Motion of Crazing in Polymers

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The motion of a circular crazing in a polymeric medium is considered on the basis of the current understanding of polymers in a uniform stress field. The polymeric system undergoes molecular orientation and inhomogenization and create a new stress field which may be analyzed by using a deflection function and the variation of the Lagrangian function in terms of various energy quantities. Through this approach an equation of motion for the behavior of crazing in the interior of a polymer is derived.

INTRODUCTION

In another paper the dynamic behavior of edge crazing has been treated [1]. Even though stress crazing incepts much more readily from surfaces than in the interior of a specimen, a better understanding of its behavior is not likely obtained without considering internal crazing. Brittle polymers may be surface strengthened so as to create internal crazing eventually as a source of weakness and failure.

In treating the problem of the dynamics of crazing, one encounters the mathematical difficulty that the determination of the deformation and the transition from non-crazing to crazing surface is not clearly known. As a result, a reasonable assumption must be made to get a function describing the deflection of the crazing surface so that relevant energy quantities for governing the behavior of crazing may be computed and the equation of motion derived.

The assumption of a relatively simple function satisfying the boundary and initial conditions is expected to be an excellent approximation for the determination of the equation of motion of crazing. In this paper a brief description of an analysis for the propagation of a circular crazing contained in a homogeneous, viscoelastic medium is given. The equation of motion is derived from the fundamental principles of mechanics. In terms of the calculus of variations, Hamilton's principle or the Lagrange equations of motion is employed for this purpose.

PRINCIPLE APPLIED TO CRAZING

As often observed, assume that a circular crazing starts from the vicinity of a point flaw usually composed of different substance than the medium itself. Under a simple tensile stress field the circular crazing is found to expand uniformly and radially. Without knowing the exact stress distribution in the neighborhood of the "crazing front," the deflection of the crazing surface may be considered according to the model of a clamped circular plate under a uniform constant stress.

To a first approximation, if a constant stress field of simple tension covers the circular crazing area of radius c, the elastic deflection curve would be [2]

$$z(c,r) = \frac{1}{64D}\left(1 - \frac{r^2}{c^2}\right)^2 c^4\sigma \qquad (1)$$

where $D = Eh^3/12(1-\nu^2)$ is the bending stiffness factor with E as the modulus of elasticity ν as Poisson's ratio, r is the radial polar coordinate and h is the constant thickness of the plate associated with the development of crazing provided that σ does not vary much. Thus the problem can be handled as (1) satisfies the following boundary and symmetry conditions:

$$z(r,c)_{r-0} = z(0) = \frac{\sigma}{64D} \qquad (2)$$

$$z(r,c)_{r=c} = 0 \qquad (3)$$

$$\left(\frac{\partial z}{\partial r}\right)_{r=0} = \left(\frac{\partial z}{\partial r}\right)_{r=c} = 0 \qquad (4)$$

where z(0) is the initial deflection. This kind of solutions can be adapted to account for time-dependent behavior. If the medium is considered to be linearly voscoelastic correspondence principle may be employed to account for the time-dependent behavior. By introducing a function such that

$$f(t) = \mathcal{L}^{-1}\,\frac{1}{64sD(s)} \qquad (5)$$

where the Laplace inverse of essentially the stiffness function in its Laplace domain s yields the function. Thus (1) may be written as

$$z(r,c,t) = f(t)\,c^4\left(1 - \frac{r^2}{c^2}\right)^2\sigma \qquad (6)$$

is the time dependent deflection. This function will be used to calculate the Lagrangian L of the nonconservative system which is

$$L = T + Q - U - S \qquad (7)$$

where

$$T = \rho\pi h \int_0^c \left(\frac{dz}{dt}\right)^2 r\,dr \qquad (8)$$

is the kinetic energy,

$$Q = -\dot{W}/2 \qquad (9)$$

is the heat dissipation function. Q plays the same part with respect to velocities in the dissipative system that potential function plays relative to the coordinates in the conservative system with

$$\dot{W} = \int_0^c 2\pi r\dot{z}\sigma\,dr \qquad (10)$$

as the work done per unit time.

$$U = \int_0^c \frac{Eh^3}{12(1-\nu^2)}\,\pi\left[\left(\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r}\right)^2 - 2(1-\nu)\frac{\partial^2 z}{\partial r^2}\frac{1}{r}\frac{\partial z}{\partial r}\right] r\,dr$$

is the strain energy (11)

and

$$S = \pi[c^2(t) - c^2(0)]\lambda \qquad (12)$$

is the increment of the "surface" energy in which c(0) is the original crazing radius and λ is the increase in the energy when crazing is formed.

After computation of these energy quantities and neglect Q for slow development of crazing, one finds that L becomes a function of $\dot{c}$,c and t. Applying the variational principle that

$$\delta \int L(\dot{c},c,t)\,dt = 0 \qquad (13)$$

and finally an equation of motion can be obtained

$$c^7 f^2(t)\ddot{c} - 8c^6 f^2(t)\dot{c}^2 - 2c^7 f(t)\dot{f}(t)\dot{c}$$
$$+ \frac{3}{4}c^8 f(t)\,\ddot{f}(t) + \frac{c^4}{2}\,\frac{f(t)}{\rho h} + \frac{8}{3}\,\frac{Eh^3 f^2(t)c^4}{1-\nu^2)\rho h}$$
$$- \frac{3}{2}\,\frac{\lambda}{\rho h\sigma^2} = 0 \qquad (14)$$

It is seen that there are quite a number of difficulties in interpreting this governing equation. First of all the dissipation of heat is not known so that the temperature rise in the vicinity of crazing is not easily estimated. Therefore the function f(t) cannot be properly represented. However, roughly a general time dependent function may be expected of the following form if τ is the relaxation time of the medium

$$\frac{1}{f(t)} = \frac{16Eh^3}{3(1-\nu^2)}\left(1 + \frac{t}{\tau} + e^{-t/\tau}\right) \qquad (15)$$

This characteristic time τ together with the tensile stress σ bring out the threshold condition for the propagation of crazing. In treating the limiting case of stability $\dot{c}$ is not of interest, thus one may let $\dot{c} = 0$ in (14) and the remaining terms may be consulted.

Aside from these an important concern is that crazing is a manifestation of the changing stress field as a result of large local deformation and molecular orientation. It may be justified to consider a modification of the model by superimposing a concentrated load with relatively high intensity uniformly distributed along the circular crazing front. Thus the stress distribution above or below crazing plane will not be uniformly distributed. The deflection function can be modified by introducing the following expressions

$$z(r,c,t)_{r<\zeta} = nf(t)\left[-(r^2+\zeta^2)\log\frac{c}{\zeta} + (r^2-\zeta^2) + \frac{1}{2}\left(1+\frac{\zeta^2}{c^2}\right)(c^2-r^2)\right]c^2\sigma \tag{16}$$

$$z(r,c,t)_{r>\zeta} = nf(t)\left[-(r^2+\zeta^2)\log\frac{c}{r} + \frac{1}{2}\left(1+\frac{\zeta^2}{c^2}\right)(c^2-r^2)\right]c^2\sigma \tag{17}$$

where n is an intensity factor of the concentrated stress round a circle of radius ζ whose magnitude must be less than c. This increases greatly the complexity in the analysis. It is hoped that additional understanding of the physics of the propagation process of crazing may be obtained through this modification.

REFERENCES

[1] C. C. Hsiao "Crazing--Yielding, Deforming or Fracturing?" to be published.

[2] A. E. H. Love, Mathematical Theory of Elasticity, 4th edition, Dover (1927) 490.