Fracture Mechanics Approach to Threshold Condition for Fatigue Crack Growth

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1. INTRODUCTION

Several papers(1~12) have recently appeared on the lower limit conditions of possibility for fatigue crack growth expressed by specific values of stress intensity factor range (ΔK), ΔK_{th}. Actually, however, the ΔK_{th} data by suitable direct measurements of crack growth are so few that it is necessary to assure the validity of the conclusion in the papers by direct measurements and theory.

The authors have obtained the ΔK_{th} values of various materials and their dependency on the material properties.

The relation between the ΔK_{th} and the fatigue limit of cracked specimens is also discussed by a model conneting non-propagating crack concepts(13) with fracture mechanics.

2. THE DEPENDENCIES OF ΔK_{th} ON MATERIAL PROPERTIES

2.1 EXPERIMENTAL PROCEDURES

The materials used include steels, Al alloys, Cu alloys and Mg alloys. Specimen is the plate ($90 \times 30 \times 5$) with a single edge notch (60°V, 0.1 or 0.5 mm root radius and 5 mm deep). Fatigue tests are carried under out-of-plane bending of stress ratio R ($= \sigma_{min}/\sigma_{max}$) = -1, at 1500 cpm in laboratory atmosphere.

Crack growth rate is expressed by $\frac{da}{dN}$ and stress intensity factor by $\Delta K = \Delta\sigma\sqrt{\pi a}$. a is surface crack length, and $\Delta\sigma$ is bending stress range (assumed here as $\Delta\sigma = \sigma_{max}$). These $\frac{da}{dN}$ and ΔK are conventional, and the relations of these

$\frac{da}{dN}$ or ΔK and actual local $\frac{da}{dN}$ or local ΔK have to be decided otherwise.

The values of ΔK_{th} are measured by a step-wise K-decreasing method. At each 0.25^{mm} increase of a, load is decreased by 10% . Accessing the ΔK_{th} level, this decrement is lowered. When the growth can not be recognized after 10^7 cycles, the load is lowered again and continued another repetition of 10^7 cycles. For one material, 2 to 4 specimens were tested and 1 to 2 months were required.

In the test range of the crack length for ΔK_{th} measurements, it was confirmed that the load is not varied by the compliance change by crack growth, and that the value of $\frac{da}{dN}$ is almost constant under a constant $\Delta\sigma\sqrt{\pi a}$ level by K-control tests, and the apparent K is a function of $\Delta\sigma\sqrt{\pi a}$.

2.3 RESULTS OF EXPERIMENTS AND DISCUSSIONS

Tendency of the dependency of the ΔK_{th} obtained for each series of the materials on the yield strength is shown by the straight lines in Fig.1. The lines can be formulated as follows,

$$\Delta K_{th} \ (= \Delta\sigma\sqrt{\pi a}\ ) = A\cdot E - B\cdot \sigma_Y \qquad [1]$$

E is Young's modulus, σ_Y yield strength ($\sigma_{0.2}$, etc.) in kg/mm^2. $A = 1.905 \times 10^{-3}$, $B = 1.655 \times 10^{-1}$ in $mm^{\frac{1}{2}}$. $\Delta\sigma\sqrt{\pi a}/E$ plotted against σ_Y/E, selecting proper E values, is shown in Fig.2. Data fall on one descending scatter band.

If one refers some narrow range of σ_Y , ΔK_{th} can be practically and approximately estimated as constant as Equ. [2], as suggested by Harrison(3), Paris(1), Speidel(9) and McEvily(2) for in-plane loading or rotary bending.

$$\Delta K_{th} \cong A\cdot E \qquad [2]$$

The descending feature of ΔK_{th} with increase of

hardness or tensile strength was recognized in the results by Ouchida(10) and Ando(7).

Comparing our results with that of in-plane loads by Paris(1) and others, the following practical formula can tentatively be proposed.

$$[\Delta K_{th}]_{des.} = n\ (A\cdot E - m\cdot B\cdot \sigma_Y) \qquad [3]$$

$$\begin{cases} n = 1, & m = 1 \quad : \text{out-of-plane bending} \\ n = 0.5 \sim 0.75, & m = 0 \sim 1 \quad : \text{in-plane loading} \end{cases}$$

If ΔK_{th} is estimated from a fatigue limit at 10^6 to 10^7 cycles of S-N diagrams by notched or cracked specimens, or from a $\Delta K - \frac{da}{dN}$ diagram of a crack initiated at notch root under constant load cycles, higher values of ΔK_{th} are often obtained. Harrison's or McEvily's estimate of ΔK_{th} seems to depend on the lower limit of these data. If mean values or upper limits are taken, ΔK_{th} possibly has a ascending tendency with increase of σ_Y, as suggested by one of the authores(5). The reason for this will be discussed in detail at the conference. The formula [1] or [3] gives the lower limit of ΔK, above which crack growth is possible if all the conditions are satisfied. It is important for the safety design.

3. RELATION OF ΔK_{th} AND NON-PROPAGATING FATIGUE LIMIT

The fatigue limit for crack initiation from a notch root, σ_{w1} , and the fatigue limit for the specimen with a non-propagating crack, σ_{w2} , for rotary bending are plotted against the stress concentration factor α , shown in Fig.3. At the branch point ($\alpha = \alpha_0$, $\rho = \rho_0$) separating the σ_{w1} line and the σ_{w2} line, the notch root radius ρ is constant (= ρ_0) for each material and ρ_0 is not affected by the notch depth, t, and the diameter, d. A non-propagating crack is observed at $\rho < \rho_0$ (13). It is the same to the case of tension or torsion.

Now let's assume that the fatigue limit of cracked specimen is equal to σ_{w2} when α tends to infinity, and that the σ_{w2} is equal to σ_{w1}, at $\alpha = \alpha_0$, as the σ_{w2} line is horizontal if machining effects have been removed. This concept is shown in Fig.4 for a wide plate under tension.

$$\alpha = 1 + 2\sqrt{t/\rho} \fallingdotseq 2\sqrt{t/\rho} \qquad (\text{for } t > \rho)$$

$$\alpha_0 = 2\sqrt{t/\rho_0} = 2\sqrt{a/\rho_0} \qquad (\text{for } a = t + a_0 \fallingdotseq t) \qquad [4]$$

$$\sigma_{w2} = [\sigma_{w1}]_{\alpha=\alpha_0} = C\cdot\sigma_{w0}/\alpha_0 = C/2\cdot\sigma_{w0}\cdot\sqrt{\rho_0/a} \qquad (C \fallingdotseq 1.1\sim1.3) \qquad [5]$$

σ_{w0} : fatigue limit of unnotched specimen

$$\Delta K_{th} = \sigma_{\infty}\cdot\sqrt{\pi a}\cdot F(a) \qquad F(a):\ \text{correction factor (e.g., 1.12)}$$
$$= \sqrt{\pi a}\cdot F(a)\cdot\sigma_{w2}\cdot(W - 2a)/W$$
$$= C/2\cdot\sqrt{\pi}\cdot F(a)\cdot\{(W - 2a)/W\}\cdot\sigma_{w0}\cdot\sqrt{\rho_0}$$
$$= C'\cdot\sigma_{w0}\cdot\sqrt{\rho_0} \qquad [6]$$

C' may be taken as almost constant for selected proper test conditions. ρ_0 is assumed to be the same for tension. Fig.5, plotted for $C' = 1.2$, seems to be able to explain the tendency of Paris' results (1).

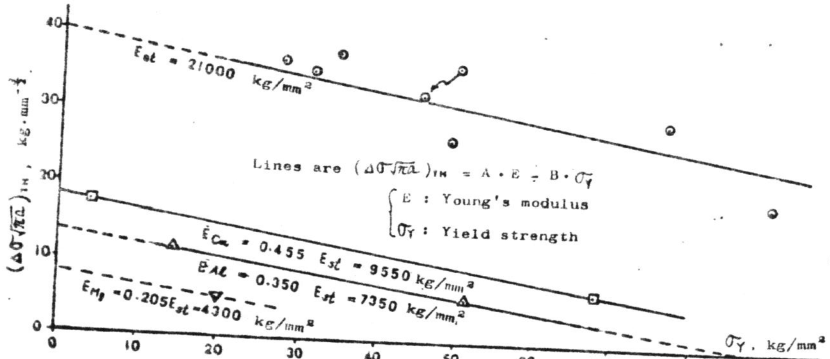

Fig. 1 Dependency of $(\Delta\sigma\sqrt{\pi a})_{TH}$ to E and σ_Y (1)

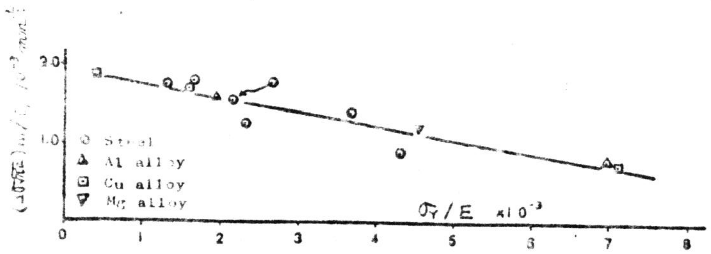

Fig. 2 Dependency of $(\Delta\sigma\sqrt{\pi a})_{TH}$ to E and σ_Y (2)

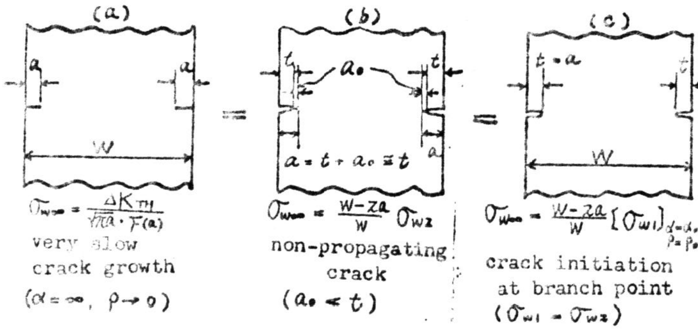

Fig.4 Proposed model for connecting stress intensity factor with the fatigue limit of notched specimen with a non-propagating crack in its root

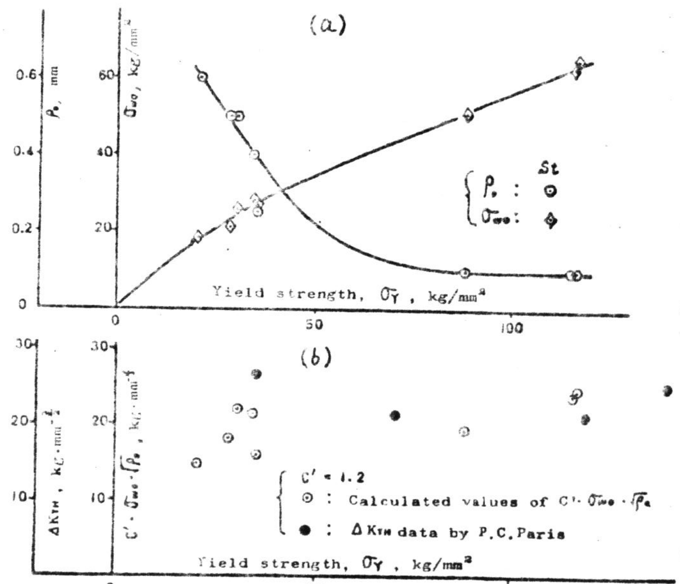

Fig. 5 Calculation of threshold stress intensity factors with branch point root radius ρ_0

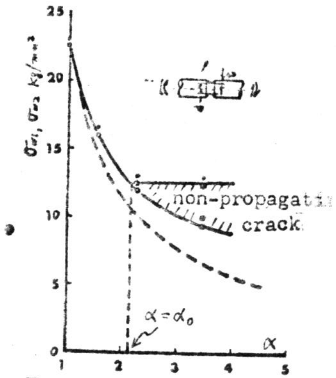

Fig.3 Retation of σ_{w1} and σ_{w2} to stress concentration factor α, and branch point $(\alpha = \alpha_0)$

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