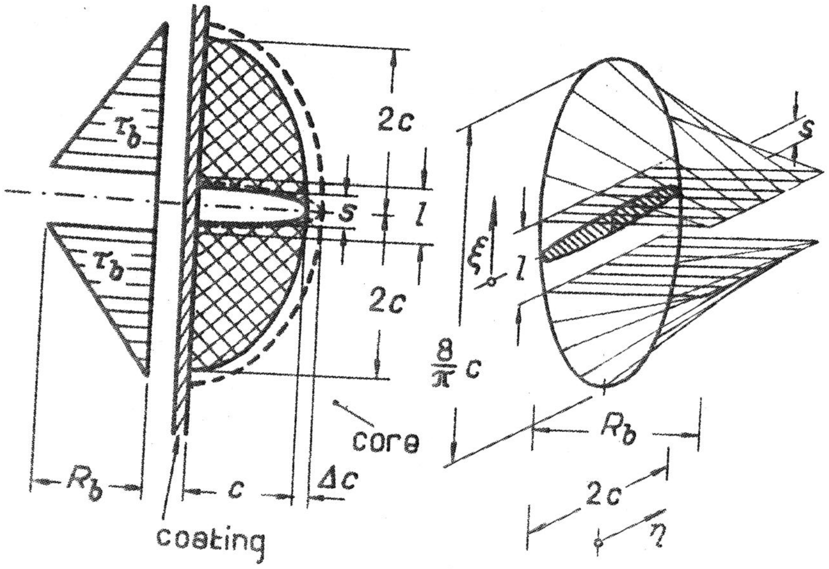
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