On the Influence of the R-Ratio on Fatigue Crack Propagation

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Introduction. The R-ratio is defined as the ratio of the minimum to the maximum stress in a fatigue cycle, and there is as yet no generally accepted approach which deals with the influence of this ratio on the rate of fatigue crack propagation. However, as can be judged by inspection of Table I there is no paucity of expressions which attempt to deal with this matter. From these and other studies certain trends are associated with mean stress effects. For example, for a given crack growth rate there is usually a decrease in the allowable amplitude as the mean stress increases.[1] Similarly, the threshold level decreases with increase in the mean stress.[2][3] These trends are analogous to those found in the Goodman diagram. However, to complicate matters, it appears that the response of materials to mean stress effects may vary.[4] Further, in view of recent considerations concerning crack-closure,[5] the role of a compressive stress, if present in a cycle, is not clear. Nevertheless, it is the purpose of this paper to assess the usefulness of a simple model based upon crack-opening displacement (COD) considerations in the prediction of the influence of the R-ratio on the rate of fatigue crack propagation.

Analysis and Evaluation. For R=0 loading and for growth rates below 10^{-4} inches per cycle, the crack growth rate has been expressed as[6]

$$\frac{\Delta a}{\Delta N} = A(COD - COD_{TH_o}) \qquad (1)$$

where Δa is the increment of crack growth over a number of cycles, ΔN; A is a material constant which also includes the effect of the environment; and COD_{TH_o} is the threshold level of the COD which must be exceeded for the crack to grow. In terms of stress-intensity quantities Eq. (1) becomes

$$\frac{\Delta a}{\Delta N} = \frac{16A}{\pi \sigma_y E}\left(K_A^2 - K_{A_{TH_0}}^2\right) \qquad (2)$$

where σ_y is the yield strength, E is Young's modulus, K_A is the amplitude of the stress intensity factor, and $K_{A_{TH_0}}$ is the stress intensity amplitude at the threshold for R=0 loading. Since the model is based upon crack advance by plastic blunting[7] the fracture toughness, K_c, which is dependent upon other separation modes, is not included in this rate equation.

In order to include the effect of positive R values on the rate of growth, we can write

$$\frac{\Delta a}{\Delta N} = A\left[COD_{MAX} - (COD_{MIN} + COD_{TH_0})\right] \qquad (3)$$

which leads to

$$\frac{\Delta a}{\Delta N} = \frac{16A}{\pi \sigma_y E}\left[K_A^2\left(\frac{1+R}{1-R}\right) - K_{A_{TH_0}}^2\right] \qquad (4)$$

For negative R values, if only Mode I loading is involved it is expected that the compressive portion of the cycle would be of little influence, and there is indeed evidence that this is the case.[8] Should Modes II and III be operative, however, a compressive stress may be of greater influence, but these modes are not considered herein.

Eq. (4), with material constants from ref. 6, is compared with experimental results in Fig. 1 (dashed curves) and it is seen that the expression overestimates the effect of the mean stress. The aforementioned variation in the response of different materials to the mean stress can also be seen.

To predict better the behavior of these materials, the following modification to Eq. (4) is suggested:

$$\frac{\Delta a}{\Delta N} = \frac{16A}{\pi \sigma_y E}\left[K_A - K_{A_{TH_0}}\left(\frac{1-R}{1+R}\right)^{1/2}\right]^2\left(\frac{1+R}{1-R}\right) \qquad (5)$$

This expression includes the influence of the R value as well as the amplitude on the effective value of the threshold level as might be expected from refs. (2), (3), and (5). A comparison with experimental results also given in Fig. 1 (solid curves) shows that better agreement between predicted and experimental values is obtained as compared to the unmodified COD expression.

Concluding Results. A general method for the prediction of the rate of fatigue crack growth which involves but two material constants has been presented based upon a mechanistic consideration of the fatigue growth process. In certain cases the agreement with experiment is better than for others. Further work along these lines will reveal the general validity of this approach.

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TABLE I. SUMMARY OF RELATIONS CONCERNING MEAN STRESS ON CRACK GROWTH

	$\frac{\Delta a}{\Delta N}$ given by	Reference	Material Constants
1.	$A(\Delta K)^m$	Paris & Erdogan (9)	2
2.	$\frac{A(\Delta K)^m}{(1-R)K_c-\Delta K}$	Foreman, Kearny, Engle (1)	3
3.	$A\left(\frac{2}{1-R}\right)^{2m} (\Delta K)^{2(m+n)}$	Erdogan (10)	3
4.	$\frac{A\left(\frac{2}{1-R}\right)(\Delta K-K_{TH})^n}{K_c-\left(\frac{2}{1-R}\right)\Delta K}$	Erdogan & Ratwani (11)	5
5.	$A[K_A\left(\frac{2}{1-R}\right)]^n - K_{A_{TH_o}}^{\;n(1-p)} [K_{TH_o}\left(\frac{1}{1-R}\right)^n]^{kp}$	Klesnil & Lukas (2)	5
6.	$A(\Delta\sigma)^m \sigma_{max}{}^n a^p$	Mukerjee & Burns (12)	4
7.	$A\left(\frac{\Delta K}{1-R}\right)^3 \exp(-BR)$	Broek & Schivje (13)	2
8.	$B\,K_{max}^m$	Miller & Throop (14)	2
9.	$B\left\{\left[\frac{2K_A}{1-\left(\frac{2R}{1-R}\right)\frac{K_A}{K_c}}\right]^2 - K_{TH_o}^2\right\}$	McEvily & Wei (3)	3
10.	$f[B(S_{max})^{1-m}\,\Delta S^m\sqrt{\pi a}]$	Walker (16)	-
11.	$f(\Delta K_{eff});\ \Delta K_{eff} = (0.5 + .4R)\Delta K$	Von Euw & Hertzberg (15) [based on Elber (5)]	-
12a.	$\frac{16A}{\pi\sigma_y E}\,[K_A^2\left(\frac{1+R}{1-R}\right) - K_{A_{TH_o}}^2]$	Present	2
b.	$\frac{16A}{\pi\sigma_y E}\left[K_A^2 - K_{A_{TH_o}}\left(\frac{1-R}{1+R}\right)^{1/2}\right]^2\left(\frac{1+R}{1-R}\right)$	Present	2

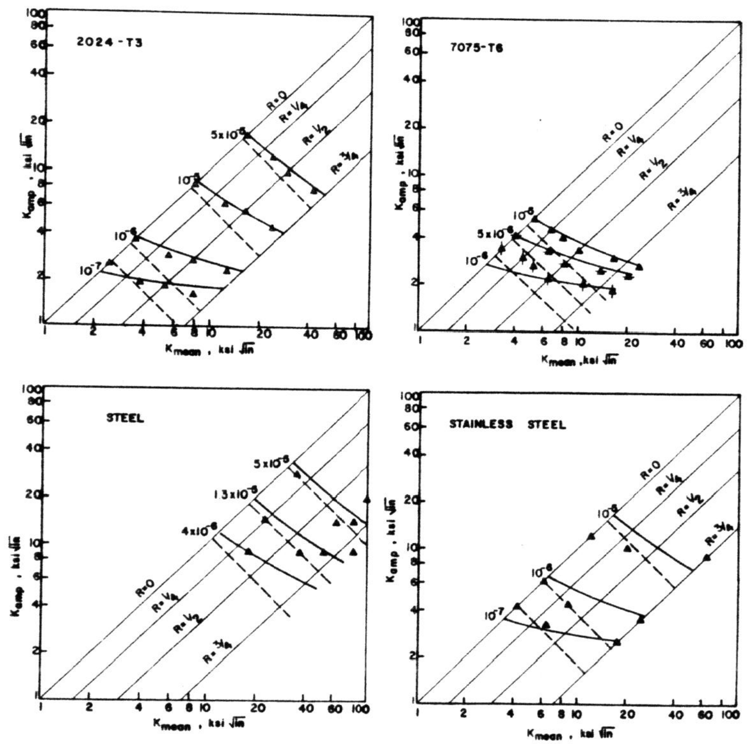

Fig. 1. Comparison of predicted and experimental rates of fatigue crack propagation as a function of R ratio. Dashed curves based on Eq. 4; solid curves based on Eq. 5. Growth rates are in inches per cycle.