Fracture in Rocks and Engineering Implications

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INTRODUCTION

Understanding of fracture phenomena is of central importance in the study of the mechanical behaviour of rock and rock masses. Large scale fractures control tectonic movements and earthquakes, fracture systems pervade most near-surface rock masses, and microscopic examination of rock specimens frequently reveals additional, small-scale fractures that control the mechanical properties of the rock. Development of fractures in intact rock is a requirement in such engineering activities as crushing of rock to release mineral particles, rock drilling and blasting, improvement of production from depleted oil reservoirs, and more recently, extraction of geothermal energy from hot rocks in the earths crust.

In general, the obscure history of a rock and the complexity of its structure are such that precise formulation of the processes of fracture is not possible. Much practical value can be gained, however, from the application of the principles of fracture mechanics to the study of rock fracture and the engineering problems associated with it.

FAILURE MECHANISMS IN ROCK

Disintegration of material is usually considered to occur in two basic ways; by shear, or by cleavage. Analysis of shear failure

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is generally based on the theory of dislocations (TAYLOR, (1934), POLANYI, (1934), OROWAN, (1934)) whereas studies of cleavage failure and crack growth have their origin in the energy balance method introduced by GRIFFITH, (1921).

Although dislocation movement, twinning, and gliding of crystals does occur at high confining pressures and temperatures (e.g. RALEIGH, 1965), in most engineering situations rock is encountered, at relatively low temperatures and stress states such that it fails in a 'brittle' manner, i.e. predominantly by crack growth and crack coalescence. Analysis of rock failure has therefore drawn heavily on GRIFFITH'S (1921) theory, modified to account for the particular conditions that pertain to rock.

ROCK AS A GRIFFITH MATERIAL, IN TENSION

Griffith essentially pointed out that two conditions were necessary and sufficient for brittle fracture to occur:

1. A 'stress' condition – the interatomic cohesive force (per unit area) must be exceeded. [Since he assumed the material to contain 'flaws' or 'micro-cracks', this 'stress' need only be reached at the crack tips and not across an entire cross-section.]
2. An energy condition. The energy necessary to form two distinct surfaces through the solid must be supplied. This energy will become avaiable if, by crack extension, the potential energy of the system [i.e. the loaded plate with crack] decreases.

Idealizing the flaws as sharp elliptical cracks Griffith demonstrated that the change (ΔP) in potential energy of the system due

to introduction of a crack length 2c, in an infinite, isotropic, elastic plate of unit thickness was: -

$$\Delta P = 4\gamma c - \frac{\pi c^2 \sigma^2}{E} \qquad \ldots (1)$$

where γ is the specific surface energy of the plate, E is the modulus of elasticity of the plate material, and σ is the average stress applied normal to the major axis of the crack, in the plane of the plate.

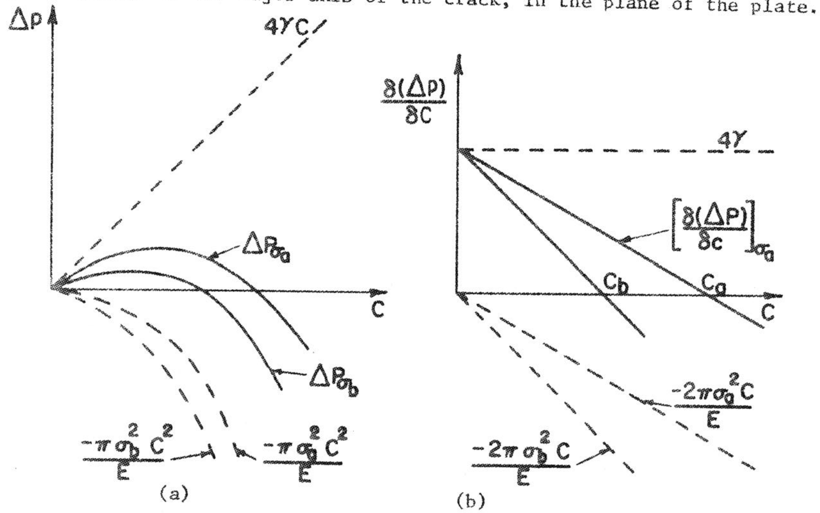


Figure 1. Graphical Representation of Griffith Criterion of Fracture.

The graphical representation of Equation (1) is shown in Figure 1(a) for two levels of applied tension (σ_a, σ_b). Figure 1(b) shows the derivative form of Equation (1), viz.

$$\frac{\partial(\Delta P)}{\partial c} = 4\gamma - \frac{2\pi\sigma^2 c}{E} \qquad \ldots (2)$$

for the two levels of applies stress. The latter [Figure 1(b)] is a more convenient form for some parts of the following discussion.

At an applied stress of σ_a, cracks greater in length than $2c_a$ would extned and, similarly, at stress of σ_b cracks greater than $2c_b$ would extend. The critical stress for a given crack length is found by setting $\frac{\partial(\Delta P)}{\partial c} = 0$ in Equation (2), from which we obtain

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi c}} \qquad \ldots (3)$$

In Griffith's model of elliptical cracks, the theoretical interatomic cohesive strength of the material (σ_m) reached at the crack tips is given by:

$$\sigma_m = 2\sigma\sqrt{\frac{c}{\rho}} \qquad \ldots (4)$$

where ρ is the (unknown) radius of curvature at the crack tip.

OROWAN (1945) and IRWIN (1948) observed that the energy absorbed in crack growth was much higher than the specific surface energy (γ), and included a significant amount of plastic work in the vicinity of the crack tip. This requires that the term 'γ' be taken to include all energy dissipated in extending the crack by a unit of area. γ is then more appropriately defined as the "Work of Fracture". Subsequent work by BARENBLATT (1959-62) and DUGALE (1960) using idealized models for the plastic yield criterion at the crack tip essentially confirms the OROWAN-IRWIN modification of the Griffith criterion.

In application to rocks, which are generally heterogeneous assemblages of crystals and discontinuities, it is likely that the required high stress concentration will result from features other than the hypothetical elliptical cracks, such as, for example, triple

points (i.e. junction of three grains). It may be assumed that the stress condition can always be satisfied so that analysis of rock fracture need only consider the energy criterion.

Stress-Strain Representation of Griffith Criterion

BERRY (1960) has derived the stress-strain locus (Figure 2a) for the Griffith criterion.

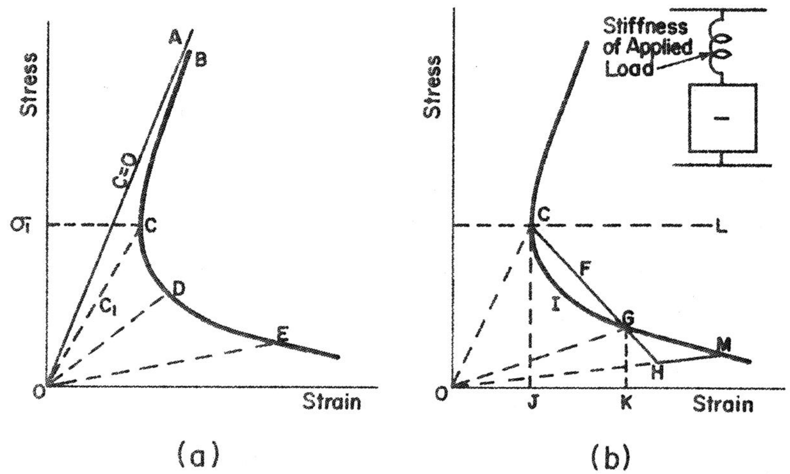


Figure 2/(a). Stress-strain Locus for Griffith Criterion, (b) Influence of Load Stiffness on Fracture Propagation.

The locus, curve BCDE is obtained by noting that introduction of a crack, length 2c, into the plate, (large) area A (unit thickness), and uncracked modulus E [represented by slope OA], increases strain energy by the amount $\frac{\pi c^2\sigma^2}{E}$ over the original $\frac{A\sigma^2}{E}$. Defining an average modulus E' for the cracked plate, such that $\frac{\sigma^2}{2E}[A+2\pi c^2] = \frac{\sigma^2}{2E'}$, we have

$$E' = \frac{E}{1 + \frac{2\pi c^2}{A}} \qquad \ldots (5)$$

For the linearly elastic plate, $\sigma = E'\varepsilon.$, so, from Equations (3) and (5), we obtain:

$$\varepsilon = \frac{\sigma}{E} + \frac{8E\gamma^2}{A\pi\sigma^3} \quad . \; . \; . \; (6)$$

as the stress-strain focus of the Griffith criterion for variable crack length.

The curve BCD is asymptotic to the uncracked line OA [actually BCD may be defined to intersect OA at the cohesive strength value σ_m, as given by Equation 4].

Thus, if a plate containing a crack length 2c is loaded to the stress level σ_1 the crack will begin to extend. If the applied load is reduced with crack extension so as to follow the locus CDE, then the crack would be slowly extended to traverse the plate i.e. the Griffith criterion $\frac{\partial \Delta P}{\partial c} = 0$ is continuously satisfied, with no excess energy to allow onset of the instability condition $\frac{\partial \Delta P}{\partial c} < 0$.

<u>Influence of Stiffness of Applied Load on Crack Extension</u>

Although Griffith derived his criterion of fracture <u>initiation</u> (Equation 3) for the case where the applied load (σ) was constant, independent of boundary displacement during crack extension, it is readily shown (OROWAN 1955) that the criterion holds independently of the stiffness (i.e. load-deformation relationship) of the applied load. However, the stiffness of the applied load does have a marked effect on fracture <u>propagation</u>.

Referring to Figure 2b, consider the situation where the plate has been loaded along OC to the point of crack extension (C). As the

crack and the plate boundaries begin to extend the applied load will tend to drop, according to its stiffness, along a characteristic such as CFGH. As the load drops to G, energy proportional to the area CFGKH is released into the plate. Slow crack extension from C to G (i.e. along the Griffith locus) requires energy proportional to the area CIGKJ [(i.e. equal to the increase in surface energy (area OCIG) plus the change in strain energy (area OGK-area OCJ)].

The energy released exceeds that required for slow extension and the crack consequently accelerates, reaching a maximum velocity at G, i.e. at a crack length corresponding to slope OG. Beyond G, the force exerted (and energy released) by the applied load is less than that required for slow crack growth. The crack decelerates, eventually stopping at H, at a length corresponding to slope OM. (Area GFGI equals area GHM.) Further crack extension will not occur until the applied load is increased to a value corresponding to M, the plate and crack stretching elastically along OM, from H to M.

Under a constant applied load, as for example, CL, crack extension is a markedly unstable process, with appreciable release of excess energy leading to rapid fracture. With testing systems of high rigidity, such that the load falls more rapidly than the Griffith locus, fracture extension is a stable process; additional energy must be supplied to the plate for continued crack extension.

<u>Velocity of Griffith Cracks in Tension.</u>

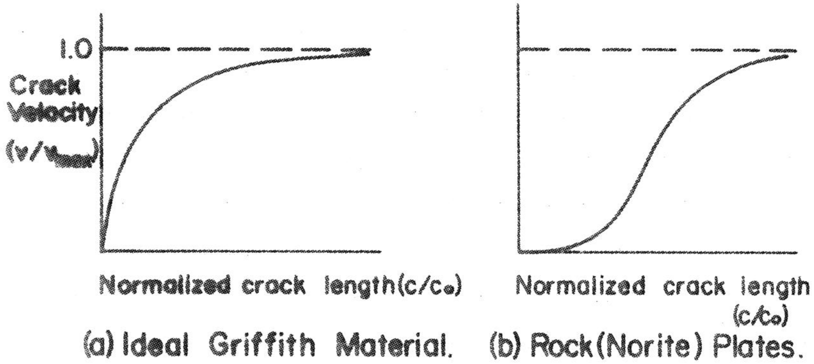
Inclusion of the kinetic energy of a crack moving with velocity V in a medium of density P in the expression for the change in potential energy (ΔP) of the system due to introduction of the crack, results in the expression, after MOTT (1948),

$$\Delta P = 4c\gamma - \frac{\pi c^2 \sigma^2}{E} + \frac{k\rho c^2 v^2 \sigma 2}{2E^2} = \text{constant} \quad . . . (7)$$

The constant k is an (unspecified) numerical factor. It has been shown BERRY (1960), DULANEY and BRACE (1960) that, for a material with Poission's ratio = 0.25, the velocity increase according to the relationship

$$V = 0.38 \frac{E}{\rho} [1 - \frac{Co}{C}] \quad . . . (8)$$

The crack length-velocity relationship is shown diagrammatically in Figure 3(a).



(a) Ideal Griffith Material — (b) Rock (Norite) Plates — Crack length (c/c_o)

Figure 3. Crack Velocity as a Function of Crack Half-Length

Thus the Griffith crack accelerates rapidly to the peak velocity which, for a material (such as granite) with a sonic velocity ($\frac{E}{\rho}$) around 4000 meters per second, will be approximately 1,600 meters per

second. At such velocities the time interval between crack initiation and complete rupture of a tensile test specimen of the order of 10 cm diameter would be less than 100 microseconds.

Velocity of Tensile Rupture in Rock

BIENIAWSKI (1968) has observed the variation in velocity of crack extension in plates of norite loaded in uniaxial tension, using a low stiffness loading system and initiating cracks from a notch in the edge of the plate. His results are shown graphically in Figure 3(b). Although data on the velocity of crack propagation in rock is sparse, the slow crack growth regime preceding rapid acceleration is consistent with other phenomena observed in rock fracture. Thus, the demonstrated ability to control rock deformation in tension [KRECH, 1972] using servo-control systems with a response (to deformation) time of the order of 5 milliseconds suggests that crack propagation occurs much less rapidly than predicted for the ideal Griffith material.

The mechanics of slow crack growth have not been investigated, but it would appear that the phenomenon results in part from the generally hetrogeneous nature of rock microstructure. WAWERSIK (1968) has loaded specimens of limestone and granite in uniaxial tension and demonstrated that substantial inelastic deformations can occur prior to rupture. Typical results are shown in Figure 4.

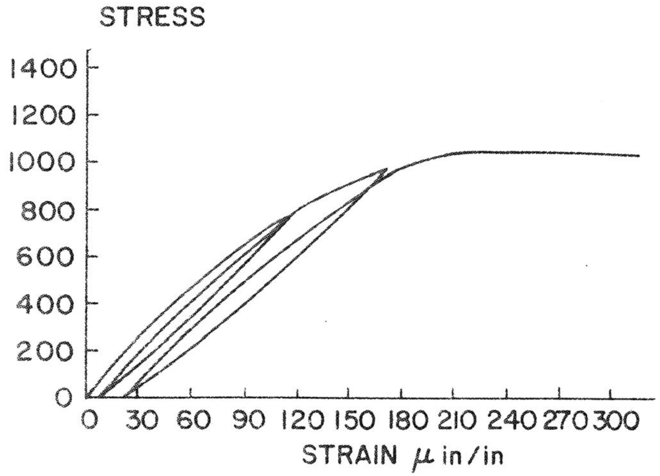


Figure 4. Experimental Load-Deformation Curves of Rock in Direct Tension.

Some specimens were loaded to 'yield', then unloaded, removed from test, and sectioned. Microscopic investigation revealed extensive grain rotation, and loosening around grain boundaries over a significant part of the specimen. Cracks were developed but had not propagated across the entire specimen.

Tensile Fracture Propagation in Rock

Most rocks consist of more than one mineral constituent, with grains of variable orientations, and pores between grains. Cracks may be present within grains or along grain boundaries, and residual (tensile and compressive) stresses may exist in the individual grains [HANDIN et. al. 1972]. Tensile rupture by crack propagation in rocks is consequently very complex. A crack may start at A, in Fig. 5., for example, and traverse the specimen along a very irregular path.

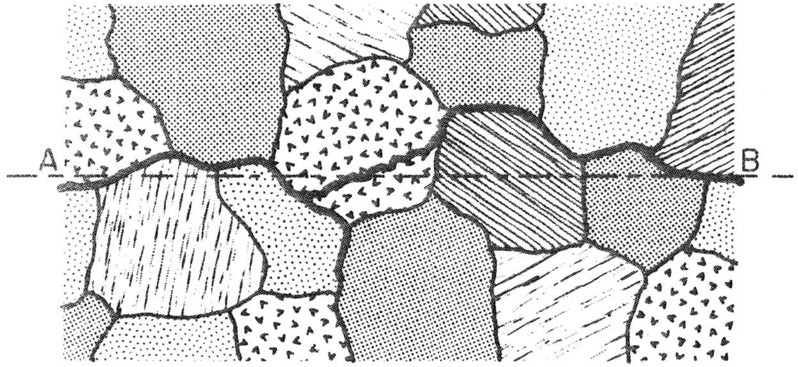


Figure 5. Irregular Crack Paths in Rock Loaded in Tensions

Propagation along the direct path AB would involve cleavage through grains of various orientations involving a greater energy dissipation than necessary to propagate the same projected length along a more tortuous path. Some cracks may cease to propagate e.g., at point D, others may be initiated ahead of and above or below the orginal crack, e.g., at point G, propagating simultaneously forward (towards H) and backwards (towards C). Grain rotations, multiple fractures, and some plastic deformation of crystals may occur [although, as mentioned, this has rarely been observed in unconfined tension tests]. SANTHANEN and GUPTA (1966), HOAGLAND et.al. (1972). With such a complicated fracture process the term 'Work of Fracture' should be used in preference to 'surface energy'. Experimental observation of the work of fracture in the tensile rupture of rock reveals that it is generally several orders of magnitude higher than that observed in pure crystals of the minerals in the rock. [See for example, PERKINS and KRECH, 1966.] Careful measurement of the crack surface area generated has indicated that this may be a factor of ten greater than the 'single-plane' length. It appears that release of residual stresses (HANDIN et. al. 1972), and slippage along favourably inclined cracks ahead of the crack tip may account for a significant portion of the remaining discrepancy. However, these dissipation mechanisms are all localized to the area of the main crack tip so, depending on the scale of the observed main crack, linear elastic analysis can be used to predict the overall response.

The rate of variation of the Work of Fracture (W_f) per unit advance in the direction AB may appear as shown by the solid line "d-m" in Figure 6. This replaces the constant, 4γ, line of Figure 1(b).

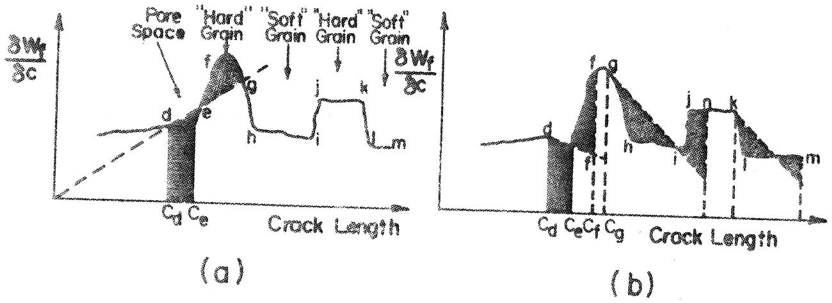


Figure 6. Energy release - Work of Fracture Interaction; (a) Constant Applied Load System, (b) Finite Stiffness Loading System

The stable - unstable nature of fracture propagation in rock is illustrated by considering the nett energy balance at various stages of propagation. Thus, for example, if the applied load in a constant

stress (i.e., dead-weight) system has been increased just to the point of fracture extension across a pore space, the rate of energy supply (indicated by the 'constant load characteristic' in Figure 6) will exceed the (zero) energy 'demand' to traverse the pore space. The excess energy will result in crack acceleration, allowing fracture to propagate across a 'hard' [i.e., high work of fracture] grain even though statically the applied stress would be insufficient to fracture the same grain. In some cases the running fracture may be arrested within the grain and the applied stress may have to be further increased to continue propagation. In most loading systems, as shown in Figure 7, the applied load is likely to drop as rapid fracture ensues due to the finite stiffness of the applied load, and excess energy will be somewhat less.

The transient instability as fractures accelerate and decelerate during passage through the rock is indicated by the phenomenon of micro-sesmic (or acoustical) energy emission that is commonly associated with rock fracture. [e.g., HARDY et.al. 1970].

Tensile Testing of Rocks

The difficulty of preparing rock specimens for 'direct-pull' tensile testing has led to the adoption of various indirect methods of determining 'the tensile strenght' of rocks. These include, for example, the beam test, diametral compression of discs (the 'Brazilian' test) and ring test, in addition to the direct-pull. The stress gradient and volume of rock subjected to tension are different for each test, so that it is perhaps not surprising that the indicated 'tensile strengths' vary considerably depending on the method used.

HUDSON (1971) has attempted to explain the variations in terms of Weibull's statistical theory of failure. As is frequently the case in applying statistical theories to failure (FREUDENTHAL, 1951) the problem of determining material properties was found to degenerate essentially into a "curve-fitting" exercise with little physical insight into the mechanisms involved. The material 'constants' obtained varied with test geometry and size. Thus, while specimen to specimen variations of strength are no doubt partially attributable to statistical effects, it is apparent that other factors may also be involved. Analysis of the energy changes associated with crack propagation for each type of test has been made by HARDY (1971) using the Finite Element Method. The results are described below.

Analysis of Crack Growth in Tension by the Finite Element Method

The Finite Element Method of analysis is now widely used in most fields of engineering. The method allows the stress and displacement distributions to be determined for any shape body under the action of internal surface tractions or internal body forces. The body is considered as a series of connected elements. The stresses are determined within each element and the displacements are prescribed at the nodal points of the elements. In determining the solution, a computer program develops the stiffness matrix [K] of the structure. The strain energy of the body U can be found directly from the displacement distribution and the stiffness matrix. In matrix notation this can be represented as

$$U = \delta^T [K] \delta \qquad \ldots (9)$$

where δ is the displacement vector for the nodes in the body.

It can be shown that the strain energy calculated in this manner

is a lower bound solution, the accuracy of which can be improved by reducing the element size in regions of high stress gradient; In fracture mechanics problems this implies small elements at the crack tip.

In studies of crack growth using the finite element method by ANDERSON et. al. (1971) and HARDY (1971), U is calculated for two bodies, one with crack length l, the other with crack length l + dl, where 'dl' represents a small change in the crack length resulting in an increase of crack area dA. Following IRWIN's (1957) terminology, the strain energy release rate G is then

$$G = \frac{dU}{dA} = \frac{U-U'}{dA} \qquad \ldots (10)$$

U strain energy of body with crack length l

U' strain energy of body with crack length (l + dl)

If F represents the external forces on the body, then the crack will extend from l to l + dl when the strain enery rate equals or exceeds the critical strain energy release rate G_c.

now $$U = \frac{F^2}{2M} \qquad \ldots (11)$$

where M is the constant of proportionately, the modulus for the particular body being studied. Therefore, substituting into Equation (10) and replacing G by G_c we find the force (F) required to cause crack extension

$$F_c = \sqrt{2\,G_c\,\frac{dA(M-M^1)}{}} \qquad \ldots (12)$$

This method is similar to that suggested by IRWIN and KIES (1954) as the "compliance" method. In previous studies, the compliance $(\frac{1}{M})$ was determined experimentally for bodies of different crack length, for substitution into the equation

$$G_c = \frac{1}{2}F^2 \frac{d}{dA}\left(\frac{1}{M}\right) \qquad \ldots (13)$$

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In studies of crack growth using the finite element method by ANDERSON et. al. (1971) and HARDY (1971), U is calculated for two bodies, one with crack length 1, the other with crack length 1 + dl, where 'dl' represents a small change in the crack length resulting in an increase of crack area dA. Following IRWIN's (1957) terminology, the strain energy release rate G is then

$$G = \frac{dU}{dA} = \frac{U'-U}{dA} \qquad \ldots (10)$$

U Strain energy of body with crack length 1

U' strain energy of body with crack length (1 + dl)

If F represents the external forces on the body, then the crack will extend from 1 to 1 + dl when the strain energy rate equals or exceeds the critical strain energy release rate G_c.

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where M is the modulus for the particular body being studied. Therefore, substituting into Equation (10) and replacing G by G_c we find the force (F) required to cause crack extension

$$F_c = \sqrt{2\ G_c\ dA/(\frac{1}{M'}-\frac{1}{M})} \qquad \ldots (12)$$

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$$G_c = \frac{1}{2}F^2 \frac{d}{dA} (\frac{1}{M}) \qquad \ldots (13)$$

Four geometric shapes were analyzed, representing the direct tension test, the three-point bending test, the Brazilian test, and the ring test. The predicted behavior is shown in Figure 7, as axial force-axial displacement curves. The force F is determined for each crack length by application of equation; the displacement is then computed from the expression

$$\delta = \frac{F}{M} \qquad \ldots (15)$$

Notice that, even in an infinitely stiff loading frame, violent collapse of the testing specimen would result for all cases after the load reached the maximum load carrying capacity of the specimen. However, for the beam test, if the initial crack were larger than the selected 0.1 inch, it is possible that a controlled failure of the beam would be observed in a stiff machine. A more detailed description of the derivation of these curves for the beam test is given in HARDY, HUDSON and FAIRHURST (1972).

The curve for the direct tension test is very similar to that developed by BERRY (1960) Figure 2(a).

The work of fracture used in this analysis [0.114 in/lbs/in^2] was determined from comparison with experimental beam tests reported by HARDY et. al. (1972). The variation in the apparent tensile stress at failure for this constant work of fracture can be seen. The stress at failure is calculated from the usual formula based on linear elastic theory for the specimen assuming no crack. These values vary from 4,300 for the ring test to 2,280 lbs. for the beam test. This type of variation in apparent tensile strength of rock is not uncommon.

Figure (8) shows the force versus crack length for each of these tests, the direct tension test and the beam test on one diagram, the Brazilian and Ring test on the other. Some very interesting conclusions

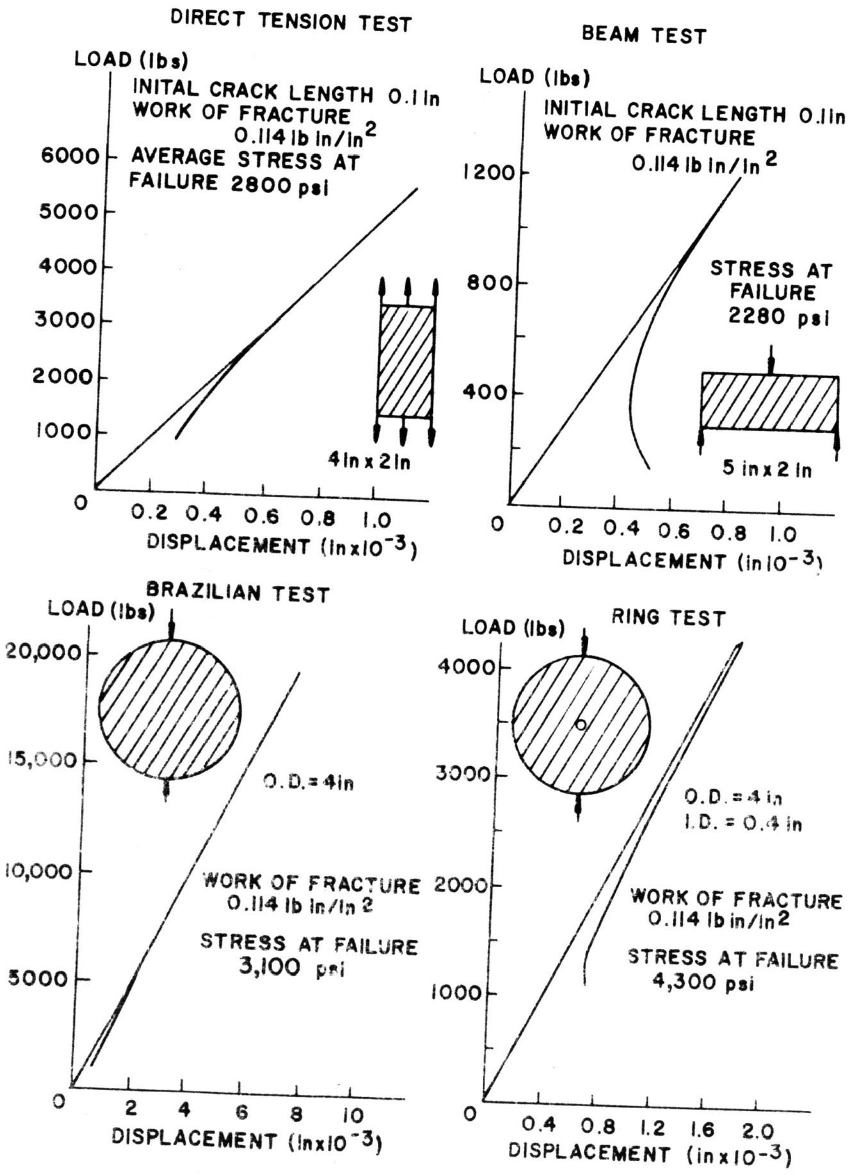

Figure 7. Predicted Load-Deformation Relationships for Various Tensile Tests

can be drawn from these figures. If the failure of the rock beams were controlled by propagation of the largest crack in the tensile stressed area, direct application of Griffith's theory would predict the strength to be inversely proportional to the square root of the crack length. Using the value of load at a crack length of 0.1 inch to calculate the constant of proportionality in the relationship,

$$F = \frac{K}{\sqrt{a}}$$

The expected value of force at a crack length of a=0.2 inches was calculated. These are shown in Table I. Notice that for the direct tensile test the agreement is excellent as would be expected because this test is closest to the original Griffith analysis. The agreement is good for the beam test and acceptable for the Brazilian test, but for the ring test the agreement is very poor. This would suggest that a Weibull-type analysis of this test would produce erroneous conclusions, as the load is relatively independent of crack length until the crack length exceeds the initial hole diameter.

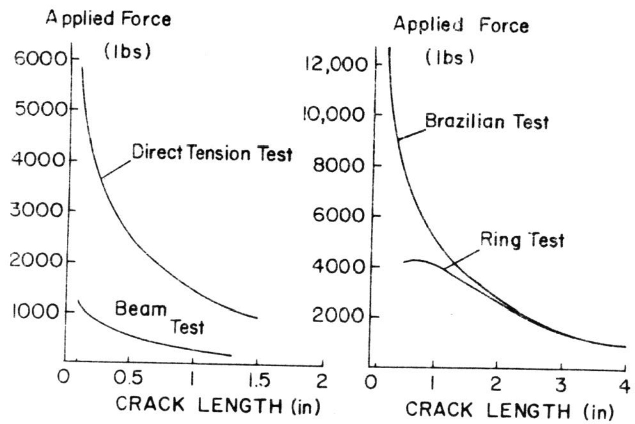

Figure 8. Predicted Force Versus Crack Length for Various Tensile Tests.

	Direct Tension	Beam Test	Brazilian Test	Ring Test
Load at crack length = .1"	2810	1212	19,497	4,200
Load at crack length = .2"	1970	854	12,772	4,220
Load at crack length = 0.2" if $F = k\sqrt{a}$	1980	857	13,750	592

Table I. Predicted Peak Loads in Various Tests on Rock for an Assumed Work of Fracture = 0.114 in. lbs/in.^{2}

Experimental Results

Experiments have been performed on the beam test and the Brazilian test. Complete force-displacement records of these failing structures have been recorded using a high speed servo-controlled testing system. (HUDSON et. al. 1972.) The (monotonically increasing) displacement over a selected width normal to the extending crack was programmed to increase monotonically with time. The applied load varied so as to maintain the constant rate of horizontal displacement. Results are shown in Figure 9. It is seen that agreement with theoretical predictions is generally good. A detailed discussion of the experimental procedure and results is presented in HARDY (1973).

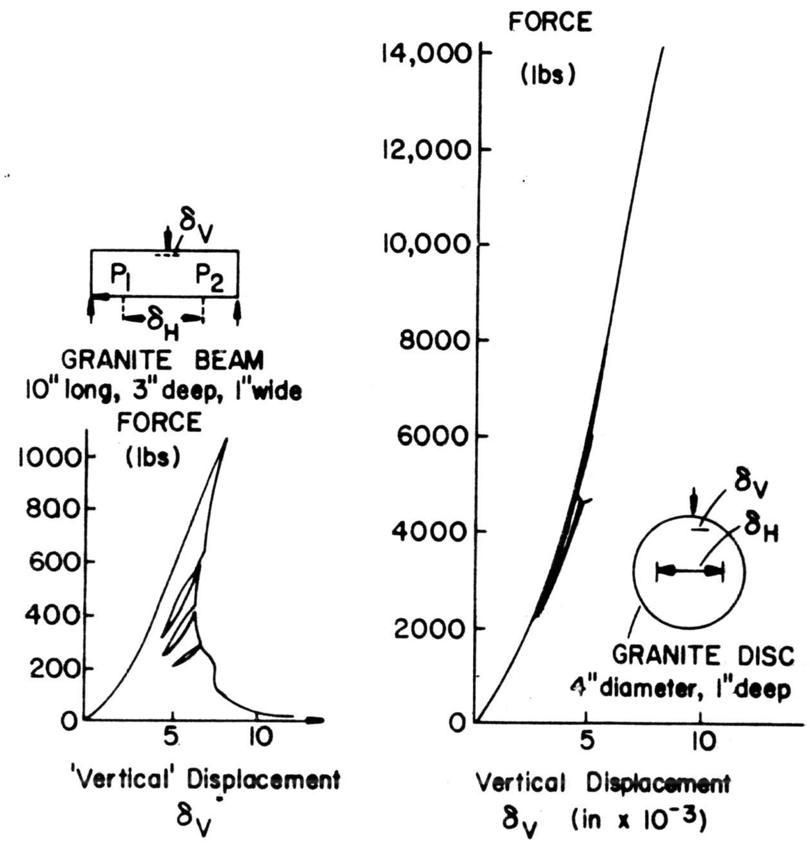

Figure 9. Typical Force Versus Displacement Experimental Results for Beam and Brazilian Test.

In summary, the theoretical method presented in this section appears to be a useful method for determining the load at which failure would occur for a crack trasversing a rock sample. The theoretical predictions suggest that the Brazilian test with a small central hole, or ring test may be a useful method of determining the engineering tensile strength of the rock, as this test appears to be less sensitive to crack dimensions than the other tests studied. This test can also be used to determine the work of fracture, without having to know precisely the initial crack length. [The peak load carrying capacity of the structure is proportional to the square root of the work of fracture from Equation (12)]. The effective crack length for a particular rock type can be uniquely determined by comparison of the experimental output from the Brazilian test, and the ring test. Figure 8 shows that for a small crack length in the ring test the load carrying capacity of the ring is independent of the crack length, but the Brazilian test is very sensitive to the initial effective crack length in the rock. Thus the ration of the two respective loads can be used to determine the crack length.

The above method of analysis of energy changes due to crack growth can be extended to the general problem of determining the path of a cleavage crack in a brittle material, and has several useful engineering applications. For example, chip formation in rock drilling or cutting results from tensile cracking. The interaction of blasts in a mining situation is the result of crack growth due to gas pressure within the blast hole and in the cracks. Energy changes due to cracking along various assumed crack paths can be analysed in this manner to determine, for a given rock type, the most efficient spacing of blast-holes. The optimum spacing will occur when cracks propagate so as to connect with the surface created by the previous explosion, rather than to form independent craters. Anisotropic effects, such as jointing, can be considered. Studies of these problems are in progress at the University of Minnesota.

ROCK FAILURE UNDER COMPRESSION

Rocks at depth are subjected to compression by gravity and tectonic forces. Certain combinations of load and rock properties may result in earthquakes and release of confinement on the rock by underground excavation may produce violent collapse such as the rock bursts reported in deep metalliferous mines (COOK ( ) BLAKE ( )) and coal 'bumps' experienced in some coal mines (Deformation and collapse of rock under pressure is thus of major significance in rock mechanics.

Compressive collapse results from an interaction of the basic modes of failure, cleavage and shear, but in such a complex and heterogeneous manner that prediction of compression strength is essentially empherical. GRIFFITH (1924) presented a simple analysis of fracture under compression loading, based on his earlier (1921) theory of tensile rupture. He again postulated the existence of elliptical flaws in the material and demonstrated that, provided the applied loads are non-hydrostatic, tensions are developed around these flaws. For given applied loads the tension maximized around flaws inclined at a specific orientation on the boundary of the flow and slightly away from the flaw tips. He then suggested that fracture would occur by extension of the original flaws when the maximum tension reached the same value as that required at the (same) crack tips during tensile rupture. In this way, Griffith predicted that the uniaxial compressive strength of a brittle material should be 8 times as large as the uniaxial tensile strength. (i.e., to produce tension around open elliptical cracks, uniaxial compression is 1/8 as effective as uniaxial tension.) Multi-axial compression, by reducing the maximum tension around the crack, increased the compressive "strength" of the rock.

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Rocks at depth are subjected to compression by gravity and tectonic forces. Certain combinations of load and rock properties may result in earthquakes and release of confinement on the rock by underground excavation may produce violent collapse such as the rock bursts reported in deep metalliferous mines (COOK (1965), BLAKE (1972)) and coal 'bumps' experienced in some coal mines (HOLLAND (1958)). Deformation and collapse of rock under pressure is thus of major significance in rock mechanics.

Compressive collapse results from an interaction of the basic modes of failure, cleavage and shear, but in such a complex and heterogeneous manner that prediction of compression strength is essentially empherical. GRIFFITH (1924) presented a simple analysis of fracture under compression loading, based on his earlier (1921) theory of tensile rupture. He again postulated the existence of elliptical flaws in the material and demonstrated that, provided the applied loads are non-hydrostatic, tensions are developed around these flaws. For given applied loads the tension maximized around flaws inclined at a specific orientation on the boundary of the flaw and slightly away from the flaw tips. He then suggested that fracture would occur by extension of the original flaws when the maximum tension reached the same value as that required at the (same) crack tips during tensile rupture. In this way, Griffith predicted that the uniaxial compressive strength of a brittle material should be 8 times as large as the uniaxial tensile strength. (i.e., to produce tension around open elliptical cracks, uniaxial compression is 1/8 as effective as uniaxial tension.) Multi-axial compression, by reducing the maximum tension around the crack, increased the compressive 'strength' of the rock.

McCLINTOCK and WALSH (1962) modified the theory to take account of the fact that the long, narrow, flaws postulated by Griffith would close under compression, so that the resolved shear stresses in the plane of the crack would be reduced at the crack boundaries due to the frictional resistance to sliding of the the two sides. Both the Griffith theory and the McClintock-Walsh modification are deficient in that they are criteria for the initiation of crack extension only. It is apparent that the energy instability condition, of Griffith's theory, is not continuously satisfied beyond the point of initiation of cracking in compression. BRACE and BOMBOLAKIS (1963) and HOEK (1965) have demonstrated that the applied load must be further increased to propagate the crack which changes orientation with extension until it parallels the direction of maximum applied compression at which point it has a very stable orientation and virtually ceases to extend. (Figure 10a.) It is also worth noting that this phenomenon of cleavage cracking towards a very stable orientation is not peculiar to Griffith flaws, but will occur for any inhomogeneity in a brittle material. Figures 10b, c shows examples of crack development around a hole and a high modulus circular (or spherical) inclusion.

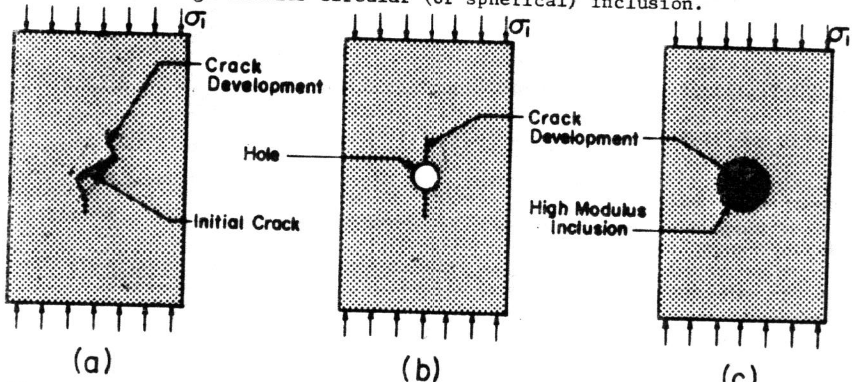

Figure 10. Stable Cleavage Crack Development Under Compression Loading

Thus, in a homogeneous non-hydrostatic compression field, crack cleavage in this way cannot lead to structural collapse. Some other mechanism is necessary. Observation of macroscopic collapse of rock in compression reveals several of the possibilities. Thus, Figure 11(a) shows a section through a cylindrical specimen of limestone compressed to almost complete collapse in a servo-controlled testing system [these systems are discussed in HUDSON, et. al. (1972)]. Sectioning of a series of specimens loaded to different levels enables the sequence of fracturing events to be established. The rock first developed essentially vertical cleavage, (predominantly away from the loaded ends) followed by shearing from the vicinity of the corners. The 'hour-glass' interior of the specimen is progressively crushed in the center, eventually giving rise to a macroscopic shear traversing the specimen. The upper and lower regions, close to the loading platens, remain relatively intact. The symmetrical collapse is somewhat forced to develop in testing systems with high lateral stiffness (NELSON and CORNET (1971)). Where lateral stiffness if low, a single macroscopic shear is more probable, since it's development is not inhibited, as by forces, generated by the platens opposing sliding of the major shear in high lateral stiffness systems.

Figure 11(b) shows fragements resulting from violent compressive collapse of a quartzite specimen. Fractures essentially parallel to the direction of maximum compression traversed the specimen. There appear to be three contributory factors to the development of "Through-going" axial fractures. viz: -

i) the restraint to lateral expansion of the specimen by the larger diameter, steel loading platens produces regions of macroscopic tension in the specimen, acting normal to the applied compression

(ii) the relatively unrestrained central portion of the specimen begins to disintegrate, with an associated lateral dilation in this region, further enhancing tension in the rock above and below

(iii) as the induced vertical cracks lengthen and coalesce they become unstable due to the presence of the macroscopic tensile regin. These cracks accelerate and, aided by local shears to produce lateral sliding and crack opening, etc. penetrate the compression region at the ends, forming the 'slivers' seen in Figure 11(b)

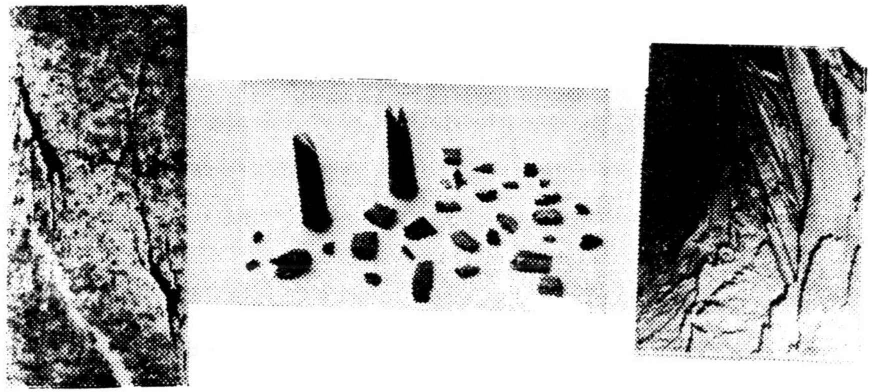

Figure 11. a) Sectional Compressed Cylindrical Limestone b) Fragments From Violent Collapse of Quartzite c) Bucking Failure of Quartzite in Underground Mine

Figure 11(c) shows buckling fractures in quartzite around the walls of a deep underground mine. The buckling occures subsequent to vertical (cleavage) splitting(FAIRHURST and COOK (1966)).

COOK (1965), has analyzed the problem of collapse in compression by unstable shear sliding along inclined cracks, by using STARR's (1928) solution for the strain energy increase in a plate due to the introduction of an inclined crack, length 2c, subjected to a uniform field shear stress (τ). Thus,

$$\Delta Us = \frac{\pi c^2 \tau^2}{E} \qquad \ldots (17)$$

Taking account of crack closure in compression and frictional sliding, COOK, in effect, replaced τ in Equation (17) by (τ=$\mu\sigma_n$), where μ is the coefficient of crack-sliding friction, and σ_n is the normal compression stress across the crack surface. The resulting energy criterion for the onset of crack extension by shear sliding in the plane of the inclined crack reduces to

$$(\tau - \mu\sigma_n) = \sqrt{\frac{2E\gamma s}{\pi c}} \qquad \ldots (18)$$

τ, σ_n may, of course, be expressed in terms of the applied principal compressions. γs is the "Work of Fracture in Shear". Since shear sliding is likely, in unconfined tests, to be preceded by axial cleavage, grain rotation, etc. There is no reason to expect γs (the energy dissipated for unit extension of a major shear crack), to be comparable to γ, the work of fracture in tension.

It is interesting to note that the GRIFFITH compression theory predicts a parabolic Mohr envelope whereas McCLINTOCK and WALSH, and the one by COOK both reduce to the Mohr-Coulomb criterion (i.e., a straight-line Mohr envelope). For most rocks the latter is a reasonable approximation at low to moderate confining pressures, although a curved envelope with slope decreasing with increased confining pressure is usually a better fit to experimental results.

Compression experiments on rocks under confining pressure generally reveal load-deformation curves as shown in Figure (12). Sections through the specimens reveal that axial cleavage fracturing is quickly suppressed as confining pressure is added. Shearing tends to become the

more dominant mode of deformation as confining pressure is raised, although volumetric dilation is not fully suppressed until relatively high (limit of curves shown) confining pressures are reached. It should be noted that, although tests at high confining pressure suggest that the rock deforms 'plastically', (some residual elasticity is observed on unloading) examination shows the rock structure to be irreversibly damaged, almost pulverized in certain rocks.

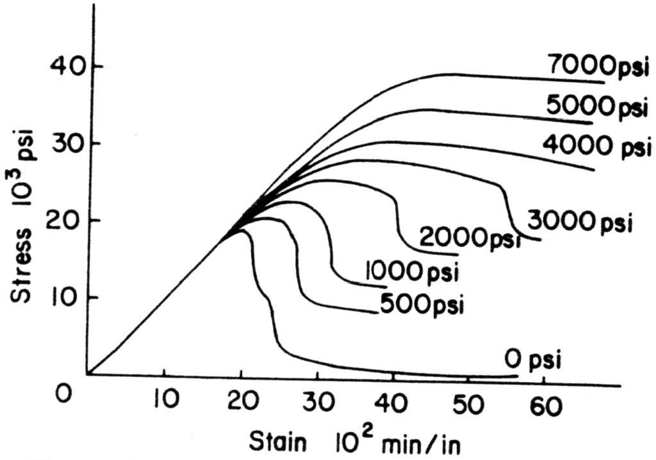

Figure 12. Typical Complete Load-Deformation Curves for Rocks Loaded to Collapse in Compression Confining Pressure is Indicated on the Individual Graphs.

Influence of Internal Pore Pressure on Strength

Rocks in-situ frequently contain pores and fractures filled with fluid under pressure. The study of the effect of fluids on the mechanical behaviour of rock is very important, particularly in view of disastrous occurences such as dam collapses and earthquakes (e.g. LONDE (1972), HEALY et. al. 1970) attributed to the action of pore fluids under pressure.

It is generally observed that the presence of fluid under pressure

(P_f) reduces the 'strength' of the rock. This may partly result from direct chemical changes (e.g. affecting γ), but there is general acceptance that the strength of a porous material depends on the "effective stress" conditions to which the material is subjected.

Simply stated, most experimental data suggest that the strength of porous brittle materials containing fluid under pressure, is a function of the effective stresses. (i.e., total applied stresses minus the applied fluid pressure) rather than of the total applied stresses. (TERZAGHI 1923). Dry tests, (solid) then become a special case $(P_f=0)$ of the general effective stress law. Thus the various criteria of failure may be modified to account for the effect of fluid pressure by replacing the total applied principal (normal) stresses, σ_1, σ_2, σ_3, by their effective stress counterparts $(\sigma_1 - P_f)$,

$$\sigma_1 \rightarrow (\sigma_1 - P_f),\ \sigma_2 \rightarrow (\sigma_2 - P_f),\ \sigma_3 \rightarrow (\sigma_3 - P_f) \qquad \ldots (19)$$

For frictional sliding of faults or joints, the criterion for slip, $|\tau| = C_o + \mu\sigma_n$, where $|\tau|$ is the shear stress along the joint or fault plane, S_o is the shear strength, μ is the coefficient of sliding friction, becomes

$$|\tau| = S_o + \mu[\sigma_n - P_f] \qquad \ldots (20)$$

Similarly, the Griffith criterion for tensile fracture which may be written as $\sigma=K$, where K is the uniaxial (straight-pull) tensile strength, become

$$(\sigma - P_f) = K \qquad \ldots (21)$$

Equations 20, 21 indicate the significance of pore pressure. Thus, increases in pore pressure may markedly reduce resistance to sliding, whereas reductions will also increase resistance. HANDIN and RALEIGH

(1972) have suggested that modification of pore pressure along fault planes may permit controlled release of energy in earthquake-faulting regions. Fluid pressure in pores may result in tensile collapse, particularly in situations of low confinement (i.e. where σ_1 approaches zero). Violent collapse due to high pressure gas in coal is a serious problem in some coal mines; similar problems occur when holes bored in oil and gas bearing rock formations encounter high fluid pressure.

The effect of fluid pressure on the complete load-deformation behaviour in compression [i.e., as shown in Figure 12] is effectively to shift the curves downwards. Thus, if loading conditions with no internal fluid pressure are such as to preduce the uppermost curve in Figure 12, then tests at various levels of fluid pressure will produce behaviour, progressively changing towards that for the unconfined condition. In other words, the results shown in Figure 12 may be generalized by stating the applied load (σ_1) and confining pressure (σ_3) in terms of effective stresses $(\sigma_1 - P_f)$ and $(\sigma_3 - P_f)$ respectively. However, variation in internal voids during disintegration produce [e.g., BRACE and MARTIN (1968), CORNET and FAIRHURST (1972)] effects due to transient changes that are not yet well understood. The subject of fluid pressure effects on rock fracture and disintegration is of great importance in view of the effects of fluids in triggering earthquakes [HEALY et. al. 1970], and the apparent increase of earthquake activity in association with the filling of high dams. The subject is currently under intensive study.

Dilatation During Rock Compression

As already noted, the development of fast response servo-controlled testing systems has facilitated detailed observation of changes associated with (controlled) rock disintegration. Figure 13 indicates schematically typical effects observed during slow strain rate (10^{-6} to 10^{-4} per second) unconfined compression of a cylindrical specimen of rock (e.g. limestone, granite). Figure 13(a) depicts typical load-deformation behaviour to complete collapse of the specimen. Figure 13(b) indicates that the specimen initally contracts in volume, but that the volumetric strain changes with applied load, eventually exhibiting a marked increase or dilation. The dilatancy is associated with the development of internal fractures. The fact that dilatancy develops well before the peak load is reached tends to confirm the conclusion reached theoretically that cleavage crack development in compression is not immediately an unstable process. It is also observed experimentally that the macroscopic shear across the specimen, commonly seen after specimens have collapsed, does not develop until well beyond the peak applied load. The onset of macroscopic shear is usually identified by a sudden steepening of the post-peak curve, such as can be seen in Figure 12 for the curves 0-3000 lb. per. sq. in. confining pressure. The practice of computing friction coefficients and angles from the macroscopic shear plane, using the peak loads, i.e. loads developed before the plane existed, thus seems incorrect.

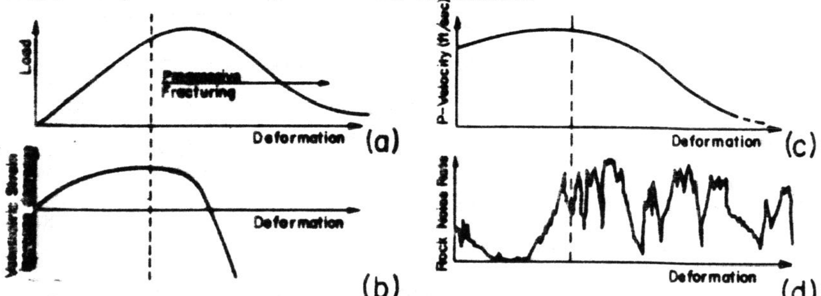

Figure 13. Typical Effects During Slow Compression of Unconfined Rock Cylinder

As noted above, the rock dilation can produce important effects when

the rock contains fluid under pressure.

Figure 13(c) illustrates the significant influence of dilation on sonic velocity (P-wave) through the rock. Again, internal voids result in substantial drop of the velocity (to a very low value as collapse is approached), after an initial increase, probably due to crack closure. The velocity shown is that normal to the direction of loading. Velocity variation parallel to the direction of loading was substantially less affected. This is suggestive of the development and opening of internal cracks that are oriented predominantly parallel to the applied load. The micro-seismic noise rate shown in Figure 13(d) is indicative of the transient instabilities produced by localised cracking.

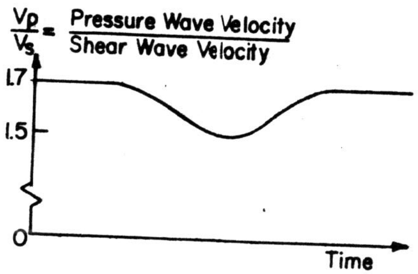

Figure 14. Typical Variation in the Ratio V_p/V_s Prior to an Earthquake

Recent results reported by Soviet scientists, and more recently confirmed by U.S. Scientists to the effect that a significant drop in the ratio V_p/V_s, [where V_p is the velocity of the dilatational or P-wave, V_s is the shear or S-wave velocity] is observed on field seismic data prior to earthquakes in regions prone to such events. The earthquake is observed to occur some time after the original ratio is again reached. The effect is shown schematically in Figure 14. It is not generally possible in field tests to determine V_p and V_s separately. This finding and the associated interest in finding means of predicting earthquakes has stimulated considerable interest in the variation of P and S waves through regions of disintegrating rock.

PRACTICAL PROBLEMS OF ROCK MECHANICS INVOLVING ENERGY INSTABILITY

Rock Bursts and coal bumps

A rock 'burst' is the term applied to the violent disintegration of rock in underground mine working, resulting from excessive gravitational and tectonic loads. Severe rock-bursts occur in deep mines, such as the gold mines of the Witwatersrand of South Africa, and the Kolar Gold Field of India. In both regions the mines are around 12,000 ft. maximum depth. Bursts do occur in shallow mines but are generally less frequent. Coal 'bumps' are essentially similar to rock bursts, but occur at shallower depths (coal is generally weaker than most rocks in metal mines), and may be associated with gas outbursts. [Note that increase in fluid (i.e. gas) pressure within coals can increase the brittleness and reduce the strength, as discussed eariiier.]

It is now generally accepted that the bursts (or bumps) result when the energy released by extension of the mined-out area exceeds the energy that can be absorbed by slow compression, to collapse, of the rock in the mineral seam or vein, i.e., the roof-floor stiffness exceeds that of the disintegrating rock between the roof and floor. The energy absorption characteristics of veins are not well known. Full scale in-situ' tests are expensive so that indirect methods such as 'back-calculation' from actual burst events may be necessary. Analog COOK and SCHUMAN (1965), digital PLEWMAN et. al. (1969) and hybrid HARDY et. al. (1972 b) systems have been developed, from the analytical method suggested by SALMON (1963), to examine the

stability of excavations for irregular mine geometries. In effect such problems represent a very large scale (and more complicated) version of the Griffith crack stability problem. The energy release - absroption method of analysis should prove very informative in improved mine design procedures.

Earthquakes

The basic problem of earthquakes due to discontinuous slip along faults may also be considered in terms of the energy released by slip of the 'elastic' or unfailed rock mass on each side of the fault plane surfaces and filling material. The onset of an earthquake represents a transient unstable energy release[probably shearing events crudely equivalent to those described in Figure 6(b),] where the resistance to sliding is a variable, as is the curve def....km, DIETERICH (1971) has suggested that slip will occur along a fault plane when the static (coefficient of) frictional resistance is exceeded and the energy will be absorbed until the lower dynamic frictional resistance of the fault plane is reached.

Blasting

As already noted, the design of patterns for effective rock blasting may be considered in terms of unstable energy changes associated with cracks extending from boreholes. The engineering problem is to arrange the layout of the holes such that the rock is broken uniformly along a line linking each row, e.g. ADFH.

The problem is to determine

(i) for the first hole, most unstable path (AB, AB' or AB"...) from the hole to the free surfaced

(ii) for succeeding holes the spacing AD, DF, ...such that fractures propagate, and along DA in preference to DE". The problem is somewhat complicated by the fact that cracks are also generated in the rock mass within the extreme paths DE and DA, but considerable insight is obtained by energy change analysis. Anisotropy can, in principal, also be included.

There are numerous other practical problems of rock mechanics that would benefit substantially from application of brittle fracture theory. It is hoped that these examples will stimulate research along these lines.

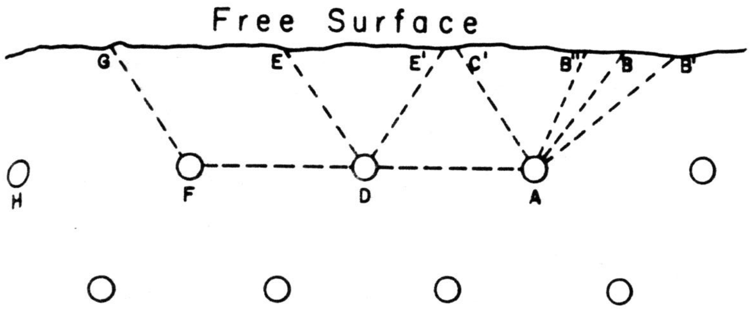

Figure 15. Plan View of Blasting Pattern

CONCLUSION

Development of new and improved procedures for fracturing rock is an increasingly important subject for research and practical application of fracture mechanics principles. The basic energy-change concept used in the Griffith fracture theory has considerable potential for providing insight into many engineering problems involving rock fracture.

Previously a serious limitation, the complicated geometries of practical situation, is now largely overcome through the advent of high speed computers. Important advances in understanding can be expected.

The subject of rock fracture is a broad one. It is impossible, within the space restrictions of this article, to do more than mention some of the details of problems and research developments. The authors regret that many topics had to be omitted, but hope that the discussion will stimulate researchers in fracture mechanics to examine some of the many practically important problems that remain.

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