Fracture Phenomena in Composites

A. Kelly
National Physical Laboratory, Teddington, Middlesex, England

1. Introduction

In this paper we shall consider fibrous composites in which cylindrical aligned fibres are introduced into an isotropic matrix and aligned parallel to the tensile stress. We shall deal with specimens which are not intentionally notched and will regard failure as termination of the stress-strain curve. It must be recognised at the outset that in a fibrous composite fracture in one or other of the components may occur, and usually does occur, during the stress-strain curve of the composite prior to this stress-strain curve passing through a maximum. Fracture may, therefore, be occurring throughout the stress-strain history of the composite and is not necessarily confined to the final stages. This means that in considering the strength of laminates containing fibres running in various directions, and in considering the fatigue and creep of composites, the fact that one of the components may be broken early in the stress-strain curve must be considered.

In a ductile material the tensile stress to produce failure is not well defined - failure occurs in tension when a point of instability is reached. This point of instability depends on both the stress system and the strain rate. Nevertheless, in order to systematise our description of events, we will suppose, first that we are dealing with an aligned fibrous composite in which both phases are continuous and each possesses a yield stress (subscript y), a UTS (subscript ult) and a fracture stress (subscript f). Thus σ_{ym} is a yield stress of the matrix and σ_{ff} a fracture stress of the fibre. For a brittle fibre $\varepsilon_{ff} = \varepsilon_{ult\ f}$ and for a ductile fibre $\varepsilon_{yf} < \varepsilon_{ult\ f} < \varepsilon_{ff}$.

Various values of ε_{ym}, $\varepsilon_{ult\ f}$, will be regarded, to a first approximation, as adequately determined in experiments before the components are joined together in a composite. In most cases accurate experiments show that the values of these parameters measured in the composite are, in fact, different from those measured outside, and interest attaches to the cases when these are markedly different. However, in many cases the differences are small. As the value of scientific work on fibrous composites grows, these differences are being clearly recognised, whereas in the infancy of the subject, it was possible to ignore them to a first approximation.

2. Continuous Fibres - Matrix and Fibre both Ductile

This classic case has been investigated by McDanels, Jech and Weeton (1965). In general, there are four stages in the composite stress-strain curve. These are: (i) Both components elastic. (ii) Matrix, plastic, fibre elastic. (iii) Both components plastic. (iv) Fracture of fibres. (v) Depending upon the volume fractions in the previous stage, one has then to consider whether or not failure of the fibres leads to immediate failure of the composite or whether (as is the case with small volume fractions of fibres) failure of the fibres is continuous during the plastic stretching of the matrix.

When both materials are plastic, the stability of the composite during continued deformation in tension parallel to the axis of the fibres must be taken into account. This was pointed out by Kelly and Tyson (1965a). However, they made no attempt to derive a quantitative result. In order to do so, one must assume a form of stress-strain curve for fibre and for matrix. This has been done by Mileiko (1969) and by Garmong and Thompson (1972). We can consider their analyses together since they are essentially the same but different empirical

forms are assumed for the stress-strain curves of the two components.

The load on a composite is given by

$$F_c = \sigma_f A_f + \sigma_m A_m . \qquad \ldots (1)$$

Stability is assured, provided for an increment of strain of the composite $d\varepsilon$, assumed the same in both components, we have $dF_c > 0$. We may write

$$dF_c = \sigma_f dA_f + A_f d\sigma_f + \sigma_m dA_m + A_m d\sigma_m . \qquad \ldots (2)$$

Dividing both sides by $(A_f + A_m)$ where A_f and A_m are the areas of fibre and matrix respectively in a composite, and noting that

$$d\varepsilon_c = -\frac{dA_f}{A_f} = -\frac{dA_m}{A_m} , \qquad \ldots (3)$$

we have as the condition for stability of the composite in tension

$$V_f\,(d\sigma_f/d\varepsilon - \sigma_f) + V_m\,(d\sigma_m/d\varepsilon - \sigma_m) > 0. \qquad \ldots (4)$$

This is a statement of the Considère criterion applied to a two-phase composite and from it an expression for the ultimate stress and ultimate strain of the composite may be obtained, provided a form of stress-strain curve is assumed both for fibre and for matrix.

Mileiko assumes a simple power law for the stress-strain curve. The true stress varies with true strain as

$$\sigma = \sigma^* \varepsilon^{1/n} , \qquad \ldots (5)$$

in which σ^* and n are constants for each component. Garmong and Thompson on the other hand, take

$$\sigma = \sigma_o + K\,\varepsilon^n . \qquad \ldots (6)$$

Garmong and Thompson consequently have a larger number of adjustable parameters than does Mileiko. Both works lead to essentially the same conclusion: these are that both the ultimate tensile strength of the composite and the strain at which this is reached vary smoothly with component volume fraction and with the values of these quantities for the two constituents. The results have been applied to the data presented by Kelly and Tyson (1965) on Cu-Mo, Piehler (1965) on silver containing stainless steel, Ahmad and Barranco (1970) on tantalum in copper, and Mileiko also applies his results to some previously unpublished work by Markov in Russia on nickel containing tungsten. A plot of the true strain at ultimate tensile stress against volume fraction is shown in Fig. 1 using Garmong and Thompson's formulation. The agreement is satisfactory and within the experimental scatter of the results. The agreement shown in Fig. 1 is typical of the agreement between either of the two theories and experiment for all the systems tested. However, it appears noteworthy that for Ahmad and Barranco's data and for nickel reinforced with tungsten, reported by Mileiko, at small volume fractions the measured ultimate tensile strength is somewhat higher than the theory predicts. Also, Bomford and Kelly (1971) find that Mileiko's equations, fitted to the results on copper-tungsten, understimate the observed strains at which the UTS of a composite is reached, and also predict a variation of UTS with volume fraction which coincides only with the lower limit of the envelope of the experimental results.

Mileiko's result enables explicit expressions for the $\sigma_{ult\ c}$ and $\varepsilon_{ult\ c}$ (note σ_{ult} is not a true stress, it is the conventionally defined ultimate tensile strength) to be written. From these it can be demonstrated that if the ultimate tensile strains of the two components are the same, then the ultimate tensile strength of the composite is given by a simple addition

$$\sigma_{ult\ c} = \sigma_{ult\ f} V_f + \sigma_{ult\ m} V_m. \qquad \ldots (7)$$

This corresponds to the line AB in Fig. 2. According to both theories the line AB can never be exceeded; it would be interesting to

investigate further the positive deviations above the line shown by Bomford and Kelly's and Ahmad and Barranco's results. It is clear that if one component fails before the other, i.e. has a smaller strain to the ultimate tensile strength, then the strength of the composite must fall below the line AB. The usual case is when the fibre has a much smaller elongation than the matrix. Under these conditions, lower limits to the strength are obtained when, for instance, the failure strain of the fibre is very small. If $1 >> \varepsilon_{ult\ m} >> \varepsilon_{ult\ f}$, Mileiko's expression may then be written as

$$\varepsilon_{ult\ c} = \varepsilon_{ult\ m} \Big/ \left[(\sigma_{ult\ f} V_f / \sigma_{ult\ m} V_m) + 1 + \varepsilon_{ult\ m} \right]. \qquad \ldots (8)$$

As $\sigma_{ult\ f} V_f$ tends to zero, $\varepsilon_{ult\ c}$ tends towards $\varepsilon_{ult\ m}$ but if $\sigma_{ult\ f} V_f$ is much greater than $\sigma_{ult\ m} V_m$ then the breaking strain of the composite is much smaller than the breaking strain of the matrix. The lower limits to the strength are given by the lines OB and AO in Fig. 2 where the curve to the line AO is

$$\sigma_c = \sigma_{ult\ m} (1 - V_f) \qquad \ldots (9)$$

and OB has equation

$$\sigma_{ult\ c} = \sigma_{ult\ f} V_f + \sigma'_m V_m \qquad \ldots (10)$$

where σ'_m is equal to the stress on the matrix at the failure strain of the fibres. If the yield strain of the matrix is equal to or greater than the ultimate tensile strain of the fibres, then $\sigma'_m = E_m \varepsilon_{ult\ f}$.

The decrease in strength shown by the line AO is only observed if fibres with a very small breaking strain are introduced into a matrix. It has been established clearly in a number of cases, e.g. by Kelly and Tyson (1965) for tungsten in copper and Calow, Robinson and Hambling (1968). The last results for small volume fractions of molybdenum in lead are shown in Fig. 3.

We will show later that there is a theoretical possibility of increasing the failure strain of the fibres at small volume fractions in a composite, provided the fibres are very small.

Equation 8 shows that the breaking strain of a composite will be much larger than the breaking strain of fibres which are essentially brittle, provided $\sigma_{ult\ m} V_m > \sigma_{ult\ f} V_f$. In the region of volume fractions to the left of O in Fig. 2, the following inequality is obeyed

$$\sigma_{ult\ f} V_f < \sigma_{ult\ m} V_m - \sigma'_m V_m. \quad \ldots (11)$$

The composite can be strained to a considerably larger breaking strain than that of the fibres and the fibres must break. This has been observed to occur in copper-chromium (Hertzberg and Kraft 1963), copper-tungsten (Kelly and Tyson, 1965b) and in low carbon steels, e.g. iron-0.02 wt % carbon which has been heat treated to produce cementite plates and then heavily deformed so as to break the plates, Lindley, Oates and Richards (1970). Fracture is a progressive one with strain. Initially the fibre or platelet is broken into long lengths and these pieces fracture further as strain proceeds. Theoretically one expects a lower limit to the size ℓ given by

$$\frac{\ell}{d} = \frac{\sigma_{ult\ f}}{4\tau} \quad \ldots (12)$$

where d is the diameter and τ an upper limit to the shearing force per unit area which may be exercised by the matrix upon the fibres. The distribution of sizes when the limit is reached is expected to be such that the lengths lie between ℓ and 2ℓ. The weighted average will lie nearer to ℓ because there will be a larger number of the smaller pieces.

It is the recognition that there is a lower limit to the size into which pieces of fibre or reinforcing plate can be broken by flow of the matrix that gives rise to the idea of a critical length, namely the minimum length of fibre which can be broken by a flow of a particular matrix. The concept is well known and extensively applied in considering the fracture of aligned composites. It is, of course, also important in the transverse deformation of aligned composites. A number of people have studied the transverse deformation of a composite consisting of ribbons, e.g. Anderson and Bode (1972). The striking pictures taken by Anderson and Bode illustrate how "pull out" develops during the fracture of a composite. It is evident that one can develop the concept of a critical length not solely from flow past the ribbons but in terms of the degree of critical overlap between the ribbons, such that if there is not sufficient overlap, the ribbons will all pull out: if there is sufficient overlap then they will break.

3. Continuous Fibres - Fibres Brittle

If fibres are brittle then a lower limit to the strength is given by equation (10), corresponding to the line OB in Fig. 2. However, at critical volume fractions greater than that given by O, failure of the fibres must lead to immediate failure of the composite if the fibres all break at the same load. It must be recognised that with a brittle material the fibres are not of equal strength and some account must be taken of the variations of strength of the population of fibres in defining $\sigma_{ult\ f}$ in (10). In addition, when a fibre fails there is a stress concentration at the neighbouring fibres. This stress concentration may be as large as a factor 2 if the matrix is incapable of bearing any load but in many cases is less than 2, see e.g. Lockett (1971). The process of failure, taking into account the variation in the strength of the fibres, and of the stress concentrating effect has been considered by Rosen and his colleagues, e.g. Rosen (1970). These theories have been discussed in detail by Hale and Kelly (1972). We give here a short account of the discussion by Hale and Kelly. The most widely used expression for the variation of tensile strength with length, of a population of fibres is the Weibull distribution, viz.

$$f(\sigma) = L\alpha\beta\sigma^{\beta-1} \exp(-L\alpha\sigma^{\beta}) \qquad \ldots (13)$$

where $f(\sigma)$ is the probability density function, i.e. the probability of the fibre stress lying between σ and $(\sigma + d\sigma)$. L is the fibre length and α and β are two parameters describing the distribution. Colman (1958) has shown that for fibres with a strength independent of the rate of loading, such a distribution of strength would be expected. This expression can be used because one can relate the mean fibre strength $\bar{\sigma}$ and standard deviation s by the following expressions:

$$\bar{\sigma} = (\alpha L)^{-1/\beta}\,\Gamma(1 + 1/\beta) \qquad \ldots (14)$$

$$s = (\alpha L)^{-1/\beta}\left[\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)\right]^{\frac{1}{2}}. \qquad \ldots (15)$$

The coefficient of variation is given by

$$\mu = \frac{s}{\bar{\sigma}} = \frac{[\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)]^{\frac{1}{2}}}{\Gamma(1 + 1/\beta)} \qquad \ldots (16)$$

and μ is hence a function of β only. Rosen points out that for $0.05 < \mu < 0.5$, which covers most fibres, then μ is of order $\beta^{-0.92}$. For glass fibres the coefficient of variation is about 0.1 corresponding to a value of 11 for β. For boron filaments μ is about 1/3. The values of the parameters α and β can be obtained directly from experimental data, e.g. from the values of the mean fibre strength (14) and the standard deviation using (15) and (16). The strengths of

bundles of fibres have been analysed by Daniels (1945). He shows that for a Weibull distribution of fibres the expected value (mean strength) of the tensile strength of a bundle expressed as an average stress per fibre in the bundle, is

$$\bar{\sigma} = (L\,\alpha\beta e)^{-1/\beta} \qquad \ldots (17)$$

where L is the length of fibres in the bundle. Colman compared this value for the strength of the bundle with that given by equation (14) for the average strength of fibres of the same length. It is then clear that when there is no dispersion of the fibre strength (β very large), then the bundle strength is the same as the mean fibre strength. As the coefficient of variation of fibre strength increases above zero, the bundle strength approaches zero in the limit of infinite dispersion. When the coefficient of variation is about 20%, the strength of the bundle is about 70% of the average strength of the fibres. This is the result often quoted in the literature Its use demands a knowledge of the variation of s with length of fibre.

Rosen (1970) introduces what is called the cumulative weakening model in which the fibres break and a length near their end becomes ineffective in bearing load. This length, the ineffective length, is related to the stress transfer length above. Rosen then considers the composite to be made up of a set of bundles of fibres, the length of the bundles being equal to the effective length and he then considers these bundles as links in a chain and considers the probability of a particular link failing. In fact, he later assumes that the number of elements in a bundle is so large that the standard deviation of the strength of a bundle is zero, and under this condition the statistical mode of the composite strength becomes equal to the average strength of a bundle, as equation (17). It follows that the most probable failure stress of the composite is

$$\sigma_c = V_f (\ell^* \alpha \beta e)^{-1/\beta} \qquad \dots (18)$$

where V_f is the fibre volume fraction and ℓ^* is the ineffective length; the strength of the matrix is neglected here. Some results of the strength of a composite consisting of aligned continuous fibres of boron-aluminium taken from the work of Kreider and Leverant (1966) are shown in Fig. 4. Kreider and Leverant state that the average tensile strength of the filaments is 425,000 psi and the diameter of the fibres is between 0.004 and 0.005 inch. Boron filaments usually have a coefficient of variation of between 0.2 and 0.4 - Kreider and Leverant say that their filaments have $\mu = 0.2$. We may thus take the value of β as one third. Boron filaments have a strength usually measured in one inch lengths; assuming that this is so for Kreider and Leverant's values, we have

$$\Gamma(1 + 3) = 6,$$

and hence can evaluate $(\alpha L)^{-1/3}$ for a given value of L. We can then calculate the ineffective length which would be necessary to account for the results. If the ineffective length is 0.71 in, the calculated line passes through the middle of the experimental points for the sintered aluminium-boron. If it is to pass through the outer envelope of the experimental results - see Fig. 4 - then the ineffective length must be 0.66 in.

It is sensible to relate the ineffective length to the value of the quantity ℓ in equation (12) by taking $\ell^* = \ell/2$. Then using the expression for ℓ in (12) we can relate ℓ^* to a value of τ. The two values of ℓ^* yield values of τ of 375 psi and 400 psi respectively. Any stress concentrating effects due to the presence of the broken ends which accumulate during the deformation of the composite

will enable the results to be interpreted using larger values of ℓ^*. For example, if a broken fibre concentrates the stress on its neighbours by a factor 7/6 = 1.166, then ℓ^* could be as small as 0.63 in and τ as large as 464 psi.

4. Continuous Fibres - Brittle Matrix

In this case we assume that the matrix is brittle and $\varepsilon_{ym} = \varepsilon_{ult\,m}$. We further assume that $\varepsilon_{ult\,f}$ is much greater than $\varepsilon_{ult\,m}$. Two cases must then immediately be considered: that in which the modulus of the fibres is greater or less than that of the matrix.

If the fibres have a lower Young modulus than that of the matrix, then when a composite attains the failure strain of the matrix $\varepsilon_{ult\,m}$, the load per unit area borne by the fibres is less than the load per unit area borne by the matrix. At small volume fractions of fibres the strength of the composite will be less than that of the matrix and the variation of ultimate strength of the composite with volume fraction will decrease with increasing V_f, following the line AB" in Fig. 5. Correspondingly, if the modulus of the fibre is larger than that of the matrix, then when the matrix fails the fibre will carry a larger load per unit area than that of the matrix and the strength of the composite will rise with increasing volume fraction of fibres following the line AB' in Fig. 5. In both cases the intercept on the ordinate at $V_f = 1$ is $\varepsilon_{um} E_f$. In one case this is smaller than the stress on the matrix at failure and in the other case it is larger. If the fibres are considerably stronger than the matrix, that is $\sigma_{ult\,f}$ much greater than $\sigma_{ult\,m}$, then failure of the matrix will only result in failure of the composite for small volume fractions. The condition that failure of the matrix leads to failure of the composite is that

$$\sigma_{ult\,m} V_m > \sigma_{ult\,f} V_f - \sigma_f' V_f \qquad \dots (19)$$

where σ_f' is the stress on the fibre when the matrix breaks and is equal to $\varepsilon_{ult\ m} E_f$. This equation defines the critical volume fraction, V_f, such that

$$V_f' = \frac{\sigma_{ult\ m}}{(\sigma_{ult\ f} + \sigma_{ult\ m} - \sigma_f')} . \qquad \ldots (20)$$

For volume fractions greater than this, the breaking stress depends only on the breaking stress of the fibres and is given by $\sigma_{ult\ f} V_f$. For smaller volume fractions, there is an increase in breaking strength of the composite above that of the matrix, only provided the modulus of the fibre is greater than that of the matrix. In the region where volume fractions are larger than that given by equation (20), the strength is <u>independent</u> of the modulus of the fibres and depends only on the breaking strength.

If $V_f < V_f'$ when the matrix fails the fibres are unable to bear the load placed upon them. If they are ductile, they neck down and break. The region over which further deformation of the fibre occurs, after failure of the matrix, is confined to a small region on either side of the initial crack. Only one crack is usually formed. The elongation of the specimen is governed by the length of region over which the fibre continues to plastically deform. This region is generally shorter than the specimen length and so the failure strain of the composite will be very much less than that of the fibres. What experiments there are, Grenier and Cooper (1969), indicate that the failure strain of the composite, or more accurately, the ultimate tensile strain of the composite is, in fact, smaller than that of either component, i.e. smaller than that of the matrix. Whether this is a real effect remains to be seen. It may be that the method of preparing the matrix in the presence of fibres is different from that

of preparing specimens of the matrix without any fibres and hence the failure strains of the matrix are not the same outside the composite and in it.

In the region of volume fraction greater than that given by (20) continued deformation of the composite after the first matrix crack must result in further cracking of the matrix. When multiple fracture of the matrix occurs, the limiting crack separation is between x' and 2x', though in this case, in contrast to equation (12) above, we have

$$x' = (V_m/V_f)\, \sigma_{ult\ m}\, r/2\tau . \qquad \ldots (21)$$

The difference between ℓ and x' in equations (12) and (21) respectively is noteworthy. It arises because in multiple fracture of the fibres, the load necessary to break the fibres and the load transferred across the break, both depend on the radius of the fibres and hence there is no term (V_m/V_f) in equation (12).

According to equation (21) the crack spacing should decrease with the decrease in radius of the fibres for a given volume fraction. A decrease in the radius of fibres corresponds to an increase in the area of fibre-matrix interface per unit volume. This can easily be seen since the area of interface per unit volume of the composite is just $(2V_f/r)$ and hence equation (21) can be written

$$x' = (V_m\, \sigma_{ult\ m}/\tau)(1/a_V) \qquad \ldots (22)$$

where $a_V = (2V_f/r)$ is the area of interface per unit volume of the composite. According to (21), the spacing of the cracks should decrease without limit as r becomes smaller. However, other effects must intervene and we deal with these below.

Fig. 6 shows the stress-strain curve of a specimen undergoing multiple fracture of the matrix. It has some features of special interest. When the failure strain of the matrix is attained, the load carried by the matrix per unit area of composite is thrown onto the fibres bridging the crack. The fibres elongate further under this additional load, which can be assumed to be transmitted back into the matrix over a distance x' on either side of the crack, by constant limiting interfacial shear stress τ, given by equation (22). Whether or not the fibre and matrix are completely debonded governs whether or not τ is independent of distance along the interface in a direction normal to the crack. This point has been discussed in detail by Aveston and Kelly (1972), who conclude that except for very special cases, the debonding will usually be complete after the appearance of the first crack. This argument will be reinforced if the fibres are a ductile material which is plastically deforming, since then the Poisson ratio of the fibre will be much larger than that of the matrix and this will tend to pull the fibre away from the matrix. According to equation (21) the additional stress in the fibres will, therefore, vary linearly between zero at a distance x' from the crack to a maximum of $\sigma_{ult\ m}(V_m/V_f)$ at the crack. The mean additional strain over distance $2x'$ is $\alpha\,\varepsilon_{mu}/2$ where

$$\alpha = E_m V_m / E_f V_f. \qquad \text{... (23)}$$

If the matrix has a well defined breaking strain, the cracking will continue at a constant stress $E_c\,\varepsilon_{um}$ until the matrix is broken down to a set of blocks, each of length between x' and $2x'$. For the lower bound to the crack spacing, x', the maximum additional stress in the fibre at the crack is still $\sigma_{ult\ m}\,V_m/V_f$ as before. The maximum length over which this additional stress can be transferred back to the matrix is now reduced to $x'/2$ and so only one half of the additional stress can be transferred to the matrix. The mean additional strain over the length x' is, therefore, increased to

$3\alpha\,\varepsilon_{um}/4$. The total strain at the limit of multiple cracking is, therefore, $\varepsilon_{um}(1 + \alpha/2) < \varepsilon_c < \varepsilon_{um}(1 + 3\alpha/4)$. A further increase of the load on the composite will result in the fibres being stretched further and slipping through the blocks of matrix which can take no further share of the load, so that the Young modulus of the specimen will become $E_f V_f$. The composite will eventually fail at stress $\sigma_{ult\ f}\,V_f$ and strain ε_{uc} given by

$$(\varepsilon_{uf} - \alpha\,\varepsilon_{um}/2) < \varepsilon_{uc} < (\varepsilon_{uf} - \alpha\,\varepsilon_{um}/4). \qquad \text{... (24)}$$

It is noteworthy that the predicted failure strain of the composite is in all cases, of course, less than the failure strain of the fibres.

5. Constrained Failure

When a single crack is formed in the matrix running completely across the specimen normal to the fibres, the fracture surface energy γ_m per unit area of matrix must be provided by the work done by the loading system. If this energy cannot be supplied at the cracking strain of the matrix, the matrix will be prevented from cracking until some larger strain is attained (Aveston, Cooper and Kelly 1971).

When a crack is formed under conditions of fixed load, the following energy changes occur:

(a) Work is done by the applied stress since the body increases in length so that it becomes more deformable. This work, if the crack is formed at a strain ε_{um} is given by

$$\delta W = E_c\,\varepsilon_{um}^2\,x'\alpha. \qquad \text{... (25)}$$

The matrix slides back over the fibres and the fibres extend. Where the displacement of the fibres and matrix differ, work is done against the frictional force τ times the difference in displacement. This work is given by

$$u_s = E_f E_m V_m/6\tau\, \varepsilon_{um}^3\, \alpha\, r\, (1 + \alpha). \qquad \ldots (26)$$

The matrix looses elastic strain energy because over a distance $2x'$ since the strain in it is reduced. This work is equal to

$$u_m = E_f E_m V_m/3\tau\, \varepsilon_{um}^3\, \alpha\, r. \qquad \ldots (27)$$

There is an increase in the elastic strain energy of the fibres. This energy is equal to

$$u_f = E_f E_m V_m/2\tau\, \varepsilon_{um}^3\, \alpha\, r\, (1 + \alpha/3). \qquad \ldots (28)$$

This work done by the applied stress δW above can be rewritten in terms of the quantities α and r by substituting for x' in equation (25) from (21). We have

$$\delta W = E_f E_m V_m/2\tau\, \varepsilon_{um}^3\, \alpha\, r\, (1 + \alpha). \qquad \ldots (29)$$

In order to form a crack at a breaking strain ε_{um} we must then have

$$2\gamma_m V_m \leq E_c E_f\, \varepsilon_{um}^3\, \alpha^2\, r/6\tau\,. \qquad \ldots (30)$$

It is clear that if $\gamma_m V_m$ is independent of fibre size, then if the radius of the fibres is decreased with other factors held constant, we enter a regime of fibre size in which cracking cannot occur at the normal strain of the matrix: the composite strain will have to be increased to a value ε_{cc} given by

$$\varepsilon_{cc} = \left\{ 12\tau\, \gamma_m\, E_f V_f^2/E_c E_m^2\, r\, V_m \right\}^{\frac{1}{3}}. \qquad \ldots (31)$$

In Fig. 6 the normal breaking strain of the matrix is 0.02%. One sees from the figure that the breaking strain has been considerably increased and no cracks appear prior to a strain of 0.09%. The evidence for this is given by the acoustic emission counts. Taking $\gamma_m = 10^5$ ergs/cm^2, $\alpha = 0.82$, $V_f = 0.084$, $E_f = 210 \times 10^{10}$ dyne/cm^2, we obtain a calculated first cracking strain of 0.9% in very good

agreement with experiment.

It is clear from the physics of the argument just given that in a composite of small V_f in a matrix with $\varepsilon_{ym} > \varepsilon_{ult\ f}$ that a brittle fibre should be extended to larger strains before failure if an equation analogous to (31) is used. This case is also investigated by Aveston et al (1971) and it is concluded that such will only be observed with small fibres.

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$$u_s = E_f E_m V_m / 6\tau \, \varepsilon_{um}^3 \, \alpha \, r \, (1 + \alpha). \qquad \ldots (26)$$

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In order to form a crack at a breaking strain ε_{um} we must then have

$$2\gamma_m V_m \leq E_c E_f \, \varepsilon_{um}^3 \, \alpha^2 \, r / 6\tau \, . \qquad \ldots (30)$$

It is clear that if $\gamma_m V_m$ is independent of fibre size, then if the radius of the fibres is decreased with other factors held constant, we enter a regime of fibre size in which cracking cannot occur at the normal strain of the matrix: the composite strain will have to be increased to a value ε_{cc} given by

$$\varepsilon_{cc} = \left\{ 12\tau \, \gamma_m \, E_f V_f^2 / E_c E_m^2 \, r \, V_m \right\}^{\frac{1}{3}} . \qquad \ldots (31)$$

In Fig. 6 the normal breaking strain of the matrix is 0.02%. One sees from the figure that the breaking strain has been considerably increased and no cracks appear prior to a strain of 0.09%. The evidence for this is given by the acoustic emission counts. Taking $\gamma_m = 10^5$ ergs/cm^2, $\alpha = 0.82$, $V_f = 0.084$, $E_f = 210 \times 10^{10}$ dyne/cm^2, we obtain a calculated first cracking strain of 0.9% in very good

agreement with experiment.

It is clear from the physics of the argument just given that in a composite of small V_f in a matrix with $\varepsilon_{ym} > \varepsilon_{ult\ f}$ that a brittle fibre should be extended to larger strains before failure if an equation analogous to (31) is used. This case is also investigated by Aveston et al (1971) and it is concluded that such will only be observed with small fibres.

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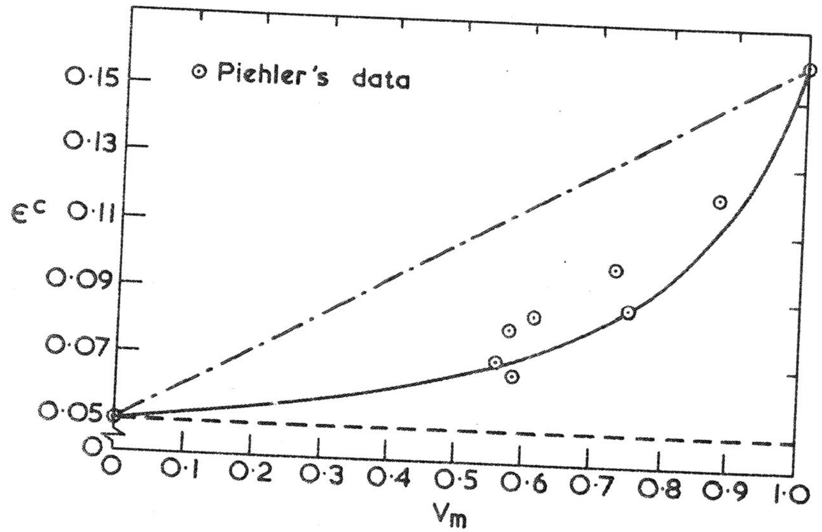

Fig. 1 Prediction of ultimate tensile strain according to Germong and Thompson's formulation of the individual stress-strain curves

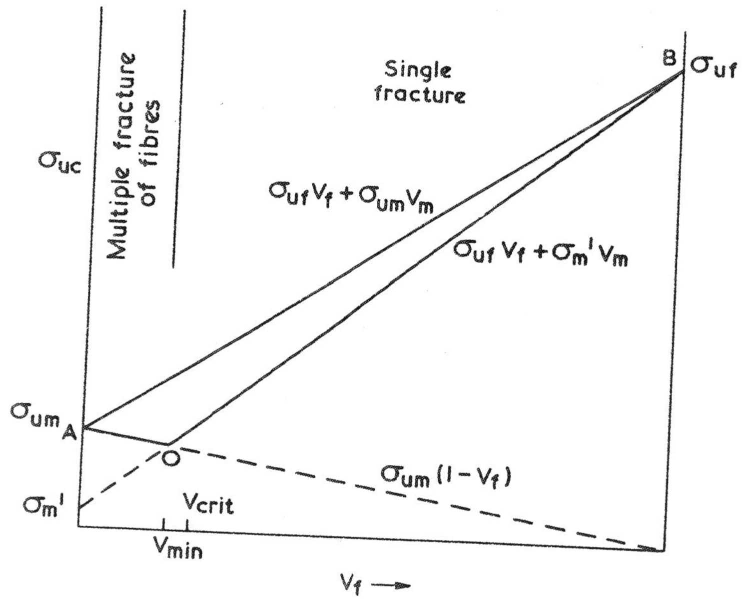

Fig. 2 Variation of tensile strength with volume fraction for a composite with ductile matrix

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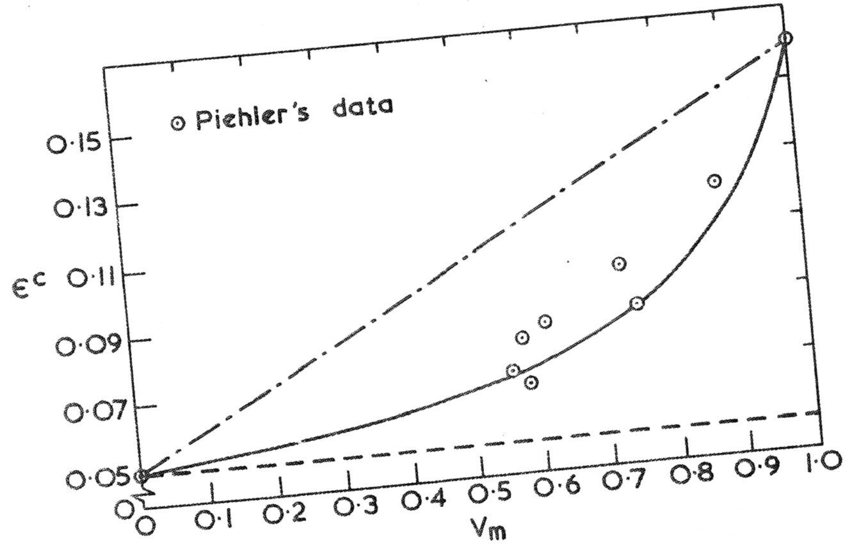

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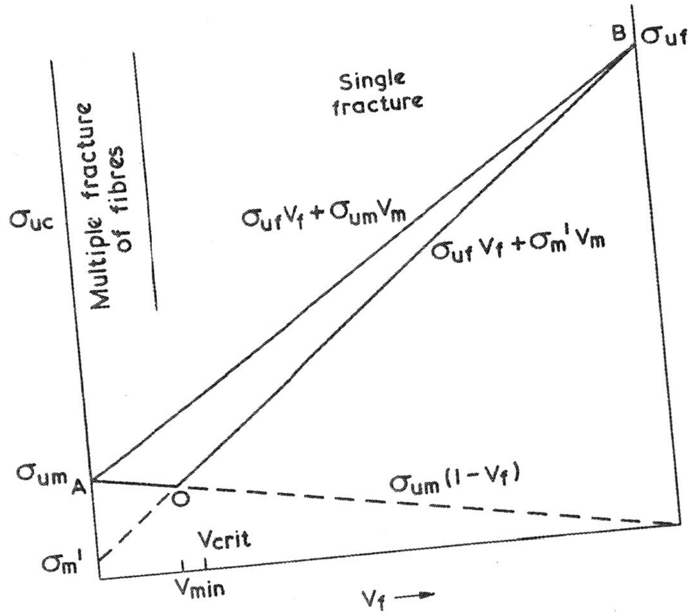

Fig. 2 Variation of tensile strength with volume fraction for a composite with ductile matrix

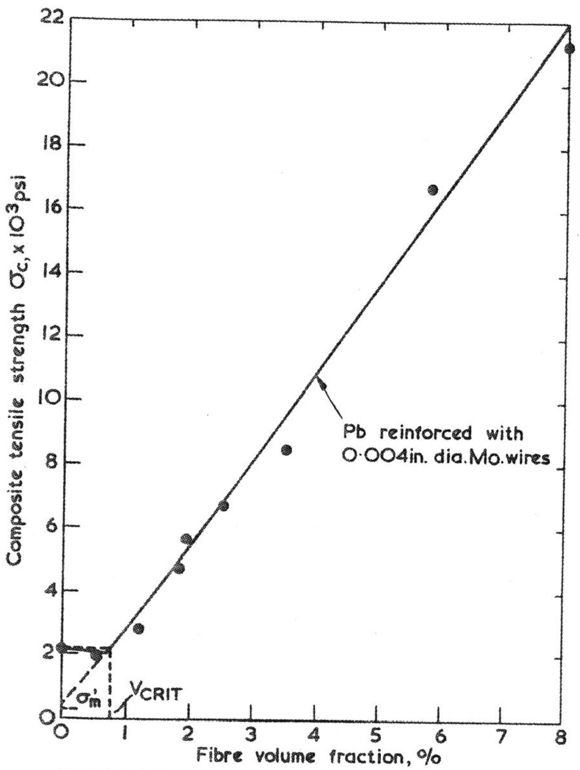

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Fig. 3

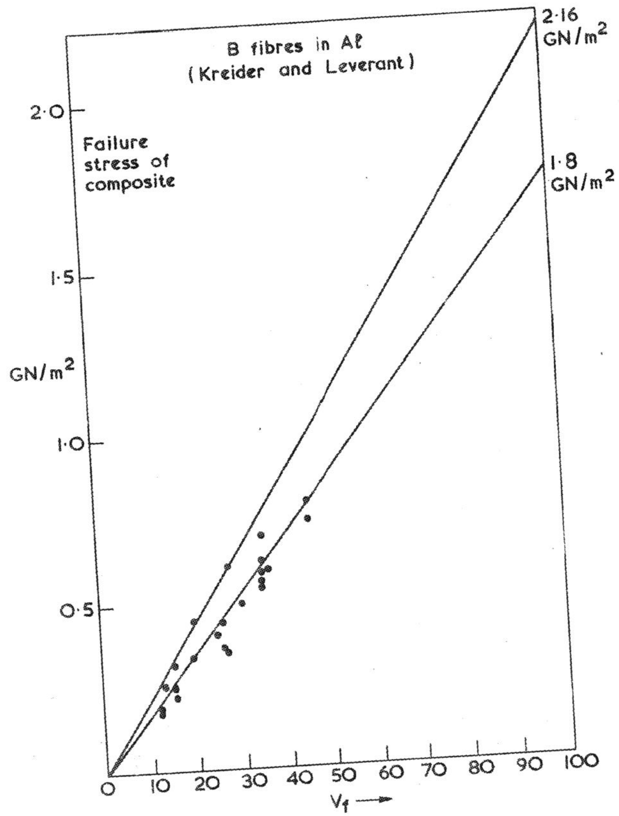

Fig. 4

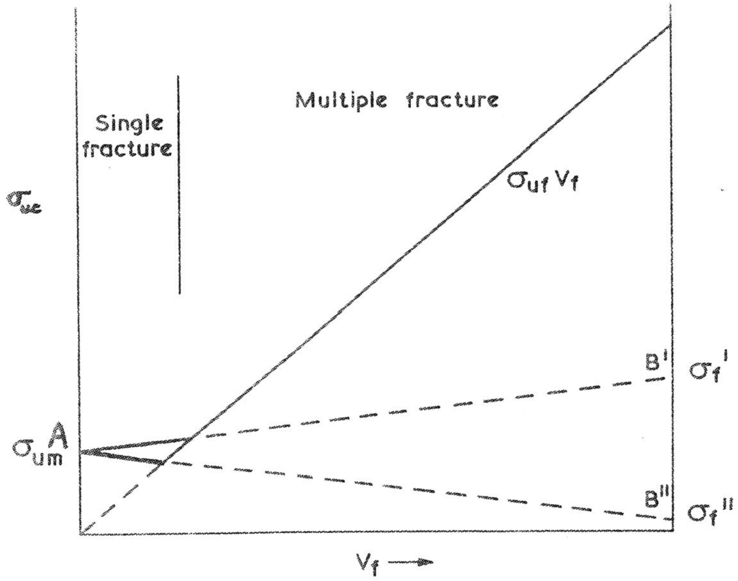

Fig. 5 Variation of tensile strength with volume fraction for a composite with a brittle matrix

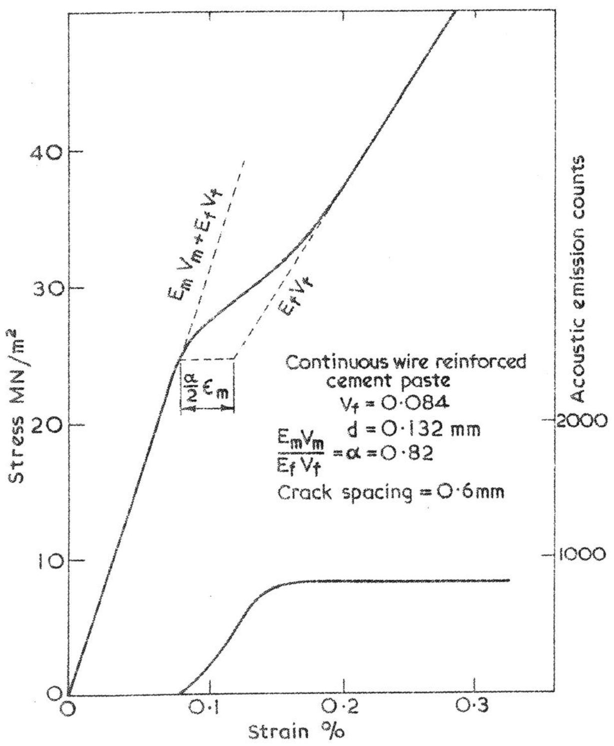

Fig. 6