Fatigue Lifetime and Reliability of Prestrain Sensitive Materials under Random Loads

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1. INTRODUCTION

Fatigue lifetime under random loads has been widely reported in literature; for a general survey it may be referred to (1) All theoretical methods have in common the assumption of either a given load spectrum or the probability distribution of certain material specimen; further, the validity of some cumulative damage law is assumed to be some single valued function. Accordingly, by means of a statistics of the load distribution on the S-N curve and the cumulative damage law the moments of the distribution of the number of cycles to fatigue is obtained.

This paper deals with the particular situation which arises when the S-N curve is branching and therefore no longer single-valued, as for instance reported by T.H. Topper and B.I. Sandor (2) see Figure 1. Then a conditional statistics has to be introduced to overcome the branching of the S-N curve.

2. THE EXPECTED NUMBER OF FATIGUE CYCLES

For convenience of versatility in experimentation a multinomial distribution of the load amplitudes is assumed; the interval S of the load amplitudes is divided into r disjoint subintervals $S_1, S_2 \ldots, S_r$ = S, S, $<S_2$ ... etc. each of the probability θ_1, θ_2, ... etc.

Further, the following assumptions are made:

(a) Applicability of Palmgren - Miner's hypothesis.

(b) For simplification, prestrain is defined if within the first N_p cycles the load S exceeds at least once the critical value S_p which is assumed to be independent of the cycle number.

(c) For a single valued S-N curve the number of fatigue cycles for the constant load amplitudes $S_1 < S_2 < \ldots < S_r$ may be denoted by $N_1, N_2, \ldots, N_r$.

(d) For a branching S-N curve two sets of N-values have to be considered, $N_i^{(u)}$ $(i = 1, \ldots, r)$ in case of no prestrain and $N_j^{(\ell)}$ $(j = 1, \ldots, r)$ for the event of prestraining.

Introducing a new random variable Y as a linear combination of S_i $(i = 1, 2, \ldots, r)$ in the form of the cumulative damage of Palmgren and Miner's hypothesis:

$$Y = \sum_{i=1}^{r} \frac{S_i}{N_i}$$

yields immediately the expected number N_F of fatigue cycles taking the expectations:

$$N_F = \frac{1}{E\{Y\}} \qquad (1)$$

$$N_F = \frac{1}{\sum_{i=1}^{r} \frac{\theta_i}{Y_i}}$$

Denoting the probability of prestraining by P_S the cumulative damage for the branching S-N curve is expressed as

$$Y_b = \sum_{i=1}^{r} \frac{S_i}{N_i^{(\ell)}} P_S + \sum_{i=1}^{r} \frac{S_i}{N_i^{(u)}} (1 - P_S)$$

and hence the expected number of fatigue cycles follows as

$$N_F = \{P_S \sum_{i=1}^{r} \frac{\theta_i}{N_i^{(\ell)}} + (1 - P_S) \sum_{i=1}^{r} \frac{\theta_i}{N_i^{(u)}}\}^{-1} \qquad (2)$$

this result is not surprising: In case of a branching S-N curve the expected number of fatigue cycles is simply shifted to the weighted mean value.

3. VARIANCE AND RELIABILITY

Two contributing facotrs are considered, first the cumulative damage function as a random variable and second the uncertainity of experimental evaluations of the S-N curve and of Miner's hypothesis, the variance of which may be denoted by σ_E^2. Instead of the cumulative damage functions Y and Y_b the corresponding functions of the relative frequencies X and X_b respectively are then considered. Probabilistic reasoning (3) yields then for the variances of cycles to fatigue:

$$V(N - N_F) = N_F V(X) + \sigma_E^2 \qquad (3a)$$

for the simple case and

$$V(N - N_F) = N_F V(X_b) + \sigma_E^2 \qquad (3b)$$

for the prestrain sensitive case respectively.

The difference in (3a) and (3b) originates in the different variances for X and X_b which are

$$V(X) = \sum_{i=1}^{r} \sum_{j=1}^{r} \frac{1}{N_i} \frac{1}{N_j} \theta_i (\delta_{ij} - \epsilon_j) \qquad (4a)$$

and with denoting

$$C_i = P_S / N_i^{(\ell)} + (1 - P_S) / N_i^{(u)}$$

$$V(X_b) = \sum_{i=1}^{r} \sum_{j=1}^{r} C_i C_j \epsilon_i (\delta_{ij} - \epsilon_j) \qquad (4b)$$

is obtained (δ_{ij} denotes Kronecker's symbol).

Numerical results exhibit then that even for small differences of $N_i^{(\ell)}$ and $N_i^{(u)}$ the variance of the number of fatigue cycles increases significantly. Further, if reliability is defined in the statistical way, i.e.

$$N_R = N_F - k\,\{V(N)\}^{\frac{1}{2}} \qquad (5)$$

then for even small k (k=3) depending on the form and position of the load spectrum and the branching S-N curve the number of reliable cycles may even become negative, which indicates a limitation of the convenient statistical definition of reliability.

4. REFERENCES

(1) S. R. Swanson: "Random Load Fatigue Testing: A State of the Art Survey"; ASTM, Materials Researchs Standards, Vol. 8, No. 4, April 1968 pp. 10-44.

(2) T. H. Topper and B.I. Sandor: "Effects of Mean Stress and Prestrain on Fatigue Damage Summation," Department of Theoretical and Appl. Mech. University of Illinois, T. & A.M. Report No. 318, August 1968 Urbana, Illinois.

(3) W. Feller: "An Introduction to Probability Theory," 2nd ed., Vol. I., John Wiley & Sons.

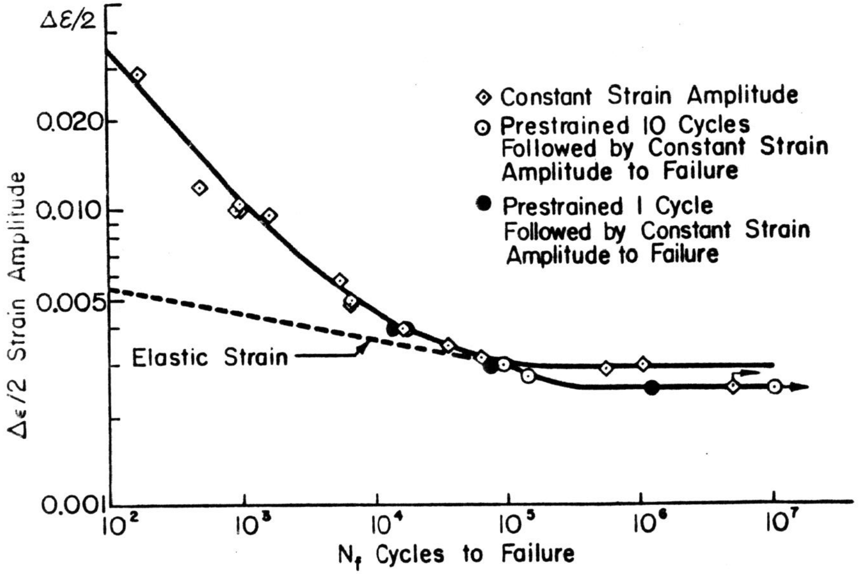

Fig.1 Effect of a Prestrain on the Strain-Life Behavior of SAE 4340 Steel See Ref. 2