Planning and Evaluation of Fatigue - Tests
a Survey of Recent Results

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Today the existence of great scattering in the results of fatigue-tests can be taken for granted /1/. Small scattering is the exception. Expensive tests with a great number of specimens are indispensable. Lowering the costs necessitates careful planning and evaluation of fatigue-tests.

Using the Wöhlerdiagram, the results may be divided into two groups, figs.1 and 2 /1/. In fig.1, which applies to structural steel and, possibly, some light metals, the full lines give the range within which almost all specimens will break. Today the diagram is divided into the ranges of finite endurance (RFE), transition (RT) and infinite endurance (RIE). In RFE all specimens will break before the greatest experimental number of cycles N_G is reached. In RT some will break, some not. In RIE none will break.

Often RT is termed the range of infinite life, and values won there are taken for fatigue strength data. This is wrong! If the existence of scattering is assumed in principle, the separation of RT and RIE is imperative.

The fatigue limit is given by the line between RT and RIE. For its correct determination $N_G = 5 \cdot 10^6$ is sufficient for steel, as shown in /2/ and numerous other publications.

The scatter of most fcc-metals is smaller, and the full lines approach the logN-axis asymptotically, fig.2. By definition there exists only one range of finite endurance. This conception, though correct, is too severe because, at least, a substitute value for the fatigue limit is needed in practice. The experiments are stopped at N_G; a vertical line there divides the diagram again into "RFE", "RT" and "RIE" which are now substitutions. This way it is possible to determine substitute values of fatigue data, the quality of which depends on N_G, usually chosen between $2 \cdot 10^6$ and 10^8 /1/.

When testing a sufficiently great number of specimens at levels of constant stress or strain amplitude, cumulative frequency distributions can be noted /1/. In RFE a distribution can be found for any level, when the order numbers "i" or mathematical derivations are plotted vs. the corresponding cycles to fracture, fig.1. In RT only one curve of a cumulative distribution can be won out of all test levels. Here the number of all fractured specimens "r" or a mathematical derivation at each level is plotted vs. the load values.

The mathematical treatment of the distributions makes it possible to calculate values of fatigue strength with certain probabilities of fractures "P_F" even if they are not covered experimentically. In the figures 1 and 2 the lines may be taken for P_F=1% and 99%.

There is no consensus on the appropriate probabilities of fracture. Restriction to 50% alone should not be permitted because in cases of differently wide ranges of transition, materials or designs cannot be compared. On the other hand, it is not advisable to give estimates of values with a 0% or 100% probability of fracture. The samples are too small and thus the probabilities of 0% and 100% will mean a juridical risk out of statistical laws /3/. Therefore a grouping of values with the probabilities of 1% or 10%, and 50% and 90% or 99% should be calculated.

The mathematical functions and the methods of planning were investigated, using the results of the one-step tests with a constant maximum and minimum of amplitude. They are supposed to apply also to tests with predetermined changes of load (programme-, random tests etc.)

In the transition range four levels, arranged in an even manner, with 10 to 12 specimens each, should be tested. From the investigated Gaussian-, arc sin$\sqrt{\ }$ - and extreme value-functions, the Gaussian seems to be the best /2,4,5,6/. It is most simple and most reliable in results. The estimates "p" of the expected values of the probabilities of fracture, which have to be plotted into

the Gaussian cumulative paper vs. alternating load or cycles to fracture, are calculated from "r" by using the formula $p=100\cdot(3r-1)/(3n+1)$ [%], according to /3,7/.

There exist two methods which were derived from the Gaussian distribution, the staircase method and the "boundary technique". The first is used all over the world but not recommendable nevertheless /8/. With acceptable numbers of specimens it allows the determination of 50% values only. The reason is that it experimentally concentrates the specimens on the median.

By means of the boundary technique, however, all specimens are concentrated at two levels of load near the borders of the transition range. The method offers ways to find those two levels /3/. The boundary technique is supposed to allow a very reliable determination of RT and requires, at present, a minimum of planning, evaluation and specimens.

A further way for the evaluation of RT is given by the methods of cumulative fatigue damage, which are much disputed. This dispute seems to be superfluous because these methods involve additional scattering as it is not possible to increase loads with absolute accuracy and as the machine will be stressed in various ranges of rigidity and resonance with every specimen. This has been confirmed by the surprisingly large scattering of Miner-values /12/ and the statement that 6 to 8 specimens are necessary to determine the median and 25 specimens to determine the extension of RT with only a small degree of reliability /5/. For better reliability the number of specimens has to be raised. Thus even the advantage of small numbers of cycles to fracture for every specimen vanishes in cumulative fatigue damage.

The German standard DIN 50100 gives the possibility to find a value in RT but without any idea of its probability of fracture. The reliability is equal to that of the staircase method if in each case only 6 or 10 specimens are used.

The finite endurance range cannot be covered at all by means of the German standard DIN 50100. It is better to concentrate all specimens at a few levels of alternating load /2/. The minimum number of specimens at each level is 10; but 15 if $N_{max}:N_{min} > 10:1$, and 20 if $N_{max}:N_{min} > 30:1$ at one level. For scientific research with a claim to a high degree of reliability these numbers have to be doubled.

Experience has shown that for technological investigations in RFE one level of load can be assumed for every half magnitude as related to N_{50}, N_{50} being the number of cycles with 50% probability of fracture; for scientific work with high reliability one level for every 1/3 magnitude.

The evaluation of the distribution of fractures at the various levels and the calculation of numbers of cycles with certain probabilities of fractures may be done by means of several functions. Investigated were the extreme-value-method, several arc sin$\sqrt{\ }$-functions and the Gaussian cumulative distribution /2,5,6/; only the first was based on theory /10,11/. Nevertheless in practice the Gaussian function gives the most reliable estimates and means a minimum of work /2/. Similar to RT also in RFE the estimates "p", which have to be plotted into the Gaussian cumulative paper, are calculated from "i" by using the formula $p=100\cdot(3i-1)/(3n+1)\,[\%]$, but $p=100\cdot i/(n+1)\,[\%]$ is acknowledged too /2/.

Points with the same probabilities of fracture can be linked by lines, as was done in figs.1 and 2. It has often been tried to find mathematical functions for this linking /5,6,9/. There are reasons not to do so, and also the amount of work does not pay off. In particular it must be warned of extrapolating into ranges not investigated.

/1/ Maennig et al., Materialprüfung 14 (1972) p.249/54
/2/ Maennig, Fortschrittber.VDI-Z. Nr.5, Reihe 5, 1967
/3/ Maennig, Materialprüfung 12 (1970) p.124/31
/4/ Dorff, Dissertation Techn. Univ. Berlin 1961
/5/ Mäiß, Dissertation Techn. Univ. Berlin 1965
/6/ Dengel, Dissertation Techn.Univ. Berlin 1967
/7/ Rossow, Z.f.wirtsch.Fertigung 59 (1964) p.596/97
/8/ Maennig, Materialprüfung 13 (1971) p.6/11
/9/ Kosteas, Aluminium 48 (1972) p.147/54
/10/ Gumbel, Mitt.Math.Stat. 8 (1956) p.97/130
/11/ Freudenthal et al., J.Amer.Stat.Ass.49 (1954) p.575/97
/12/ Jacoby, Fortschrittber.VDI-Z. Nr.7, Reihe 5, 1969

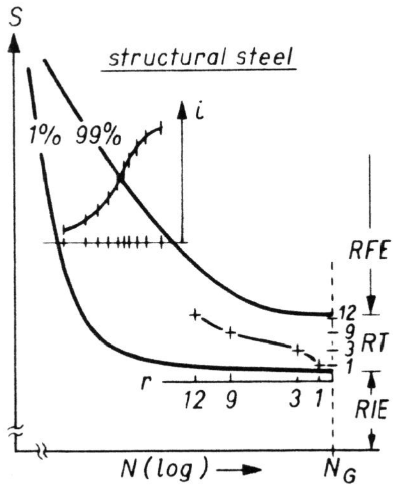

fig.1: Wöhlerdiagram typical for structural steel

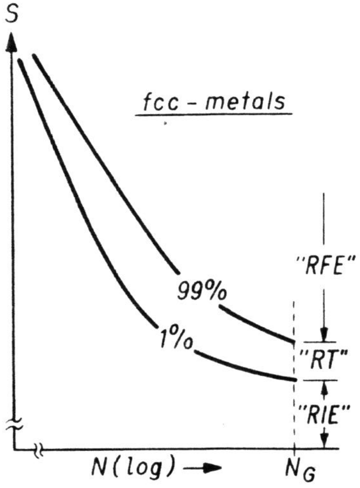

fig.2: Wöhlerdiagram typical for fcc-metals

S = amplitude of alternating stress or strain; maximal amplitude in case of programme-or randomtests
N = number of cycles
N_G = greatest experimental number of cycles
1%,99% = probabilities of fracture
RFE = range of finite endurance
RT = range of transition
RIE = range of infinite endurance
| = positions of fractured specimens in RFE
i = order number of the positions of fractured specimens
-9 = level of alternating load and number r of fractured specimens in this level out of n=12=constant specimens in this fig.; range of transition
r = number of fractured specimens plotted vs. the levels of alternating load (a constant number of n=12 specimens at each level in RT is assumed in this figure)