The Conditions Low Cycle Crack Initiation and Propagation in Stress Concentration Zones

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SUMMARY The low cycle fracture initiation in the stress concentration zones is determined by cyclic and one-side plastic strain accumulation; the crack propagation is determined by elastoplastic strain intensity. The calculated elastoplastic strain fracture kinetics are experimentaly confirmed.

INTRODUCTION The number of cycles N_f needed for fracture in the stress concentration zones under low cycle loading is equal to the sum of numbers of cycles till crack initiation N_c and for crack propagation N_p $(N_f = N_c + N_p)$. The value N_c is determined /1/ taking into account the cycle-by--cycle changes of plastic strains on the base of linear addition of quasistatic d_s and fatigue d_f damages

$$d_s + d_f = 1 \qquad (1)$$

The value d_s is equal to the ratio of one-side accumulated strain $\bar{e}^{(k)}_{pmax}$ to the corresponding critical value under simple tension e_f, the value d_F denoting the linearly accumulated fatigue damage under cyclic strain of variable amplitude $\bar{e}^{(k)}_{pa\,max}$ being determined on the base of power type fatigue curve equation by Manson-Coffin. The values of stress $\bar{\sigma}$ and strain $\bar{e}$ denote the ratio of the respective values to the values of the yield point σ_Y and e_Y $(\bar{\sigma} = \sigma/\sigma_Y$, $\bar{e} = e/e_Y)$. The value N_p is found by integrating the expression for the crack propagation rate $d\ell/dN$ depending on the level of nominal stress $\bar{\sigma}_n$, on the elastic stress concentration factor α_σ and on cyclic properties of the material. The strain criteria of fracture are used for evaluation of N_c and N_p.

RESULTS The maximum strain values $\bar{e}^{(o)}_{max}$ in the stress concentration zones in the zero half-cycle are equal:

$$\bar{e}^{(o)}_{max} = \bar{e}_n \cdot K_e \qquad (2)$$

where $\bar{e}_n$ is the nominal strain; K_e is the strain concentration factor. The stress-strain diagram is approximated by the power equations:

$$\bar{6}_n = \bar{e}_n^{m_o} \quad \text{and} \quad \bar{6}^{(o)}_{max} = \bar{e}^{(o)m_o}_{max} \qquad (3)$$

where m_o is the hardening factor in the elasto-plastic region. The value K_e in elasto-plastic condition is calculated /2/ on the base of the following modifications of the Neuber formula:

$$K_e = \frac{\alpha_6^{2/(1+m_o)} \cdot \bar{6}_n^{(1-m_o)(1+m_o)}}{(\alpha_6 \cdot \bar{6}_n)^{n(1-m_o)[1-(\bar{6}_n-1/\alpha_6)]/(1+m_o)}} \quad \text{for } \bar{6}_n \leqslant 1$$

$$K_e = \frac{\alpha_6^{2/(1+m_o)}}{(\alpha_6 \cdot \bar{6}_n)^{n(1-m_o)[1-(\bar{6}_n-1/\alpha_6)]/(1+m_o)}} \quad \text{for } \bar{6}_n \geqslant 1 \qquad (4)$$

in which n =0.5. The maximum values of stresses $\bar{S}^{(k)}_{max}$ and strains $\bar{\mathcal{E}}^{(k)}_{max}$ in the stress concentration zones at half-cycle k are determined /1/ from the stress-strain diagram according to (3): $\bar{S}^{(k)}_{max} = \bar{\mathcal{E}}^{(k)m(k)}_{max}$ ($m(k)$ is the cyclic hardening factor). The values $\bar{S}^{(k)}_{max} = S^{(k)}_{max}/S_Y$ and $\bar{\mathcal{E}}^{(k)}_{max} = \mathcal{E}^{(k)}_{max}/\mathcal{E}_Y$ are measured in coordinates with a centre at the point of unloading. The value $m(k)$ found from experimental data for $S_Y = 2 6_Y$ and $\mathcal{E}_Y = 2e_Y$ is:

$$m(k) = \lg \bar{e}^{(o)m_o}_{max} / \lg\left[\bar{e}^{(o)m_o}_{max} + \frac{A}{2}(\bar{e}^{(o)}_{max} - 1)F(k)\right] \qquad (5)$$

where A is the material characteristic. The function $F(k)$ for cyclic stable, hardening and softening materials are /1/:

$$F(k) = 1; \quad F(k) = (k-1)^{-B(\bar{e}^{(o)}_{max}-1)}; \quad F(k) = \exp C(\bar{e}^{(o)}_{max} - 1)$$

respectively (B and C are parameters of the cyclic stress-strain diagram). Then $\bar{\mathcal{E}}^{(k)}_{max} = \bar{\mathcal{E}}_n \cdot K^{(k)}_{\mathcal{E}}$ and $\bar{\mathcal{E}}^{(k)}_{apmax} \cong \frac{1}{2}(\bar{\mathcal{E}}^{(k)}_{max} - 2) = \frac{\bar{e}^{(k)}_{apmax}}{2}$

The strain concentration factor $K^{(k)}_{\mathcal{E}}$ is evaluated by (4) substituting $m(k)$ for m_o . The one-side accumulated strain $\bar{\Delta}^{(k)}_{max}$ during the half-cycle k is equal to the difference of maximum strain for half-cycles k and $k-1$ ($\bar{\Delta}^{(k)}_{max} = \bar{\mathcal{E}}^{(k)}_{max} - \bar{\mathcal{E}}^{(k-1)}_{max}$). The total strain $\bar{e}^{(k)}_{pmax} \cong 2\bar{\mathcal{E}}^{(k)}_{pmax} = 2\int_0^k \bar{\Delta}^{(k)}_{max}\, dk$.

The damages d_s and d_f in accordance with /1/ are as follows: $d_s = \bar{e}_{pmax}/\bar{e}_f = \frac{2}{\bar{e}_f}\int_0^k \bar{\Delta}^{(k)}_{max}\, dk$; $d_f = \frac{1}{2(\bar{e}_f/4)^{1/m}}\int_0^k \bar{e}^{(k)1/m}_{apmax}\, dk$ (6)

where m is the power index of the Manson-Coffin equation ($m \cong 0.5$) and $\bar{e}_f = \frac{1}{\bar{e}_Y}\ln\frac{1}{1-\psi}$ (ψ is the reduction in area). The number of half-cycles $k_c = 2N_c$ till the crack initiation is determined /1/ by (1) as the upper limit of integrals (6). The following cyclic loading is accompanied by crack propagation and a variation of the strain at the crack tip. The evaluation of local strains at the crack tip is carried by using (4) and value of

$$\alpha_6 = \bar{K}_I/(\sqrt{2\pi\tau})\bar{6}_n \qquad (7)$$

where $\bar{K}_I$ is the stress intensity factor; τ is distance from crack tip. Neglecting $1/\alpha_6$ for small τ in (7), we obtain:

$$K_e = \frac{1}{\bar{6}_n}(\bar{K}_I/\sqrt{2\pi\tau})^{[2-n(1-m_o)(1-\bar{6}_n)]/(1+m_o)} \quad \text{for } \bar{6}_n \leqslant 1$$

$$K_e = \frac{1}{\bar{6}_n^{2/(m_o+1)}}(\bar{K}_I/\sqrt{2\pi\tau})^{[2-n(1-m_o)(1-\bar{6}_n)]/(1+m_o)} \quad \text{for } \bar{6} \geqslant 1 \qquad (8)$$

The strain near the crack tip can be determined using (2):

$$\bar{e}^{(o)}_{max} = \bar{e}_n \cdot K_e = \bar{K}_{Ie}(2\pi\tau)^{P_{\tau e}} \qquad (9)$$

where $\bar{K}_{Ie}$ is the strain intensity factor; $P_{\tau e}$ - index of power. From (8) and (9) it follows that

$$\bar{K}_{Ie} = \bar{K}_I^{P_{ke}} \quad \text{for } \bar{6}_n \leqslant 1; \quad \bar{K}_{Ie} = \bar{6}_n^{(1-m)(1+m_o)} \quad \text{for } \bar{6}_n \geqslant 1 \qquad (10)$$

$$P_{ke} = [2 - n(1-m_o)(1-\bar{6}_n)]; \quad P_{\tau e} = P_{ke}/2 \qquad (11)$$

The crack propagation condition is assumed as the reaching of local strain $\bar{e}^{(k)}_{max}$ at the crack tip to a critical value $\bar{e}_{fc}$. The fracturing zone length τ_f is obtained from (9) for $\bar{e}^{(k)}_{max} = \bar{e}_{fc}$, taking into account the increasing of the initial crack length with the nominal stress to $\bar{6}_n$ (by the cut and try method):

$$\tau_f = \frac{1}{2\pi}(\bar{K}_{Ie}/\bar{e}_{fc})^2\left[1 - \frac{1}{2\pi\ell_o}(\bar{K}_{Ie}/\bar{e}_{fc})^2\right]^{-1} \qquad (12)$$

The crack propagation rate can be calculated from (12) in the case of cyclic loading with a strain intensity factor range $\Delta\bar{K}_{Ie}$ (in coordinate "$\bar{S}-\bar{\mathcal{E}}$" for $d\ell/dN = \tau_f$ and $m_o = m(k)$)

$$d\ell/dN = \frac{1}{2\pi}(\Delta\bar{K}_{Ie}/\bar{\mathcal{E}}_{fc})^2\left[1 - \frac{1}{2\pi\ell_o}(\Delta\bar{K}_{Ie}/\bar{\mathcal{E}}_{fc})^2\right]^{-1} \qquad (13)$$

If the crack length is subcritical $\frac{1}{2\pi\ell_0}(\Delta\bar{K}_{Ie}/\bar{\mathcal{E}}_{fc})^2 \ll 1$,

$$d\ell/dN = (1/2\pi\bar{\mathcal{E}}_{fc})^2 \Delta\bar{K}_{Ie}^2 \qquad (14)$$

The value $d\ell/dN$ from (9) and (14) is propartional to the square of the range of elasto-plastic low-cycle strain at the crack tip $(d\ell/dN \sim (2\bar{e}_{amax}^{(k)})^2$. Then, from (10), (13) and (14):

$$d\ell/dN = C_{16}\,\Delta\bar{K}_I^{\gamma}\left[1-\frac{C_{16}}{\ell_0}(\Delta\bar{K}_I^{\gamma})\right]^{-1} \qquad (15)$$

where $\gamma = 2p_{ke}$ and $C_{16} = 1/2\pi\bar{\mathcal{E}}_{fc}^2$. For the values $m(k)$ =0.02 to 0.3 and $\bar{\sigma}_n$ =0.3 to 1 the value γ =2.7 to 3.93, this being in agreement with results of /3,4,5/.

In this connection cyclic softening Cr-Mo-V and low-carbon (0.21%C) cyclic stable steels were tested under constant loading amplitudes for different asymmetry factors τ_σ within -1.2 and +0.6 and values α_σ. The strain measurements were made by the grid method with a grid pitch of 0.1mm. The interrelation between the values $d\ell/dN$ and $2\bar{e}_{amax}^{(k)}$ independent on loading conditions is shown in Fig.1. The experimental data are in agreement with (9) and (14). A comparison of calculated data and experimental results at the stages of crack initiation $(N \leq N_c)$ and crack propagation $(N_c \leq N \leq N_f)$ are shown in Fig.2 and Fig.3. The final fracture is determined by reaching at the crack tip. $\bar{e}_{max}^{(k)} = \bar{e}_{fc}$ Critical value of $\bar{e}_{fc}$ remains constant for different concentration levels and fracturing numbers of cycles.

REFERENCES

1. Серенсен С.В.,Махутов Н.А.,Шнейдерович Р.М. Машиноведение, 1972, № 5.
2. Махутов Н.А. Машиноведение, 1971, № 6.
3. Paris P., Erdogan F. Trans. ASME, Ser. D, 1963, Dec.
4. Liu H.W., Iino N., Fracture, Chapman and Hall, 1969.
5. Черепанов Г.П. Прикладная механика и техническая физика, 1968, № 6.

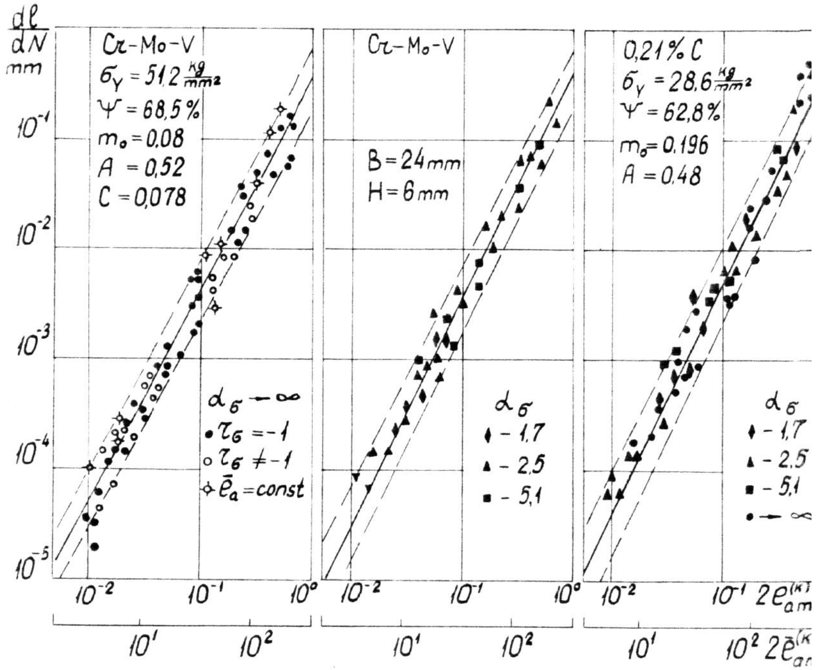

Fig.1 Crack propagation rate versus local strain ran

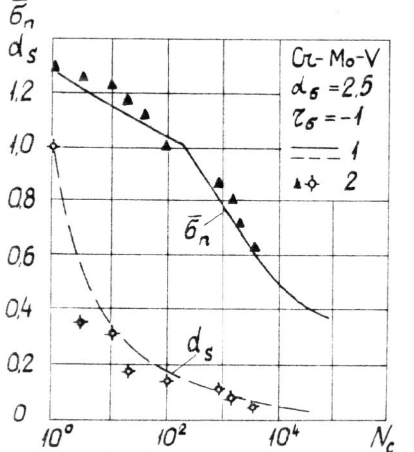

Fig.2 Characteristics of crack initiation (1 - analytical, 2 -experimental).

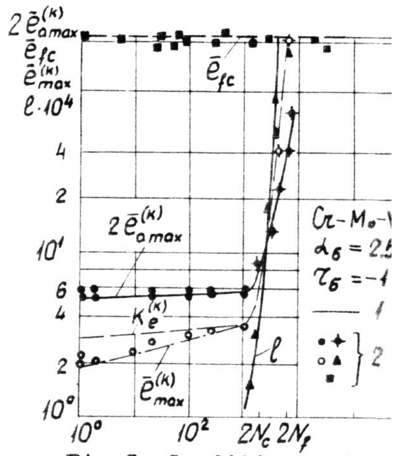

Fig.3 Conditions of crack initiation and propagation (1 -analytical, 2- experimental).