A Fracture Model for Fiber Reinforced Cementitious Materials

By A. E. Naaman,
Assistant Professor of Civil Engineering, California State University, Los Angeles.

A. Argon,
Professor of Mechanical Engineering,
Massachusetts Institute of Technology.

F. Moavenzadeh
Professor of Civil Engineering,
Massachusetts Institute of Technology.

Fiber reinforced cementitious materials are made with Portland cement matrices and with discontinuous randomly oriented and distributed fibers. Cementitious matrices show in general similar mechanical characteristics, i.e., relatively high compressive strength, poor tensile strength and brittleness at failure. The addition of fibers by amount of generally less than 10% is meant to enhance their tensile properties, delay cracking and increase their toughness.

The main differential characteristic of these composites, as compared to fiber reinforced polymeric and metal matrices, is that the ultimate tensile strain of the matrix is much lower than the yield or ultimate strain of the fiber. (Fig. 1). This implies that at some level of loading the matrix will crack and the resistance to full separation is to be opposed by the fibers bridging the cracked surfaces. In general, in this postcracking state and with current materials characteristics and properties, the fibers pull out under a load much below their load carrying capability. Further, depending on the ratio of the fiber length to the member length, the shape of the load elongation curve suggests that the composite fracture may be brittle or ductile-like. (Fig. 2).

A probabilistic model predicting the strength of fiber reinforced cementitious materials, with particular emphasis on fiber reinforced concrete has been developed. The model takes into consideration the statistical nature of most variables involved and recognizes the extreme value characteristic of tensile strength as stated in the chain weakest link hypothesis. In order to account for the fiber size effect on the ductile or brittle-like fracture response, two approaches were applied.

The first approach, simulating ductile failure, is based on the mechanics and statistics of composite materials. It explores in detail all that is going on at the one fiber level and extrapolates results to the macroscopic response of the composite. Simple expressions for the expected characteristic values of the composite cracking strength, maximum postcracking strength and surface energy for the one link chain are then derived and given as follows:

$$\bar{\sigma}_{cc} = \bar{\sigma}_{mu}(1-V_f) + 0.12\,\bar{\tau}\,V_f\,\frac{\ell}{\phi}$$

$$\bar{\sigma}_{cu} = \frac{1}{\pi}\,\bar{\tau}^*\,V_f\,\frac{\ell}{\phi}$$

$$\bar{\gamma}_c = \frac{1}{2}\left[\,\bar{\gamma}_{mu} + \bar{\sigma}_{cu}\,\frac{\ell}{6}\,\right]$$

Where $\bar{\sigma}_{mu}$ = matrix ultimate strength

$\bar{\gamma}_{mu}$ = matrix surface energy

V_f = volume fraction of fibers

ℓ = fiber length

ϕ = fiber diameter

$\bar{\tau}$ & $\bar{\tau}^*$ = bond strength, and bond strength modified by density of fibers.

Corresponding values for the N link chain tensile member are numerically determined using the mathematical apparatus of the chain weakest link concept.

The second approach simulates the brittle-like failure and uses the fracture mechanics criterion relating nominal strength, fracture toughness and critical crack radius in plane, i.e., $K_{IC} = \sigma_{nom.crit.}\sqrt{\frac{4a}{\pi}}$. The definition of crack length, however, introduces a new concept in discontinuous fiber reinforced composites where randomness of the fibers distribution is assumed. The definition takes into account the statistical distribution of largest inherent weak areas - i.e., areas without fibers - in a random cross section and recognizes the potential effect of these areas on the observed strength. The critical crack radius, a, is then computed as the sum of the radius δ of the largest inherent weak area and R the pseudo plastic zone radius (Fig. 3). The distribution of δ is assessed by a Monte Carlo simulation technique but can also be bounded by a mathematical lower bound solution. The pseudo plastic zone refers to an area where the matrix is cracked and the fibers are in a state of pull-out. Its radius R depends on the fiber length and is assumed to be determined experimentally as well as K_{IC} from double cantilever beam tests. The importance of the ratio δ/R is pointed out, and the above mentioned fracture mechanics relation is used to determine $\sigma_{nom.crit.}$ knowing K_{IC} and a. Figure 4 shows schematically the stress distribution around a typical crack.

An example of application to fiber reinforced concrete is presented and compared to experimental observations. It is concluded that the overall model leads to rather realistic bounds in predicting the strength characteristics of the composite.

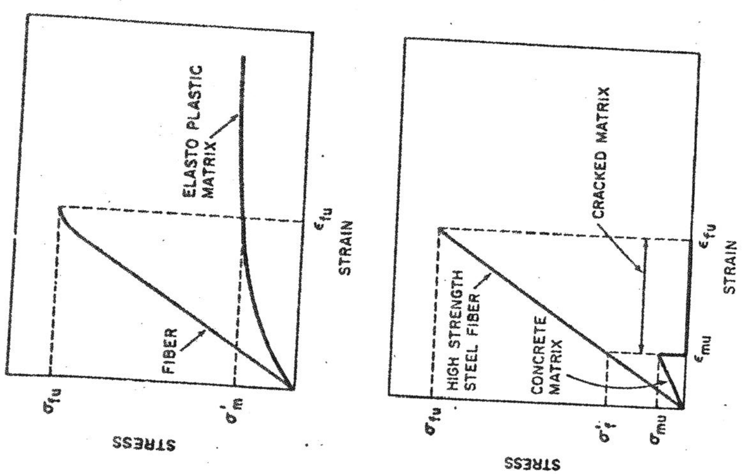

FIG. 1. TYPICAL STRESS-STRAIN DIAGRAMS OF FIBER-REINFORCED CONCRETE AS COMPARED TO MOST FIBER-REINFORCED PLASTICS.

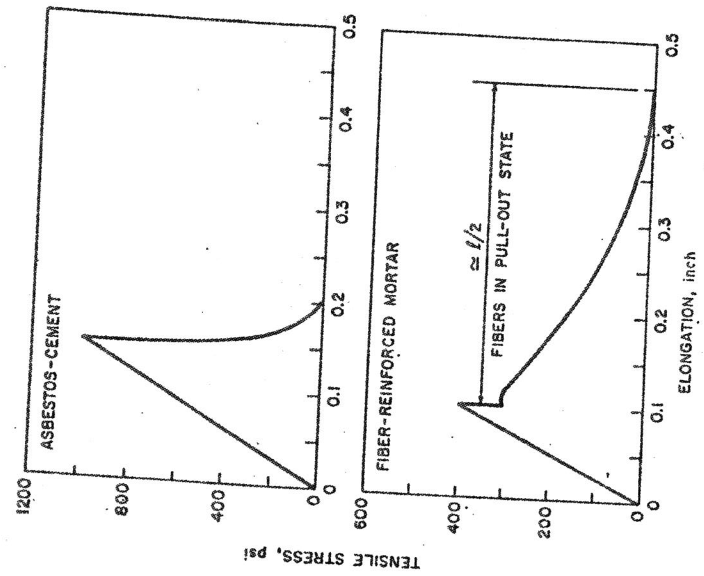

FIG. 2. TYPICAL STRESS ELONGATION CURVES AS INFLUENCED BY FIBER LENGTH.

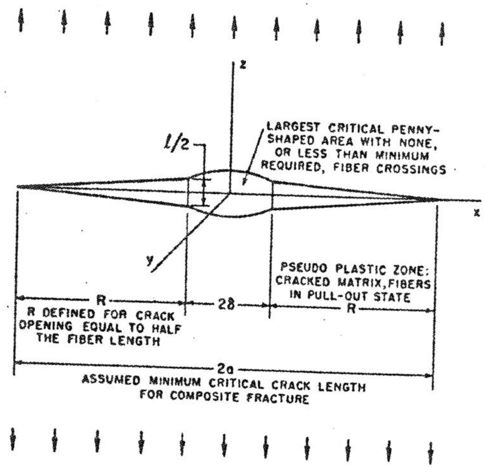

FIG. 3 ASSUMED CRITICAL CRACK MODEL CONTROLLING FRACTURE OF FIBER-REINFORCED CONCRETE.

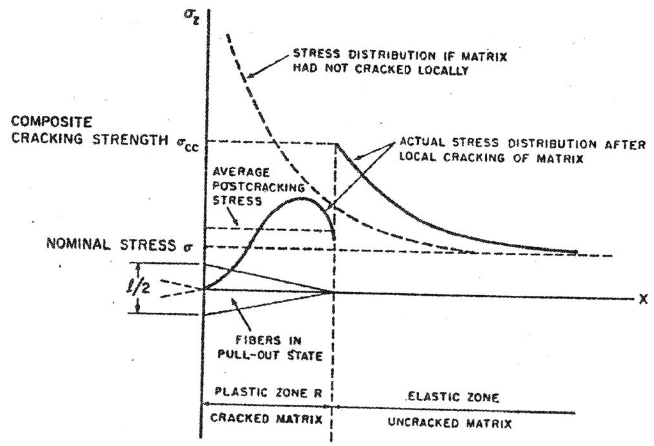

FIG. 4 DISTRIBUTION OF LONGITUDINAL STRESS AHEAD OF A CRACK IN A FIBER-REINFORCED CONCRETE COMPOSITE.