Growth of Fatigue Cracks in Thermoplastics

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1. Introduction Empirical studies of fatigue failure in polymeric materials have revealed that the failure of these materials under cyclic loading conditions takes place as a consequence of the occurrence of one of the following three phenomena:

1) Creep (strain accumulation) due to cyclic loading
2) Propagation of flaws up to a critical length
3) "Softening" of the material due to an excessive rise in temperature within its bulk. (Cyclic thermal softening).

In the present paper fatigue failure in the absence of high material temperature will be discussed; the failure process is divided into initiation and propagation.

2. Determination of Crack Growth Initiation Life Cherepanov (1968) analysis of failure in solids which is founded upon thermodynamic energy balance concepts, resulted in the following criticality condition for viscoclastic solids:

$$\pi \int K \frac{1-\nu(t)}{E(t)} \dot{K}\, dt + \frac{\pi}{2} \frac{K}{E(t)} [3+\nu(t)]\, K = 2\gamma_* \qquad (1)$$

where K is the stress intensity factor, γ_* is the specific fracture energy and $E(t)$ and $\nu(t)$ are time dependent modulus and Poisson's ratio. As discussed by Williams (1971), for an incompressible material ($\nu=\frac{1}{2}$) with a simple 3-element material model the creep compliance is given by

$$D_{crp}(t) = D_e - \frac{D_e - D_g}{e^{xt}} \qquad (2)$$

where $x = \frac{1}{\tau}$, D_g is the glassy value and D_e the rubbery value of the compliance.

Let $$K(t) = \sigma(t) \left[\frac{a(t)}{2}\right]^{\frac{1}{2}} \quad (3)$$

with an initial crack length of a_o and an external disturbance of the form

$$\sigma(t) = \sigma_o + \sigma_1 \sin \omega t \quad (4)$$

From the above equation, the time to failure $t_f (= \frac{2\pi}{\omega} N_i)$ is calculated (Arad 1972) where N_i = the number of cycles for initiation:

$$N_i = \frac{\omega}{2\pi x} \ln \Delta$$

where $$\Delta = \frac{3\pi\sigma_o^2 a_o D_e (x^2 + \omega^2 + x)}{3\pi\sigma_o^2 a_o D_e (x^2 + \omega^2 - \omega x) - 8 \gamma_* (x^2 + \omega^2)} \quad (5)$$

with $N_i = \infty$, the level of maximum applied stress intensity factor corresponding to fatigue strength is found to be:

$$K_* = \frac{4\gamma_*}{3\pi D_e} \left(\frac{x^2 + \omega^2}{x^2 + \omega^2 - \omega x} \right) \quad (6)$$

Fig.1 shows a plot of N_i against σ_o. Some experimental data obtained from tests on Polymethylmethacrylate were superimposed on the theoretical curve and reasonably good correspondence was observed.

3. Macroscopic Crack Propagation Under Cyclic Loading Conditions

3.1. The effect of the amplitude and the mean level of stress intensity factor:

A comprehensive programme of testing on the three materials (PMMA, PC, N 6.6) revealed very significant effects on the cyclic rate of crack propagation, $\dot{a}_N$, of the mean level and amplitude of the stress intensity factor (K_m and ΔK). It was shown that by an appropriate choice of the maximum and minimum limits of the load cycle, it was possible to obtain higher crack growth rates at

smaller ΔK values. Based upon experimental data, and from a general analysis of the crack propagation process (in terms of the relationships between $\dot{a}_N$, the COD and crack tip plasticity) the following model for crack propagation was proposed:

$$\dot{a}_N = \beta\lambda^n \quad (7)$$

where $\lambda = K_{max}^2 - K_{min}^2$ ($= 2\Delta K \, K_m$) and β and n are constants.

It was thus demonstrated that irrespective of values of K_{max} and K_{min} the rate of crack propagation may be described by parameter λ. This model has been shown to be applicable to fatigue failure process in some metals (Arad et al 1973). Such data on metals, which are normally presented in the form of graphs of $\dot{a}_N$ related to ΔK at various values of $R = K_{min}/K_{max}$, were shown to fall on a single line when presented on the basis of λ model.

3.2. The effect of loading frequency:

The effect of frequency on the value of $\dot{a}_N$ was investigated at 0.1, 5 and 20Hz. It was found that, contrary to the case in most metals, in general in polymers the loading frequency had a significan effect on $\dot{a}_N$. The cyclic rate of crack growth decreased as frequency was raised. The effect of frequency may be attributed to two sources:

1) The effect of the external parameters: Here the change in fracture toughness level of the material with increase in crack speed (or rate of load application) may be considered. It has been shown that for example for PMMA, K_{1c} rises with crack speed in the stable growth region. This is expected to result in a smaller rate of growth per cycle under high frequency fatigue conditions.

In association with this effect, the influence of load rate on the extent of crack tip plasticity is important. The existence of strain and time limits for development of crazes of a certain size in some glassy polymers have been demonstrated. A stress applied for a shorter time will result in a crazed zone smaller than one which will result from application of the same stress for a longer time. This aspect of craze formation in polymers (not yet fully documented) will be immensely useful in the analysis of the fatigue failure process.

2) The effect of parameters associated with molecular structure: Analysis of relaxation data for thermoplastic materials yields information on the specific frequency levels which result in achievement of maxima in loss modulus (E") values at a certain temperature. Clearly, depending upon the position of the test frequency range in relation to such peaks, the value of E" will increase or decrease with increasing frequency. The extent of viscous energy absorbed per cycle $\dot{W}_v$ is related to the loss modulus parameter:

$$\dot{W}_v = \frac{\pi E''}{E''^2 + E''^2} \left(2\sigma_m^{\ 2} + \sigma_a^{\ 2}\right) \qquad (8)$$

For a constant total input energy, if $\dot{W}_v$ increases with increasing frequency, the amount of energy available for crack propagation per cycle will be reduced. Hence a reduction of $\dot{a}_N$ is expected.

3.3. <u>The effect of cyclic waveform</u>: This effect was studied from tests with sinusoidal and triangular waveforms. It was found that the value of $\dot{a}_N$ under sinusoidal loading conditions were slightly higher than those under triangular ones. It is expected that, under the same loading conditions, as the waveform is varied from triangular via sinusoidal and trapozoidal to a square wave, the extent of damage per cycle is increased. Such an effect may be attributed to an increase in 'hold time' under higher loads, as the waveform is changed in the above manner.

3.4. <u>The effect of orientation</u>: Tests were carried out on Nylon 6.6 specimens cut from extruded sheets, in two perpendicular directions

(one in the direction of extrusion). The effect of molecular orientation in the direction of extrusion was observed in the form of a 10-15% smaller rate of growth for fatigue cracks propagating in a direction perpendicular to the direction of extrusion.

<u>References:</u>

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FIG. 1: DETERMINATION OF CYCLIC CRACK GROWTH INITIATION LIFE.

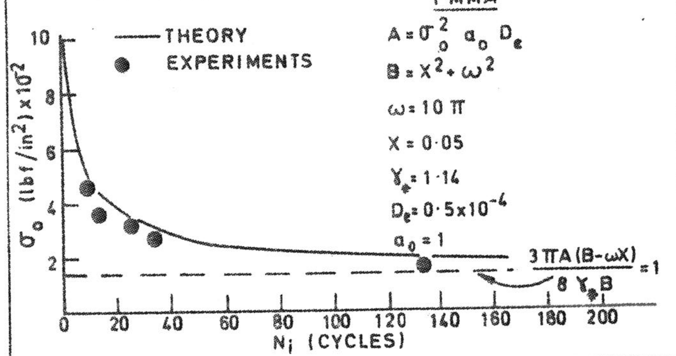

FIG. 2: FATIGUE CRACK GROWTH DATA RELATED TO PARAMETER λ.