

# An Analysis of the Non - Propagating Fatigue Crack Using the Finite Element Method

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## INTRODUCTION

The non-propagating fatigue crack is one of the most important phenomena in the fatigue strength of notched specimens. However, the real mechanism for this phenomenon is still left unknown in the present time, mainly because of the lack of information about the stress and strain behavior around the crack tip.

In this paper, an emphasis was placed on the determination of the real stress and strain behavior around the tip of crack under cyclic loading. The purpose of this study is to obtain a quantitative method of prediction on the non-propagating fatigue crack in combination of the stress and strain behavior with a proper fracture criterion.

## METHOD OF ANALYSIS

The method of the analysis employed was the finite element elastic plastic analysis which was essentially the same as that developed by Yamada et al.<sup>(1)</sup> The computer program was modified in this study so that the reversed loading can be applied. This program can automatically detect whether the crack is opened or closed and can select the proper boundary conditions corresponding to the opening and the closure of each node along the crack. The plane stress condition was assumed together with the Mises yield criterion. A bi-linear stress-strain curve was employed. The characteristic values used in this study in relation to this stress-strain curve are as follows: Young's modulus  $E=20000\text{kg/mm}^2$ , Yield stress  $Y=30\text{kg/mm}^2$ , and Work hardening rate  $H'=0.06E$

$\approx 1200 \text{ kg/mm}^2$ . In order to characterize the hysteresis loop under cyclic loading in the steady state, a constant value of  $Y=30 \text{ kg/mm}^2$  was used both for tensile and compressive yield stress.

The geometry of the specimen analysed is a double-edge notched plate with a crack of several lengths at the root of the notch as shown in Fig.1. The usual triangular finite element was used. The total number of the elements was about 270. The minimum size of the element was 0.2mm in side length; the ratio of which to the specimen width was 0.008.

#### RESULTS AND DISCUSSION

Referring to the experimental work by Frost and Dugdale<sup>(2)</sup> as shown in Fig.2, the analysis was made for two kinds of notch severity; the stress concentration factors of which were 9.35 and 3.46. The former was severe enough to cause the non-propagating fatigue crack. Several stress levels as shown in Fig.2 by open circles were selected in which the non-propagating fatigue crack was expected to exist and not to exist. They were  $\pm 2.00 \text{ kg/mm}^2$  (Case A),  $\pm 3.33 \text{ kg/mm}^2$  (Case B),  $\pm 4.81 \text{ kg/mm}^2$  (Case C) and  $\pm 5.43 \text{ kg/mm}^2$  (Cases D and E).

Figs.3,4,5,6 and 7 show the results of the analysis expressed in terms of the strain range against the crack length for Cases A,B,C,D and E, respectively. The arabic numerals 1,2,3 and 4 in these figures specify the numbers allocated to the elements ahead of the crack tip as shown in Fig.1. Therefore, the curve 1, for example, in Figs.3 to 7 indicates the behavior of the strain range in the element no.1 ahead of the crack. It is noted that for a sharp notch of  $K_t=9.35$  the behavior of the strain range ahead of the crack showed a minimum at a certain crack length, while for a less sharp notch of  $K_t=3.46$  no minimum value was observed in the strain range versus crack length relation and the strain range increased monotonously with increasing crack length.

Now we combine the strain range behavior analysed in the foregoing

with a critical strain fracture criterion, which may be stated in this study as "A crack propagates when the magnitude of the strain range in an element at a certain distance in front of the crack, say in the element no.3, exceeds a critical value." (See Ref.3 in detail.) Taking the strain range corresponding to the fatigue limit of unnotched specimens as the critical value, which was shown in Figs.3 to 7 by the dashed line, the behavior of the crack may be predicted by examining the relative position of the curve 3 and the dashed line in each figure. For example, in Figs.4 and 5, the curve 3 extends upward above the dashed line in a range of short crack length, but it decreases with the increase of crack length and intersects the dashed line at a certain crack length beyond which the curve 3 remains below the critical strain range value. This means that a crack is initiated and propagated until the crack length is reached to a certain length, but no further crack growth will occur. This results in the non-propagating crack as expected from the experimental results in Fig.2. Other cases could also be well interpreted in the same way.

The same analysis was also extended to the case of the cyclic stress superposed on the mean tensile stress or the mean compressive stress. Figs.8 and 9 illustrate the strain range behavior analysed for Cases D and E, respectively, under cyclic loading with a compressive mean stress of  $1.525 \text{ kg/mm}^2$ , while Fig.10 illustrates the same plot for Case C under cyclic loading with a tensile mean stress of  $1.525 \text{ kg/mm}^2$ . Applying the hypothesis proposed in the foregoing to these cases, it is predicted that the non-propagating fatigue crack exists in the case of Fig.8, while no non-propagating fatigue crack exists both in the cases of Figs.9 and 10. These predictions are consistent with the experimental results by Frost and Dugdale. Again the proposed method was successful to explain the non-propagating fatigue crack.

On the other hand, one may expect that the governing condition for

the non-propagating fatigue crack is expressed in terms of the range of stress intensity factor  $\Delta K$ . In this study,  $\Delta K$  was calculated precisely on the basis of the real behavior of the crack opening and closure analysed in the foregoing. But  $\Delta K$  increased monotonously with the increase of crack length, and such a minimum as observed in the strain range versus crack length relation was not observed. The non-propagating fatigue crack could not be explained by the threshold stress intensity factor range  $\Delta K_{th}$ .

REFERENCES

- (1) Y.Yamada et al, Int. J. of Mech. Sci.,10,343(1968).
- (2) N.E.Frost and D.S.Dugdale, J. of Mech. Phys. Solids,5,182(1957).
- (3) K.Ohji et al, Proc. of the 15th Japan Congr. on Mat. Res.,91(1972-3)

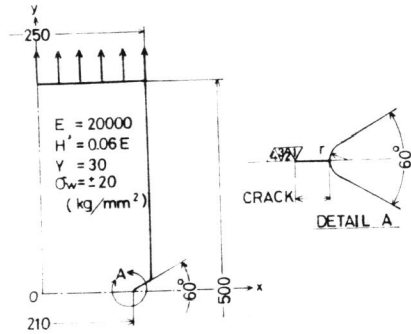


Fig.1 Geometry of the specimen analysed

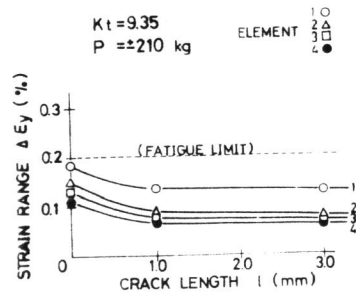


Fig.3 Case A ( $\sigma = \pm 2.00 \text{ kg/mm}^2$ )

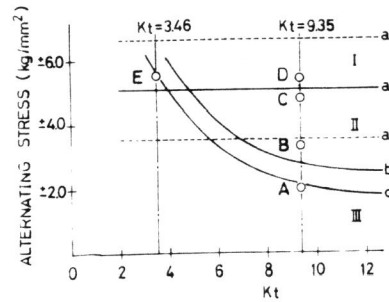


Fig.2 Experimental results on the non-propagating fatigue crack by Frost and Dugdale (Ref.2)

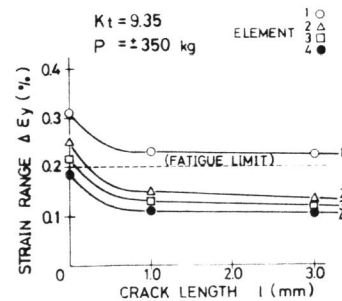


Fig.4 Case B ( $\sigma = \pm 3.33 \text{ kg/mm}^2$ )

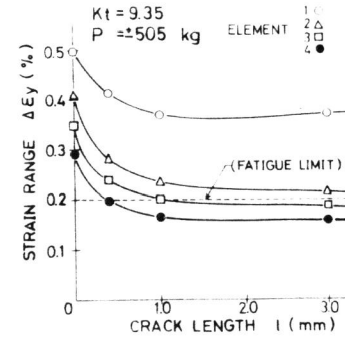


Fig.5 Case C ( $\sigma = \pm 4.81 \text{ kg/mm}^2$ )

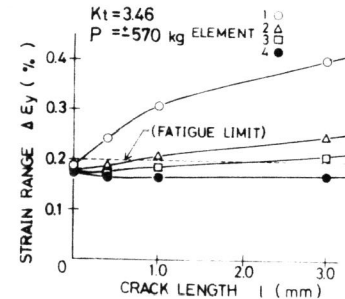


Fig.7 Case E ( $\sigma = \pm 5.43 \text{ kg/mm}^2$ )

Figs.3 to 7, strain range versus crack length relation under cyclic loading with zero mean stress

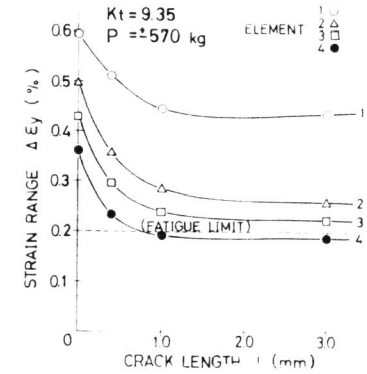


Fig.6 Case D ( $\sigma = \pm 5.43 \text{ kg/mm}^2$ )

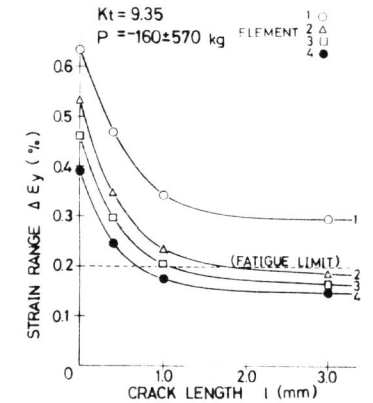


Fig.8 Case D ( $\sigma = -1.525 \pm 5.43 \text{ kg/mm}^2$ )

Figs.8 to 10, strain range versus crack length relation under cyclic loading with tensile or compressive mean stress

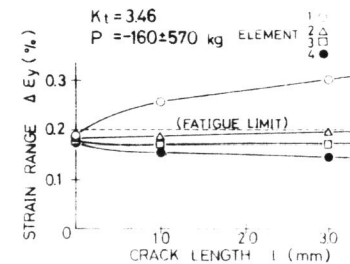


Fig.9 Case E ( $\sigma = -1.525 \pm 5.43 \text{ kg/mm}^2$ )

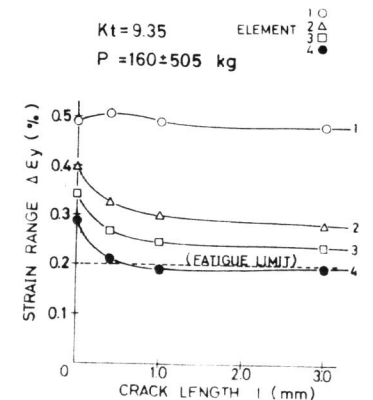


Fig.10 Case C ( $\sigma = 1.525 \pm 4.81 \text{ kg/mm}^2$ )