Analytical Approach to Notch Size Effects in Fatigue

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1. INTRODUCTION - As is well known notched parts in fatigue are less sensitive to stress concentration than results from theory of elasticity solutions and the fatigue notch factor $K_F = \sigma_d/\sigma_{dn}$ is in general smaller than the theoretical stress concentration factor K_T. This phenomenon has been explained by macro-support and micro-support (geometrical and physical) effects [1]. One well-established reason of the support effects is the positive influence of the less stressed material at some distance from the notch. It may be assumed that a fatigue crack appears only if stresses are higher than a critical stress $\sigma_k = A\,\sigma_{dn}$ in a surface material layer, at the root of the notch, of a critical thickness h[2,3]. The critical stress σ_k at the distance h from the notch root can be higher than the corresponding fatigue limit of the unnotched specimen σ_d ($A \geq 1$), because of local strain hardening, deviation from linear $\sigma - \varepsilon$ dependence and other local support effects. If we assume, that h and A are material constants independent of the size of specimen, then we have a simple model which fits not only the notch effect but also the size effect.

2. DERIVATION OF A TWO-PARAMETER FORMULA - Fig. 1 illustrates the use of the proposed model for derivation of a two-parameter equation which describes the notch-size effect . According to the assumptions of Fig. 1 (h=const, σ_k = const) the stress gradient is $\frac{d\sigma}{dy} = \frac{\sigma_{max}-\sigma_k}{h}$

According to the well-established theoretical formula, $\frac{d\sigma}{dy} = C\,\frac{\sigma_{max}}{r}$ where r is the notch radius and C a proportionality factor given in theoretical solutions.

Since $\frac{\sigma_{max}-\sigma_k}{h} = C\,\frac{\sigma_{max}}{r}$, $\sigma_{max} = \frac{\sigma_k}{1-C\frac{h}{r}} = \frac{A\sigma_d}{1-C\frac{h}{r}}$,

$\frac{\sigma_{max}}{\sigma_n} = \frac{A\sigma_d}{\sigma_{dn}(1-C\frac{h}{r})}$ because in the case of fatigue limit stress level $\sigma_n = \sigma_{dn}$ (Fig.1),

$K_T = \frac{AK_F}{1-C\frac{h}{r}}$ since $K_F = \sigma_d/\sigma_{dn}$ and $K_T = \sigma_{max}/\sigma_n$.

Because of the limitation, that the stress gradient should not tend to infinity when $r \to 0$, we supposed

$$K_F = \frac{K_T}{A}\left(1 - C\,\frac{h}{r+r_0}\right) \qquad (1)$$

and determined the value of r_0 from the condition, that for $r=0$, $K_F = 1$ and $K_T = K_{T\,max}$. In the case of axially loaded sheet specimens with central holes [2] $K_{T\,max}=3$, $C = 2,1$, $r_0 = \frac{6.3h}{3-A}$,

$$K_F = \frac{K_T}{A}\left[1 - \frac{4.2h}{d+d_0}\right] , \qquad (2)$$

where $d = 2r$ is the hole diameter, $d_o = 2r_o$ and B the width of the specimen.

Formula (2) for the central hole case and formula (1) for the general case are two-parameter equations, since the third parameter r_o is expressed by h and A. The two parameters take into account the support effect of the stress gradient and of the local cyclic strengthening.

3. VERIFICATION OF VARIOUS FORMULAS FOR LARGE NOTCHES - In the case of a large notch radius $r \gg h$ is the notch factor $K_F=K_T/A$ according to (1) and (2). If cyclic strenghtening occurs (A>1) then $K_F<K_T$. The parameter A can therefore be found from fatigue tests for specimens with large central holes as $A=K_T/K_F$. All one-parameter formulas of K_F(Neuber's, Peterson's, Heywood's, Siebel's, etc) assume that for a large notch radius K_F is nearly equal K_T [5]. Table 1 shows that this conclusion is in general incorrect for aircraft materials, and that sometimes the ratio $A=K_T/K_F$ is much higher than 1 and therefor the one-parameter formulas overestimate the notch-size effect. From Table 2 it is also seen that for the same material, the ratio K_T/K_F is always higher in the case of pulsating tension (R=0) than in the case of alternating tension-compression(R= -1). The one-parameter formulas cannot explain this fact. The model of Fig.1 gives a simple explanation. In the case of $R = 0$ ($\sigma_{max}=2\sigma_a$)the fatigue limit level is much higher than in the case of R= -1($\sigma_{max}=\sigma_a$). The cyclic strengthening and the deviation from linearity of the $\sigma - \varepsilon$ dependance is more developed for a higher stress level. Therefore the support effect, described by the material parameter A should be stronger for R=0, than for R= -1. For Alclads and middle strength Al-alloys 6061 and AG5(5056) the material parameter A is evidently higher than for high strength alloys 7075-T6 and 2024-T3, where A=1.06 in the case of R= -1.

4. COMPARISON OF EXPERIMENTAL RESULTS WITH RESULTS OF ANALYTICAL FORMULAS - The author investigated aircraft materials sheet specimens with internal notches and obtained good agreement of the test results with the two-parameter equation (Fig.2). The well known one-parameter formulas, which use parameters estimated for small laboratory specimens result in an evident overestimation of the negative effect of large notches. Neuber's or Peterson's function q=f(r) is in qualitative and quantitative disagreement with the experimental points of Fig. 3. The formula recommended in some books [7,2] for the fatigue limit of notched specimens $\sigma_{dn}=\frac{\sigma_d}{K_F}C_d$, where K_F is calculated according to the classical formula of Neuber and C_d=0.85 is the average size factor [8], leads to a further overestimation of the notch-size effect. This formula can be used when K_F is calculated according to the more accurate two-parameter equation because in this case the factor 0.85 takes into account the nongeometrical size effect, the scatter of results and the fact that the notch effect is stronger for external than for internal notches (for the same K_T and r values).

5. LIMITATIONS - It should also be taken into account that the two-parameter formula (2), the parameters A and h, and the corresponding graphs in Fig. 2 were obtained for central holes (when for $r \to 0, K_T \to 3$). The triaxiality of the stress field near the notch, especially for deep edge notches, for thick flat specimens and for round specimens with sharp notches will lead to fatigue limit values different from those calculated according to formula (2). In case of high stress concentrations the two-parameter formula (2) (similar as the one parameter formulas), cannot be used, because the assumptions of Fig. 1 will not be fulfilled. The increase of the stress concentration above $K_T>4\div5$ may not increase the notch factor K_F further. This fact is in contradiction to all analytical formulas.

The material parameters A and h may be different for flat and round specimens and for R= -1 and R=0, because for a given material the support effect is connected with the value of the fatigue limit stress level.

The use of Eq. (2) for notches other than a central hole may not lead to a great error (Fig.2), if the fatigue limit is dependent only on the uniaxial stress concentration and stress gradient, and the notch radius is not very small.

The application of formulas (1) and (2) is approximately limited to $r > 0{,}5$mm and $K_T < 3\div4$ and, contrary to the one-parameter formulas, will lead to more accurate results for large specimen and notch sizes, because the parameter A is estimated for a large specimen with large central hole.

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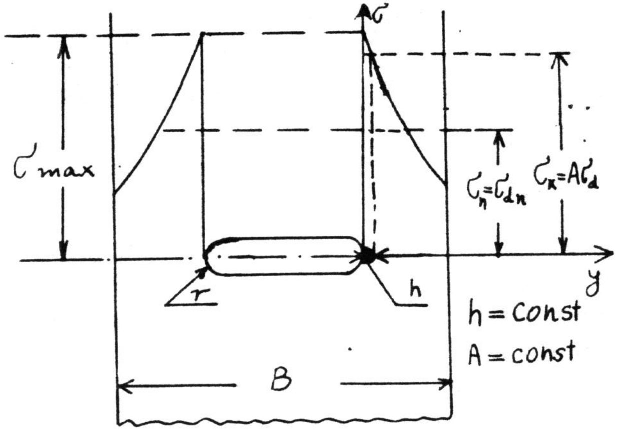

Fig.1 Pro... of the notch-size effect

Table 1 Values of K_F, K_T and $A = K_T/K_F$ for aircraft materials in case of axial cyclical loading /R = 0 or R = -1/ of specimens with large central holes.

Material	R	B mm	r mm	K_T	Fatigue Notch Factor K_F Exp.	Neuber	Peterson	$A = K_T/K_F$
7075-T6	0	101.6	25.4	2.12	1.72	2.00	2.10	1.24
2024-T3	0	101.6	25.4	2.12	1.81	2.00	2.10	1.18
2024-Alclad	0	60	20	2.07	1.53	1.97	2.04	1.35
6061-T4	0	60	20	2.07	1.38	1.87	2.04	1.50
2024-T3	-1	101.6	25.4	2.12	2.00	2.00	2.10	1.06
7075-T6	-1	101.6	25.4	2.12	2.00	2.00	2.10	1.06
2024-Alclad	-1	60	20	2.07	1.72	1.97	2.04	1.20
7075-Alclad	-1	60	20	2.07	1.92	1.97	2.04	1.08
6061-T4	-1	60	20	2.07	1.65	1.87	2.04	1.26

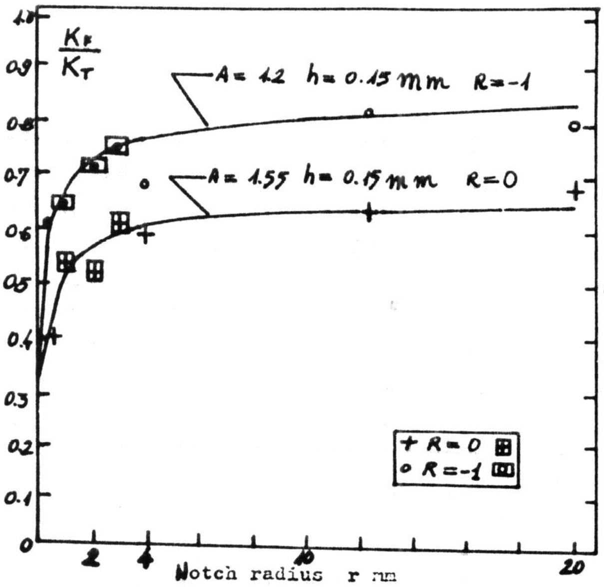

Fig.2 Experimental points for internal notched sheet specimens 6061-T4 and graphs acc. two-parameter formula (2).

● ✚ central holes $2.07 < K_T < 2.88$

▣ internal notches, K_T acc. [9]

⊞ $K_T = 2$ r=3, $K_T = 3.6$ r=2, $K_T = 4.6$ r=1 mm

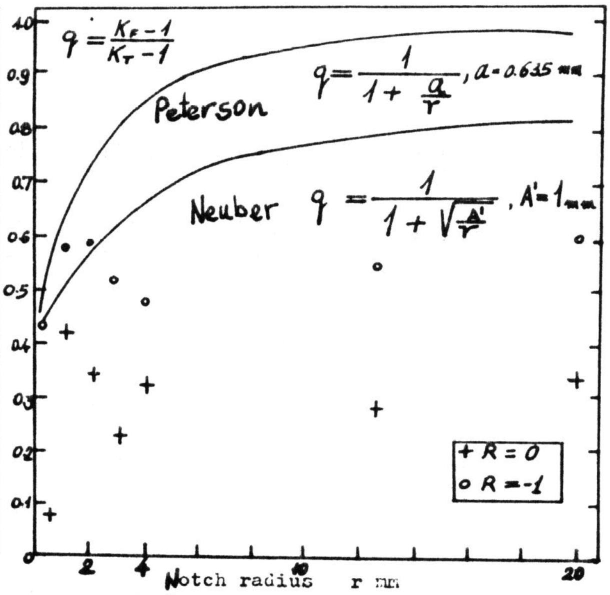

Fig.3 Experimental points for internal notched sheet specimens 6061-T4 and graphs acc. one-parameter formulas.