Fatigue Crack Growth in Pure Shear

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INTRODUCTION:- In large flight vehicles of stringer/skin construction it is possible for fluctuating pure shear loading to occur. This type of loading results in a biaxial state of stress which is tensile: compressive, and it is of importance to know what effect such loading may have on crack growth.

THEORETICAL ANALYSIS:- The Tresca and Von Mises yield criteria were used to examine the influence of applied biaxial stresses on plastic zone size (PZS) and crack opening displacement (COD). The stress S_y, applied normal to the crack plane results in a local stress σ_y which varies along the crack plane. The stress S_x, applied parallel to the crack plane results in a local stress σ_x, but because the influence of the crack is non-singular $\sigma_x = S_x$. For biaxial loading, it was considered that yielding along the crack plane would be influenced by the stress S_x, and that, at the elasto-plastic interface, a stress σ_Y would exist which could be defined by either of the yield criteria.

To examine the influence of biaxial loading, the Dugdale model has been employed see Figure (1). The influence of S_x(non-singular) is introduced through its effect on the variation of σ_Y, the stress at yield defined by a yield criteria for a biaxial stress state.

For a biaxial state of stress the yield criteria are

$$\sigma_o = \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 \qquad \text{(Von Mises)}$$

$$\sigma_o = \sigma_2 - \sigma_1 \text{ if } \sigma_1 < 0, \ \sigma_2 > 0 \qquad \text{(Tresca)}$$

where σ_1 and σ_2 are the principal stresses.

The crack plane represents a plane of symmetry and on this plane the principal stresses are $\sigma_1 = S_x$ and $\sigma_2 = \sigma_y$. At a point along the crack plane a stress of magnitude σ_Y is required at the elasto-plastic interface which satisfies the yield condition, when the applied stress S_x is introduced parallel to the crack plane.

From the Dugdale analysis, the PZS is given by $\frac{p}{a} = \sec\left(\frac{\pi S_Y}{2\sigma_o}\right) - 1$ and COD by $\frac{\delta}{a} = \frac{8\sigma_o}{\pi E}\log_e \sec\left(\frac{\pi S_Y}{2\sigma_o}\right)$

In order to examine the influence of applied biaxial stress on the PZS and COD, σ_o in the above equations can be replaced by σ_Y as defined in the previously stated yield criteria. The results are presented in terms of the ratios p_b/p and δ_b/δ, where the subscript indicates a biaxial stress state. Since pure shear results in a 1:1 applied biaxial stress ratio, only this particular condition is illustrated. In Figure (2) the variation in PZS ratio as a function of S_Y/σ_o is shown for the two yield criteria, and in addition the variation in PZS calculated using the ASTM correction $r_y = \frac{1}{2\pi}\frac{K^2}{\sigma_o^2}$ is shown for the changing yield condition. In Figure (3) the variation in COD is shown, calculated using the Dugdale type analysis.

EXPERIMENTAL PROCEDURE:- Two series of fatigue crack growth tests were conducted on an Aluminium alloy L70 sheet material, 0.036 ins thick, having a 0.1% proof stress of 45 ksi. Constant amplitude (R = 0.25) tests were carried out in tension and pure shear. Crack growth against cycles of load was recorded using a foil gauge and automatic print-out technique. The stress in the specimen was measured using a strain gauge attached to every specimen at the same position relative to the centre lines. Tests were conducted on both types of specimens at a maximum stress level normal to the crack plane of 8 ksi ($S_y/\sigma_o = 0.177$) and 18 ksi ($S_y/\sigma_o = 0.4$), four specimens of each type, at each stress level being tested. Because of Poisson's effect it was felt of interest to test specimens in pure shear at an equal applied strain to the tensile specimens. This was carried out

at a strain equal to 8 ksi in uniaxial tension and results in a stress of 6.15 ksi ($S_y/\sigma_o = 0.136$) normal to the crack plane in pure shear.

Due to the inherent buckling instability of thin sheets subject to pure shear, the test specimens were run, clamped between flat lubricated plates, and in order for the comparison to be made between the two types of loading, the restraining plates were also used on the tension specimens.

DISCUSSION OF RESULTS:- The results of the theorectical analysis clearly indicate that compressive stress applied parallel to the crack plane leads to a significant increase in PZS and COD. The changes resulting from the use of Tresca's criteria being more significant than for the Von Mises criteria. It should be borne in mind that, because of the secant form of the equations for PZS and COD these two quantities approach infinity (ie total panel yielding) as $S_y/\sigma_o \rightarrow 0.5$ for Tresca and a slightly higher value for Von Mises yield criteria.

In order to compare the results for both types of tests, crack growth rate has been plotted against stress intensity range ($K = S_y\sqrt{\pi a}$), Figure (4).

Since crack growth rate is COD dependent, the theoretical analysis would indicate that pure shear loading will result in an increase in crack growth rate, as both COD and PZS increase markedly as the applied stress approaches yield.

It can be seen that the present fatigue results are not completely conclusive, since it was only at the higher applied stress level that the crack growth rate became faster for pure shear than for uniaxial tension, and then only by a small amount. However, the effect of restraining the shear panels may affect the results because of the higher normal forces involved.

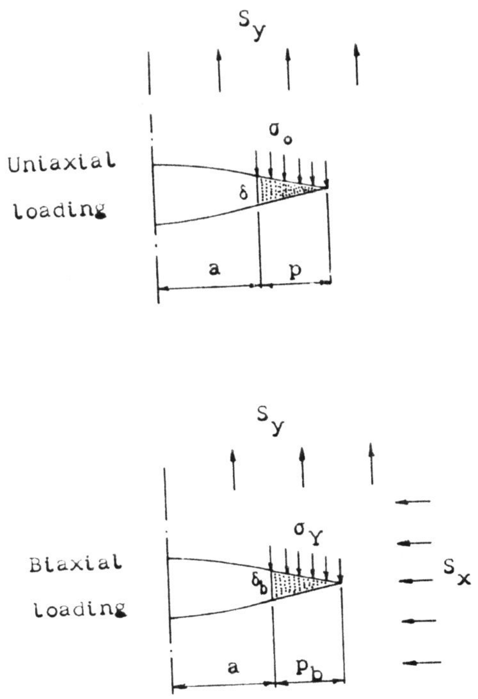

Figure 1. Yield model for uniaxial and biaxial loading.

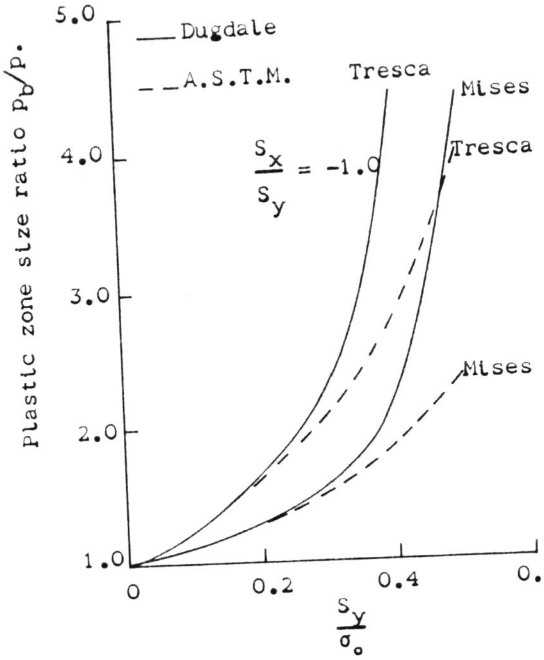

Figure 2. Variation in plastic zone size ratio with applied/yield stress ratio.

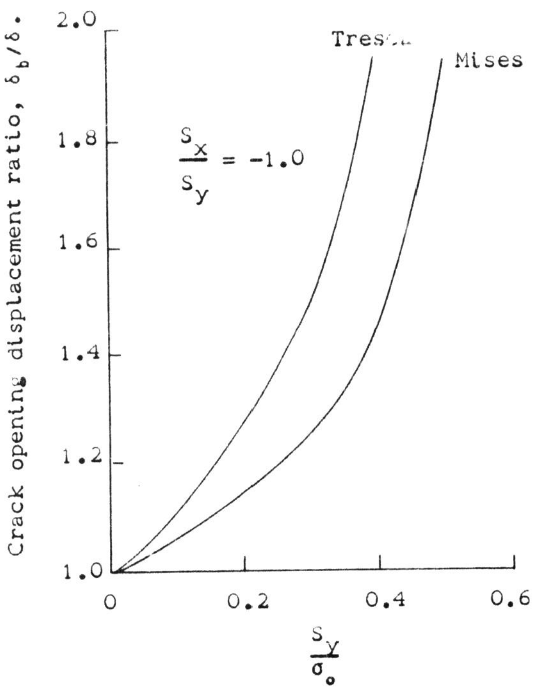

Figure 3. Variation in crack opening displacement ratio with applied/yield stress ratio.

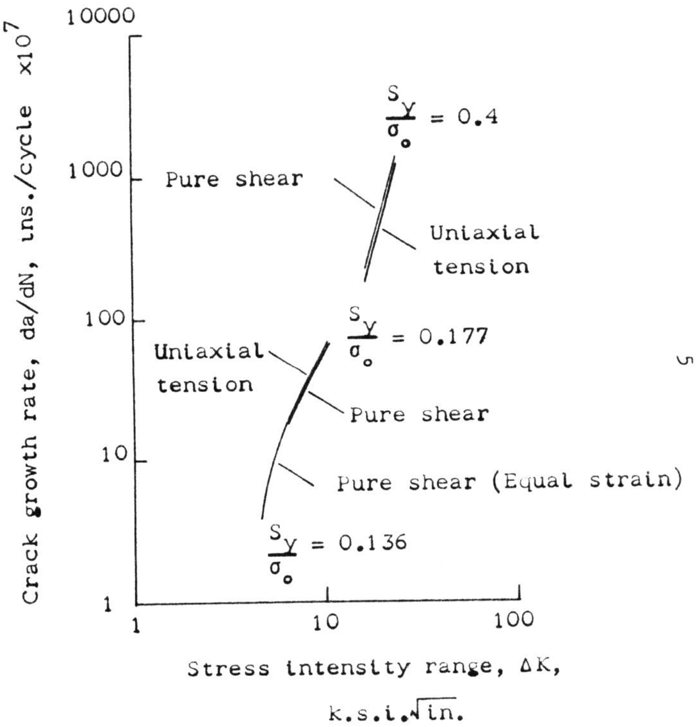

Figure 4. Crack growth rate vs stress intensity range.