To the Problem of the Ductile - Cleavage Transition of Fracture

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Cottrell has done an important step in direction of solution of this problem, proposing the known model (1) and deriving that cleavage occurs, if

$$\sigma n a = 2\gamma \qquad [1]$$

where σ is the tensile stress, na is the crack opening displacement, produced by coalesced slip dislocations at the starting point of the crack and γ is the surface energy. Replacing na by macroscopic quantities Cottrell obtained, that cleavage fracture appears at the yield stress σ_y, if

$$\sigma_y = \sigma_{yt} = \sigma_{FB} = \frac{8\gamma G}{k_y} D^{-1/2} \text{ and therefore } T = T_t\,,\ \dot{\varepsilon} = \dot{\varepsilon}_t. \qquad [2]$$

σ_{FB} is the brittle fracture stress or better the cleavage fracture stress, G is the shear modulus, k_y is the Hall-Petch constant, and D is the grain diameter. The index t denotes the state of transition. In this case the transition is defined by equality of σ_y and σ_{FB}, that means by the lowest limit of the transition range, where before fracture only one Lüders band has passed the cross section.

Some results of experimental investigations to test equation [2] are summarized in fig.1 (2 to 4). These results demonstrate, that:

1. σ_{yt} plotted versus $D^{-1/2}$ fits a nearly single straight line for all the experiments with different steels but the extrapolation of this line towards $D^{-1/2} = 0$ does not provid $\sigma_{yt} = 0$ as equation [2] predicts.
2. Within the temperature range concerned the investigated steels exhibit k_y -values between 15 to 60 $N/mm^{3/2}$, whereas the slope of the σ_{yt} -curve is independent on k_y.
3. The value of the surfade energy γ, calculated by using experimental data, is more than one order of magnitude larger than the true value of $1.5 \cdot 10^3 erg/cm^2 \approx 1.5 \cdot 10^{-3} mmN/mm^2$, because it includes the deformation work at the crack tip. So it has been called γ', the effective surface energy.

Supposed that the critical crack length is the grain diameter, one could describe the experimental results by a Griffith relationship with $c = D$ (c =half the whole crack length) in the shape of

$$\sigma_{yl} = \sigma_{FB} = \sqrt{\frac{2\gamma' E}{\pi(1-\mu^2)D}} \qquad [3]$$

μ is the Poisson constant, introduced for plane strain, E is the elasticity modulus. Doing so one gets a γ' depending on the grain size. In the log-log plot of the γ'-values, calculated by equation [3] from the experimental results, versus D one gets a slope of 0.5. That means, there should be a relationship $\gamma' \sim D^{1/2}$ as fig.2 demonstrates.

To find an appropriate relationship analytically, one has to look for a suitable model. For example, a crack is nucleated at a grain boundary by the Cottrell mechanism. The grain in which the crack is nucleated may be free of active slip bands, so that the crack tip cannot be blunted. The crack becomes unstable, if equation [1] is performed, and now its opening displacement na at the grain boundary, where the crack has been originated, cannot become larger than the surface energy permits. There the displacement for a crack of the length $2c$ is given by the dislocation theory of cracks (5) by about

$$na = 4\sqrt{\frac{2\pi(1-\mu^2)\gamma c}{E}} \qquad [4]$$

and so many dislocations run into the end of the crack (Fig. 6 b).

When the unstable propagating crack tip joints the grain boundary opposite its starting point, it cuts up dislocation pile ups in the neighbouring grain. In this moment its displacement at the other end, where it has started, is

$$na = 4\sqrt{\frac{2\pi(1-\mu^2)\gamma D}{E}} \qquad [5]$$

and the same plastic displacement now suffers the original crack tip. In the language of crack dislocations one would say, there are so many slip dislocations dragged into the crack tip as crack dislocations there have been before.

The crack is now blunted at both sides. The blunting is rather small, for example for a grain size of 0.025 mm the displacement is about 0.13 μm or n = 450. What happens afterwards depends on the level of the stress produced ex-

ternally. If the stress is high enough, the crack propagates over the whole cross section, if it is not so, it blunts further on.

A blunted crack becomes unstable, if (6)

$$\sigma = \sigma_{FB} = \sqrt{\frac{\gamma E}{4(1-\mu^2)D} \cdot \frac{\rho}{a}} \qquad [6]$$

where ρ is the crack tip radius, a is the lattice spacing and half the crack length $c = D$ regarding the grain size distribution and the fact, that for cracks of the length of grain size the fracture stress is lower in the larger grains than in the grains of medium size. This equation equals eq. [3], if (6)

$$\frac{\gamma'}{\gamma} = \frac{\gamma + \gamma_p}{\gamma} \approx \frac{\gamma_p}{\gamma} = \frac{\pi\rho}{8a} \qquad [7]$$

when $\gamma_p \gg \gamma$, where γ_p is the plastic part of the effective surface energy γ'.

The crack tip radius ρ may be half about the crack opening displacement at both ends of the crack

$$\rho = \frac{na}{2} = 2\sqrt{\frac{2\pi(1-\mu^2)\gamma D}{E}}. \qquad [8]$$

Replacing ρ in eq. [6] and [7] by eq. [8] one gets

$$\sigma = \sigma_{FB} = \sqrt[4]{\frac{\pi\gamma^3 E}{2(1-\mu^2)a^2 D}}, \text{ for iron } = 340 \cdot D^{-1/4} \quad \text{N/mm}^2 \qquad [9]$$

and

$$\gamma_p \approx \gamma' = \sqrt{\frac{\pi^3(1-\mu^2)\gamma^3 D}{8Ea^2}}, \text{ for iron } = 0{,}79 \cdot D^{1/2} \quad \text{mmN/mm}^2 \qquad [10]$$

having used the elastic constants and $\gamma = 1{,}5 \cdot 10^{-3}$ mmN/mm^2 for iron. Eq. [10] is just the relationship between γ' and D as it has been found by analysing experimental data in fig. 2. The straight line in this figure represents eq. [10] and fits the experimental data rather well.

Because slip is a necessary prerequisite to nucleate a cleavage crack one can write eq. [9] including the yield criterion concerned as a condition for ductil-cleavage transition as defined above:

$$\frac{\sigma_y(\dot\varepsilon, T)}{1-\kappa} = \sigma_{FB} = \sigma_1 \quad at \quad T = T_t, \; \dot\varepsilon = \dot\varepsilon_t \qquad [11]$$

where $\varkappa$ is a multi-axiality factor of yield stress given by $\sigma_1 - \sigma_M / \sigma_1$ (5) where σ_1 is the largest principal stress and σ_M is the stress of the von Mises criterion. Experimental data from fig.1 including values from notched specimens (3) with $\varkappa = 0.3$ are plotted now in fig.3 with respect to the derived relationship between $\sigma_{yt} = \sigma_{FB}$ and D in the shape of $\sigma_{yt}^2 \sim D^{-1/2}$. The strainght line again represents eq.[9] and fits the experimental data quite well too. Eqs.[9] and [11] confirm also results from Knott (7) who got almost equal cleavage fracture stresses for different values of $\varkappa$ and therefore also for different transition temperatures.

Fig.4 showes the dependense of σ_y from (3) and σ_{FB} on the temperature for a steel with a grain size of 0.025 mm. The dashed lines indicate the transition temperature for different values of $\varkappa$. Because σ_{FB} contains only elastic quantities its temperature dependence is small but since this is true for σ_y near room temperature too, in this range the transition temperature raises rather sharp depending on $\varkappa$ as fig.5 demonstrates. The transition temperatures for $\varkappa = 0$ and $\varkappa = 0.3$ are just these, which Kochendörfer and Schreiner (3) found.

References

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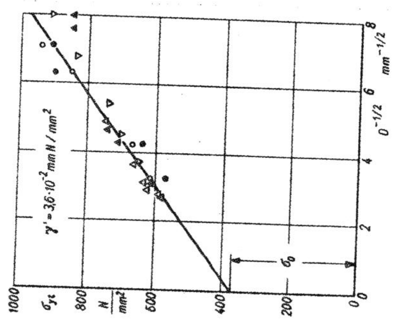

Fig.1: Dependence of the cleavage fracture stress on $D^{-1/2}$

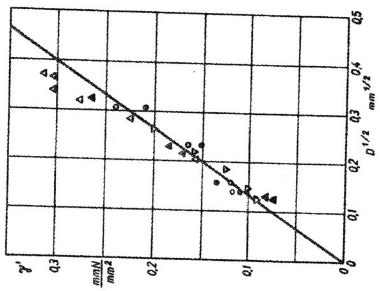

Fig.2: Dependence of the effective surface energy on $D^{1/2}$

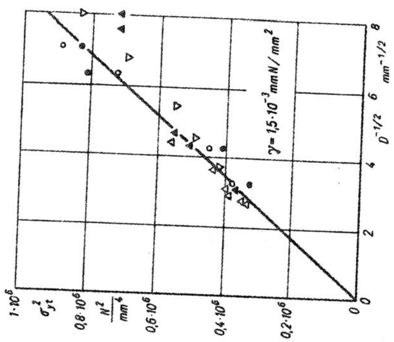

Fig.3: Dependence of the square of the cleavage fracture stress on $D^{-1/2}$

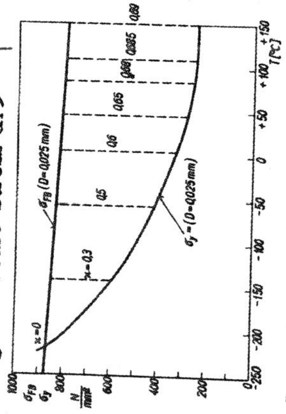

Fig.4: Dependence of the yield stress and and cleavage fracture stress on the temperature; T_t at different $\varkappa$ ----

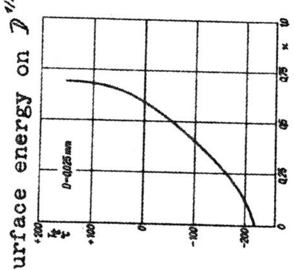

Fig.5: Dependence of the cleavage fracture temperature on $\varkappa$

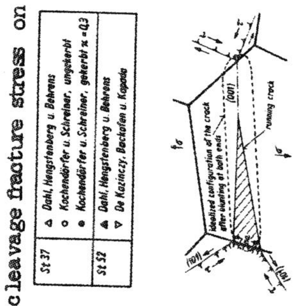

Fig.6: a) Meaning of symbols in Fig.1 to 3 b) Model