Constant-Cycle and Block-Programme Testing Needs S-N Curves Looked upon Statistically - A Job for Computer

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The computing programme in FORTRAN IV for MINSK 22 is based on the mathematics presented.

Each level $u=1,2,\dots t_1$ of testing stress σ_u contains different numbers M_u of valid results N_u, where $m_u \leq M_u$ - specimens failed, thus $(M_u - m_u)$ - specimens stopped before failure. The least number of cycles endured N_{u1} must belong to a failed specimen, the other ones are ordered in ascending size according to $N_{ux} \leq N_{u(x+\Delta x)}$. The first increase is $\Delta x=1$ and has to be changed at crossing any group of unfinished results in the way proposed by Johnson

$$\Delta x_u = \frac{(M_u+1) - (\text{order of the foregoing failed specimen})}{1 + \begin{pmatrix}\text{number of all remaining results beyond} \\ \text{the treated group of unfinished ones}\end{pmatrix}} \quad (1)$$

Having attributed the order numbers x_u to all failed specimens the order probabilities of survival can be derived

$$P_{ux} = 1 - \frac{x_u - 0.5}{M_u} \quad (2)$$

The first computing step is closed by the determination of the <u>level log mean life</u> $\bar{N}_u$ and the <u>level standard deviation</u> S_u (for all levels where $m_u \geq 2$) due to the log-normal distribution of lives N_{ux} assumed with P_{ux}.

The second step takes full advantage of these acquired values $\bar{N}_u$, S_u, m_u for the best estimate of the average S-N curve in its mostly used range from the yield point to the endurance limit in the form

$$(\sigma_u - \sigma_o)^w . \bar{N}_u = \text{const} \quad (3)$$

where σ_o - chosen stress level (e.g. endurance limit σ_e).

This relation after conversion to logarithm is able to be solved by the method of least squares even on condition of the deviation s_u varying with the stress level σ_u in accordance with the expression introduced by Serensen

$$s_u^2 = s_o^2 \frac{1}{\omega_u} = s_o^2 \frac{1}{(\sigma_u - \sigma_e)^{2v}} \qquad (4)$$

where σ_e - endurance limit. Thus the unbiased estimate of the parameter v (i.e. also of ω_u) by the method of least squares is followed by the determination of the complex weighted factor $\omega_u . m_u$ for all levels u=1,2,...t_2 having $m_u \geq 1$. (For v=0 or ω_u=1 becomes s_u constant). At this moment the relation (3) is ready for the analysis resulting in calculation of the log mean lives $\bar{N}_{Ru}$ on the regression line (i.e. S-N curve for probability of survival P=0.5) as well as of the exponent w. The degree of coincidence is expressed by the weighted residual variance

$$s_R^2 = \frac{1}{\sum_{u=1}^{t_2} \omega_u m_u - 2} \sum_{u=1}^{t_2} \omega_u \left[(m_u - 1) s_u^2 + m_u (\log \bar{N}_u - \log \bar{N}_{Ru})^2 \right] \qquad (5)$$

Its value helps to describe the weighted and smoothed standard deviation on the regression line (i.e. S-N curve for probability of survival P=0.5) as

$$s_{Ru} = \frac{s_R}{\sqrt{\omega_u}} \sqrt{\frac{\sum_{u=1}^{t_2} \omega_u m_u - 2}{\sum_{u=1}^{t_2} m_u - 2}} \qquad (6)$$

In addition to it the best choice of the stress level σ_o in relation (3) is characterized by the minimum value of s_R.

The estimation of the life N_{Ru} with a certain probability of survival P is possible with only a certain probability of the right prediction, i.e. with a certain confidence Q, due to the limited number of results. This last computing step makes use of the relation

$$\log N_{Ru} = \log \bar{N}_{Ru} - u_{PQ} . s_{Ru} \qquad (7)$$

where the factor u_{PQ} is being described in the form

$$u_{PQ} = \frac{u_P + u_Q \sqrt{\frac{1}{2}\left[1 - \frac{u_Q^2}{2(\bar{m}-1)}\right] + \frac{u_P^2}{2(\bar{m}-1)}}}{1 - \frac{u_Q^2}{2(\bar{m}-1)}} \qquad (8)$$

and

$$\bar{m} = \frac{1}{t_2} \sum_{u=1}^{t_2} m_u \qquad (9)$$

is the average number of failed specimens from the considered levels, u_P - quantil corresponding to the desired probability of survival P, u_Q - quantil corresponding to the desired confidence of prediction Q.

It is evident that the probability of survival P= 0.5 and the confidence Q= 0.5 are attributed to the values $\bar{N}_{Ru}$ on the curve of average life.

According to the assumed log-normal distribution of lives it is to recommend the use of the extrapolation of the results by means of the factor u_{PQ} only for P, Q in the interval 5 - 95 per cent.

The size of this computing programme is limited up to 10 levels and the total number of results up to 60.

It is possible to give into the computer also the information about invalid specimens. Their output data are printed in brackets, being not used for the calculation of $\bar{N}_u$, s_u. If the condition to have a valid finished result for the order number x_u=1 is not accomplished at any level, the output data for the whole said level are printed in brackets, the calculation of $\bar{N}_u$, s_u being not performed.

At the levels where m_u=1, $M_u > m_u$, there is the calculation of $\bar{N}_u$, s_u performed approximatively. In this case all the valid unfinished results are considered as the valid finished ones and are added to the mentioned calculation, what is fixed in print by the fact that the values x_u, P_{ux}

are given here even for the unfinished results.

This programme can be exploited not only for the evaluation of the standard S-N curve just described but also for the mixed-cycle S-N curves introduced by Gassner. In this case only the maximum amplitude of the considered load spectrum should be inserted for the stress level σ_u.

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