# Elastic - Plastic Approach to Fatigue Crack Propagation and Fatigue Limit of Material with Crack

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## INTRODUCTION

In this paper, elastic-plastic mechanical models are proposed in determining fatigue crack propagation and fatigue limit of cracked specimen and verified by fatigue tests conducted with plate specimens.

## APPROXIMATE ANALYSIS OF ELASTIC-PLASTIC STATE AT CRACK TIPS

If effect of finite width appears in the elastic-plastic condition as it does in the elastic stress condition, plastic zone size $r_p$ at crack tip in the plane stress condition is expressed by the following formula which is obtained by modifying the Dugdale model.

$$r_p = a_e(\sec \pi\sigma_n/2\sigma_y - 1) \qquad (1)$$

where; $\sigma_n$ is net section stress, $\sigma_y$ is yield stress of material, and $a_e$ is equivalent crack length of a plate with infinite width obtained by modifying finite width effect elastically by the following equation, and is termed "effective crack length."

$$a_e = \frac{K^2}{\pi \sigma_n^2} \qquad (2)$$

where; $K$ represents a stress intensity factor of a plate with finite width. Effective crack lengths, one is of a plate with double edge crack and the other is of a plate with center crack under tensile condition, are expressed by Eqs. (3) and (4).

$$a_e = a(1 - 2a/W)^2 \sec \pi a/W \qquad (3)$$

$$a_e = h(1 - 2a/W)^2 \cdot W/\pi \cdot \tan \pi a/W \qquad (4)$$

where; $a$ is half crack length, $W$ is plate width, and $h$ is factor representing edge effect.

## TESTING PROCEDURES

Chemical compositions and mechanical properties of materials used for experiments are given in Tables 1 and 2, respectively. In the crack propagation test are used specimens made of refined low alloy steel (HT80) whose details are shown in Fig. 2. Fatigue tests were carried out under the net section stress control with 1 Kg/mm$^2$ minimum stress. Its control accuracy was within ±2%. In the test to determine the fatigue limit of cracked materials were used mild steel specimens with details shown in Fig. 3. A test specimen such shown in Fig. 3(a) was subjected to ±6 Kg/mm$^2$ cyclic stress thereby to cause 0.35 - 10 mm fatigue cracks. Then, excluding ones with 10 mm crack, all these cracked specimens were cut of notched sections as shown in Fig. 3(b) and made into specimens with 0.35, 1.0 and 3.0 mm cracks. The fatigue limit of a plane specimen was determined by using (c) type specimen. These test specimens were annealed in vacuum at 650 °C for two hours and their surfaces were electro-polished. The fatigue test was carried out under alternating load.

## FATIGUE CRACK PROPAGATION

The relationship between fatigue crack growth rate $da/dN$ and effective crack length $a_e$ of HT80 steel is shown in Fig.4. From this Figure the following equation is derived.

$$da/dN = k \cdot a_e \qquad (5)$$

In some cases of low stress, the value of $k$ which represents gradient of the lines shows a transition, and this transition point is found to be corresponded with the point at which shear lip occures. Fig. 5 shows the relationship between stress range $\Delta\sigma$ and normalized crack growth rate $k = (da/dN)/a_e$. Curves in the figure are represented by the following equation, and, being $\Delta\sigma/\sigma_y \lesssim 0.9$ they agree with experimental values.

$$da/dN = C_1 \cdot \gamma_{p(c)} \qquad (6)$$

where; $C_1$ is a constant, and it turns out to be 1.25x10$^{-4}$ at the tensile type fracture, and 2.5x10$^{-4}$ at the shear type one. $\gamma_{p(c)}$ is a parameter obtained by substituting $\Delta\sigma_n$ for $\sigma_n$ in Eq. (1), and is termed "cyclic plastic zone size".

$$\gamma_{p(c)} = a_e(\sec \pi\Delta\sigma_n/2\sigma_y - 1) \qquad (7)$$

Eq. (6) represents that cyclic plastic strain inflicts fatigue damages on the tip of crack contributing to the propagation of crack$^{(2)}$. The experimental value $\gamma_{p(c)}^*$ which represents the size of cyclic plastic zone where persistent slip band is originated, is smaller than $\gamma_{p(c)}$ and there holds the following relation between them.

$$\gamma_{p(c)}^* = C_2 \cdot \gamma_{p(c)} \qquad (8)$$

where; $C_2$ is a constant, being about 0.1 in Fig. 9. Accordingly, if $C_3 = C_1/C_2$

$$da/dN = C_3 \cdot \gamma_{p(c)}^* \qquad (9)$$

The relationship expressed by Eq(6) is, as shown in Fig. 5, if $\Delta\sigma_n/\sigma_y \lesssim 0.2$ agrees with Frost's formula$^{(1)}$, $da/dN \propto \Delta\sigma_n^3 a$ and if $0.2 \lesssim \Delta\sigma_n/\sigma_y \lesssim 0.7$ agrees with Liu's formula$^{(2)}$, By rearranging the results of the above experiments by using $\Delta K$ it can be seen that elastic approach is not enough in the field where $\Delta K$ is large.

## FATIGUE LIMIT OF MATERIAL WITH CRACK

The results of fatigue tests with mild steel, shown in Fig. 7, agree with the following equation.

$$\gamma_{p(c)w} = a_e(\sec \pi\sigma_w/2\sigma_y - 1) = const \qquad (10)$$

where; $\sigma_w$ is fatigue limit and $\gamma_{p(c)w}$ is $\gamma_{p(c)}$ at fatigue limit. If its value for mild steel is 0.27, it can be simply expressed as $K_w = 21\ Kg/mm^2 \cdot \sqrt{mm}$, when $\sigma_w/\sigma_y \lesssim 0.3$. Fig. 8. shows the condition of the crack tip below and above the fatigue limit, when $a \fallingdotseq 0.35$ mm. From this figure it can be seen that representing the area of persistent slip band at the fatigue limit is 0.02 - 0.03 mm, which is about one tenth of $\gamma_{p(c)w}$ and is nearly equal to the measured mean grain size, 0.025 mm.

## REFERENCES

(1) N.E. Frost and D.S. Dugdale, J. Mech. Phys. Solids, 6 (1958), 92.

(2) H.W. Liu, Appl. Materials Res., (Oct. 1964), 229.

Table 1 Chemical analyses of materials.

%

| MATERIAL | C | Si | Mn | P | S | Cu | Ni | Cr | Mo | V | Aℓ | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| HT80 STEEL | 0.14 | 0.25 | 0.77 | 0.015 | 0.009 | 0.25 | 0.86 | 0.51 | 0.46 | 0.051 | 0.026 | 0.004 |
| MILD STEEL | 0.18 | 0.01 | 1.04 | 0.016 | 0.026 | — | — | — | — | — | — | — |

Table 2 Mechanical properties of materials.

| MATERIAL | Y. S. $kg/mm^2$ | U. T. S. $kg/mm^2$ | EL. % | R.A. % |
|---|---|---|---|---|
| HT80 STEEL | 82.0 | 87.7 | 25.9 | 67.9 |
| MILD STEEL | 25.6 | 49.3 | 43.2 | 66.4 |

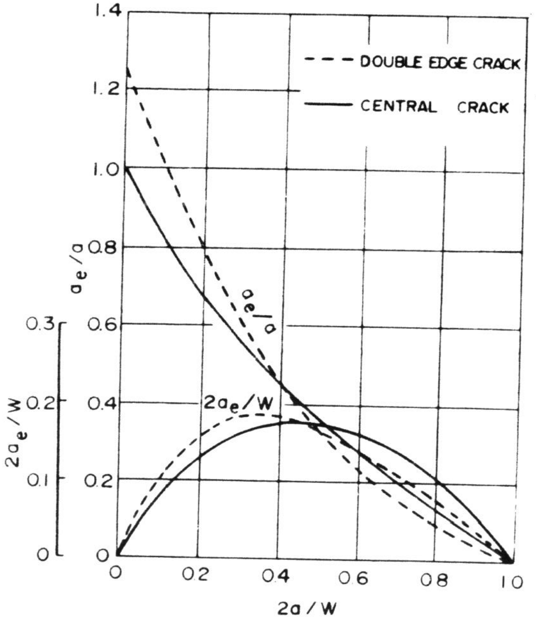
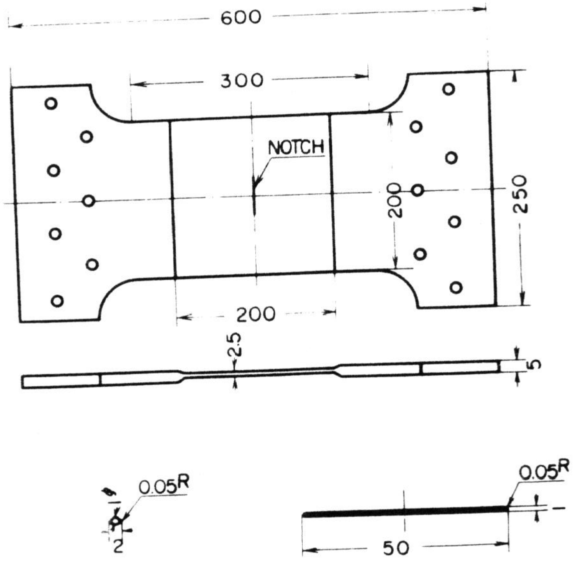


Fig. 1 Effective crack length in cracked plates subject to tension.



NOTCH DETAILS

Fig. 2 Details of test specimens (HT 80 steel).

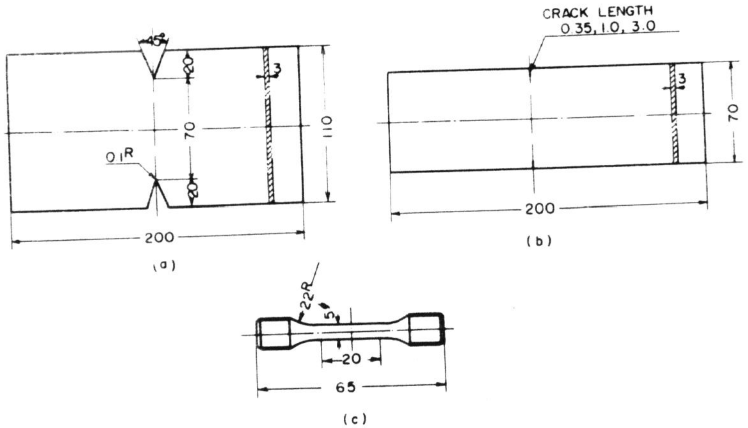


Fig. 3 Details of test specimens (Mild steel).

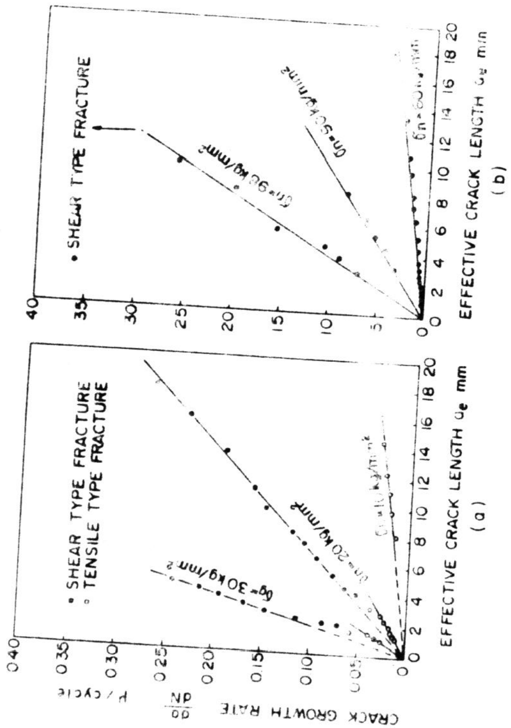
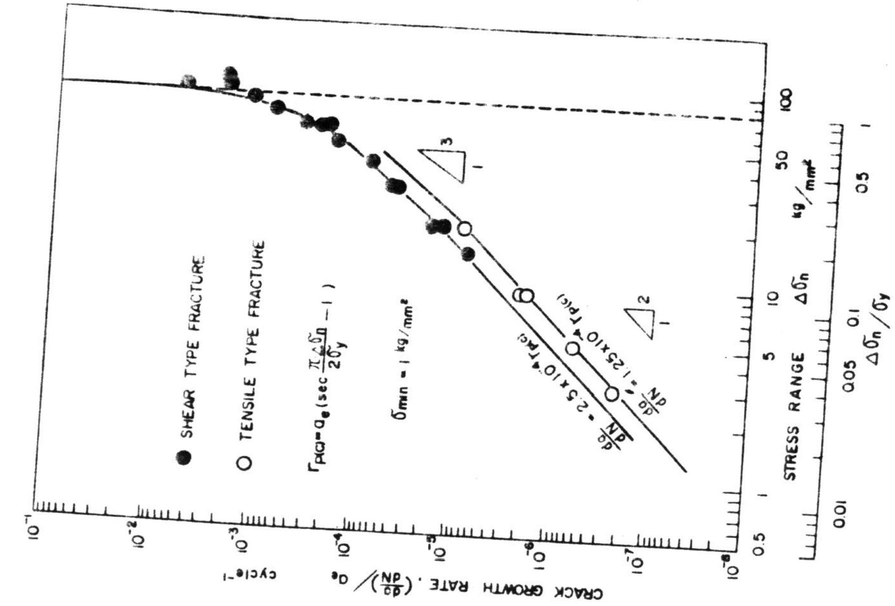


Fig 4 Relationship between effective crack length and crack growth rate



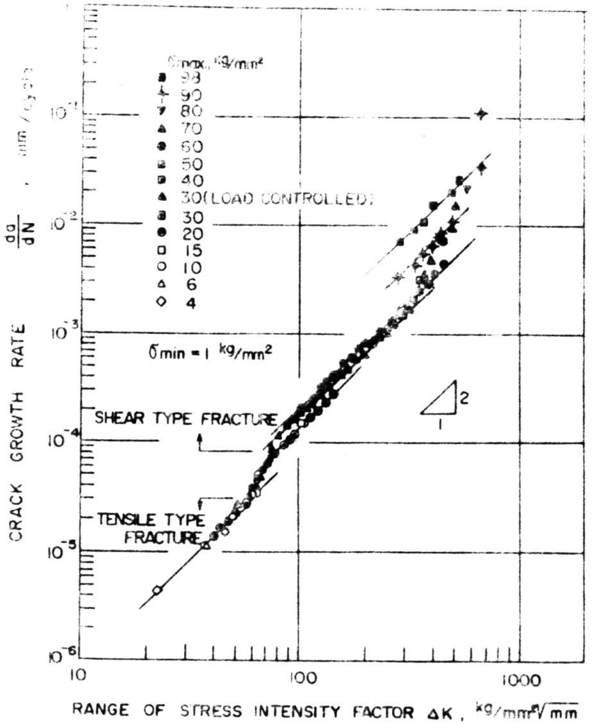


Fig. 6 Crack growth rate versus range of stress intensity factor (HT 80 steel)

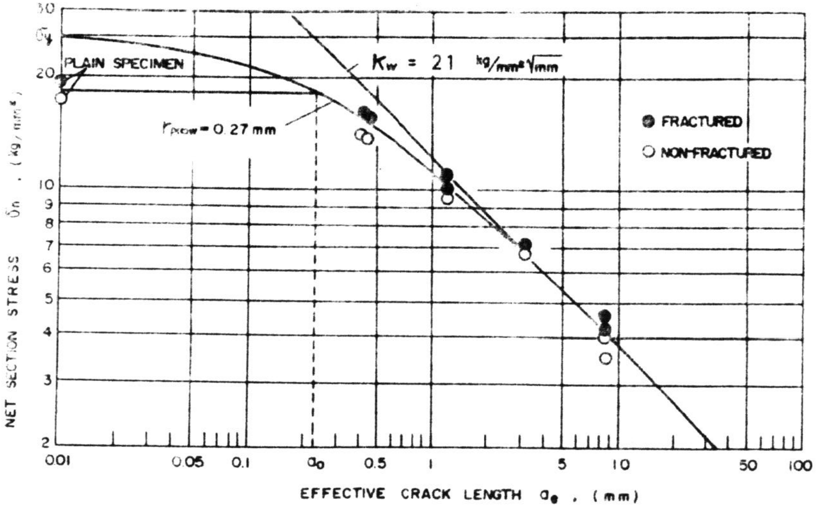
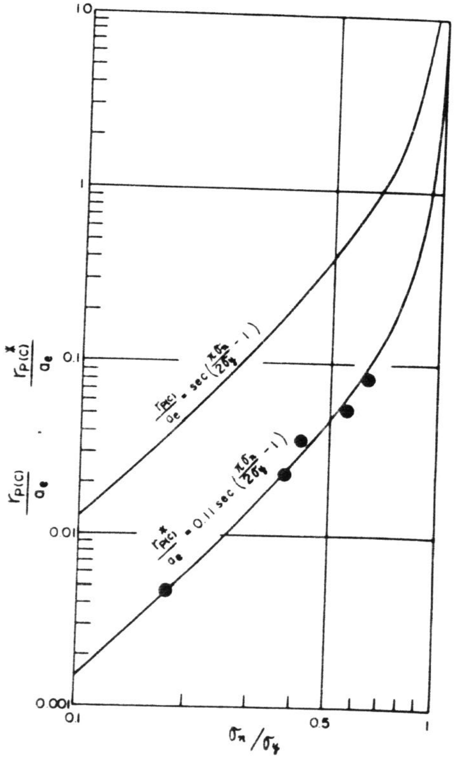


Fig. 7 Effect of effective crak length on fatigue strength (Mild steel)

(a) NON PROPAGATING CRACK
($\sigma n = \pm 14{,}0\,{}^{kg}/mm^2$; $N = 10^7$)

(b) PROPAGATING CRACK
($\sigma n = \pm 16{,}0\,{}^{kg}/mm^2$; $N = 1{,}4 \times 10^5$)

Fig. 8. The tip of propagating and non-propagating fatigue cracks on the surface of mild steel specimen ($a \fallingdotseq 0{,}35$ mm).



9 Comparison with theoretical cyclic plastic zone size and experimental values obtained by fatigue test (Mild steel)