An Interdisciplinary Approach to Fracture and Fatigue Fracture of Solids

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Synopsis

A concept of an attempt is described to combining macroscopic fracture mechanics with microscopic fracture mechanics or dislocation mechanics. Then, it is related with a thermaly activation process approach to time dependent fracture. Further a stochastic approach is described to combine macroscopic variability with microscopic one. Finally an example is shown of an interdisciplinary approach of this type for fatigue fracture.

§ 1 Why is fracture strength of real solids weaker than ideal ones ?

Real materials fracture at applied stress level 10 to 100 times below the ideal fracture stress. This discrepancy became a matter of concern for researchers. Nowadays, defects or factors of two types are known for this.

Defect of first type is stress concentrator as crack or notch which has been proposed historically earlier than second one. For elastic body, the fracture stress σ_c has been given by Griffith[1] as follows:

$$\sigma_c = \sqrt{\frac{2E\gamma}{\pi c}}, \qquad (1)$$

where E = Young's modulus, γ= specific surface energy and c = heaf length of crack. The theory was based on energy unstable criterion. For elastic body, however, Eq.(1) can be derived based on local tensile stress criterion. The local stress field in the neighbourhood of the crack tip under applied tensile stress σ is as follows [2)3)]:

$$\sigma_{ed} = \sqrt{\frac{c}{2x}}\,\sigma \,, \qquad (2)$$

or

$$\sigma_{ed} = \frac{K}{\sqrt{2\pi x}} \,, \qquad (2a)$$

where x = the distance from the crack tip in the direction of the length, K =$\sqrt{\pi c}\,\sigma$= stress intersity factor. Using Eq.(2a) and other continuum concepts, the approach has been improved for the crack of the type which accompanies plastic deformation with its propagation, which is usually called linear elastic fracture mechanics[4)] or simply fracture mechanics. The defect of this type may be called a macroscopic one. This theory may be called as the continuum or macroscopic fracture mechanics theory. According to this theory, however, smooth specimen of metals such as steel must always contains a priori crack of the length of the order of some mm, and it is unrealistic.

Thus the defect of second type was later proposed. This corresponds to stress concentrator as dislocation.[5)6)] The defect of this type is atomic scale and thus may be called a microscopic one. When edge dislocations move on the slip plane under applied shear stress τ and pile

up at some obstacle such as grain boundary, large local tensile stress σ_{ed} will occur near by the piled up, that is, near the end of the slip line. σ_{ed} is given by as follows[7)]:

$$\sigma_{ed} = \sqrt{\frac{d}{2\xi}}\,(\tau - \tau_i) \,, \qquad (3)$$

where

2d = length of slip line (in simple structure, equal to grain diameter)

ξ = the distance from the piled-up

and

τ_i = frictional force opposed to the movement of dislocation.

Since $\sqrt{d/2\xi}$ is very large, atomic cohesive force will be exceeded and energy unstable condition may also be fulfild, crack may initiate and propagate from the pile-up. Thus fracture stress is given as follows[7)]:

$$\tau \simeq \sqrt{\frac{E\gamma}{2d}} \,. \qquad (4)$$

The stress appears to be in accord with the order of magnitude of fracture stress of rather pure metals for physical studies. This theory may be called as the atomistic or microscopic fracture mechanics theory.

§ 2 A concept of combined micro-and-macro fracture mecanics approach

Returning to the real materials and problems, we notice that they contain ineviably both stress concentrators as cracks or notches and as dislocations, that is, both macroscopic and microscopic one. For example, in the case of engineering materials for construction or structure the macroscopic defect in terms of weld defect or external artificial notches are usually contained as well as the microscopic defect in terms of dislocation. Thus I would like to propose a general concept of combined macro and micro fracture mechanics approach.

In general, the following two requirents should be fulfiled for the crack to propagate : The first requirement is that the local tensile stress at the tip of the crack exceeds the ideal fracture strength, and the second requirement is that free energy decreases associated with the extension of the crack, that is, the energy unsable condition. Thus the fracture stress will correspond to higher value of the two.

2.1 Local critical tensile stress criterion

Let us denote local tensile stress σ_ℓ in the body under simple tension. Then whatever the detailed model may be, local tensile stress criterion is written as :

$$\Psi_s \left\{ |\sigma_\ell|_{max} \right\} = 0 \quad . \tag{5}$$

On the other hand, σ_ℓ may be expressed as :

$$\sigma_\ell = \sigma_{\ell n} + \alpha\, \sigma_{\ell d} \quad , \tag{6}$$

where α is the orientation factor. $\sigma_{\ell n}$ and $\sigma_{\ell d}$ are local tensile stress at the concerned site caused by macroscopic and microscopic defects, respectively, which

have the similar meanings to Eqs.(2) and (3) respectively. Putting Eq.(6) into (5), we get

$$\Psi_s \left\{ |\sigma_{\ell n} + \alpha\, \sigma_{\ell d}|_{max} \right\} = 0 \quad . \tag{7}$$

That is,

$$|\sigma_{\ell n} + \alpha\, \sigma_{\ell d}|_{max} = R_{s1} \quad . \tag{8}$$

This is general argument. Then, in the present section, for example, let us assume a model of plastically induced propagation of the crack as shown in Fig.1. That is, at the crack tip dislocations are initiated and spread into the body and the tip stress is relaxed. On the other hand, dislocations are formed by multiplication near by and move toward the crack tip, and may pile up at the obstacle, such as a grain boundary.

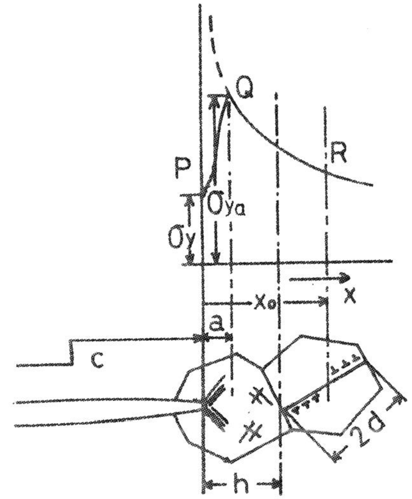

Fig.1 Schematic illustration of an example of a model of combined micro-and-macro fracture mechanics approach

Then high local tensile stress appears near the dislocation pile-up, and the crack extends to this point. The process will be repeated, and thus the crack propagation will occur. The model is shown in Fig.1. In this case of materials showing strain hardening, when the plastic zone is very small compared with the crack length, the plastic tensile stress distribution in the x direction at a point $(x \ll c, 0)$ near the crack tip is as shown in Fig.1*. On the other hand, stress relaxation will occur at the crack tip. Let us denote by **a** the zone of stress relaxation, then the real plastic tensile stress distribution is as shown in Fig. 1. It is to be noted that σ_ℓ is not obtained by simple sum of $\sigma_{\ell n}$ and $\sigma_{\ell d}$ when the crack tip and the piled up point are close[8] as in this case, although it may be understood that $|\sigma_\ell|_{max}$ will be near the piled-up. Taking into interaction of the two, and using approximation[10], we get finally the brittle fracture stress σ_c of this type as follows :

$$\sigma_c = \frac{\alpha^*}{\sqrt{c}}\left\{\frac{R^*}{\sqrt{d}} + \tau_i - \frac{Z}{2}\right\}, \qquad (9)$$

where $\alpha^* = 2\sqrt{2x_0}/A_0$, $R^* = \sqrt{2(h-x)}R_{S1}$. Z and A = constants dependent on material and loading mode. It can be seen that the calculated value of σ_c (by Eq.(9)) exceeds atomic cohesive force when using reasonable values of parameters.

Eq.(9) is also rewritten in terms of fracture toughness Kc as :

$$K_c = \alpha^*\left\{\frac{R^*}{\sqrt{d}} + \tau_i - \frac{Z}{2}\right\}. \qquad (10)$$

It is, however, noted that Kc in Eq.(10) or in this line of consideration is included as the local concentrated

* For the case the plastic zone size is far much smaller than the crack length, elastic approximation was used in the previous articles.[8)9)]

stress, but not as the stress field[4] as in usual macroscopic fracture mechanics. Further in macroscopic fracture mechanics it is fundamentally assumed that the stress at the crack tip could not exceed the yield stress,[11] σ_Y, and thus as for the fracture model, a slipping-off mechanism,[12] say, rupture named by Orowan[13] has been used. Therefore, as the fracture criterion, the critical displacement such as COD (crack opening displacement) has widely been used,[11] instead of either critical stress or energy unstable criterion. However, as can be seen from the line of consideration mentioned above, the stress at the crack tip may exceed atomic cohesion, much larger than the yield stress, and a slipping-off mechacism or critical displacement criterion might not be necessary. Eq.(9) is at least qualitatively in good agreement with the data on the effect of notch length **c**, and ferrite grain size, **d** on low-stress brittle fracture of one and the same low carbon steel.[10]

2.2 Energy unstable criterion

Knowing the condition fulfiled as described in §2.1, let us examine the energy unstable condition. Now consider the case the plastic enclave is much smaller than the crack length. Then the stress intensity factor at the crack tip is given as follows[8] :

$$K_I = \sigma\sqrt{\pi c}\, g_{I1} + \left(\frac{\sigma}{2} - \tau_i\right)\sqrt{\pi c}\, g_{I2}, \qquad (11)$$

$$K_{II} = \sigma\sqrt{\pi c}\, g_{II1} + \left(\frac{\sigma}{2} - \tau_i\right)\sqrt{\pi c}\, g_{II2}, \qquad (12)$$

where g is the coefficient of the interaction between the crack and the slip line, and, a function of **c** and **d**.
Aussuming the crack extension to occur in the direction

of the crack axis, energy unstable condition is given as follws :

$$\frac{1}{(1+\nu)G}\left(K_I^2+K_{II}^2\right)+\frac{\delta W_P}{\delta c}-4(\gamma+\gamma_P)=0, \quad (13)$$

where

W_P = the energy dissipated by the plastic deformation in the zone[14] by DM model,[15][16]

γ_P = the energy dissipated by the plastic deformation in the area[17] outside the zone by DM model.

For convenience, when we neglect the interaction of the defects of the two types, then g except g_{II} in Eqs.(11) and (12) is zero, and we get[9]:

$$K_c \simeq \sqrt{E(\gamma+\gamma_P)}, \quad (14)$$

which is the same type as Griffith's formula[1], and also the formula frequently used in macroscopic fracture mechanics.[12]

It is also interesting to consider the brittle fracture after general yielding of smooth specimen of low carbon steel.[8] In this case it is well known that an initial cleavage microcrack has been formed before fracture after yielding. Thus in this case it is assumed as $c \simeq d$. Including the energy $\delta W_P/\delta c$ into γ_P and denoted them by γ_e, we get the fracture stress as :

$$\sigma_c = \alpha_2\left[\sqrt{\frac{M_1}{d}-\beta_2\tau_i^2}+\tau_i\right], \quad (15)$$

which is in good agreement with the experimental data.[8]

α_2, β_2 and M_1 are constants. The criterion by Eq.(15) might be also adequate for the brittle fracture of notched specimen after general yield.

Which of the two conditions described in §2.1 and 2.2 is critical is future problem needed to be study.

§ 3. Kinetic theory approach

In the case of time dependent fracture such as fatigue or creep fracture, thermal agitation will help in attaining both critical local stress condition and energy unstable condition with equivalent effect to enhancing the local concentrated tensile stress σ_{en} and σ_{ed} mentioned in § 2. It can be easily understood, for instance, to remember that the higher the temperature in creep fracture, the lower the stress sustained up to some definite life. Thus in time dependent fracture, we see that the statistical mechanical approach, that is, kinetic approach is necessary. In the fracture of this type, fracture criterion is given by one equation including the two requirements mentioned in § 2.

For the case of the crack nucleation or for the case in which the crack propagation is caused by the mechanism of another crack nucleating in the neighbourhood of the main crack, whatever the detailed model may be, the nucleation rate μ of the crack is given by the following equation of Arrhenuius type:[18][19]

$$\mu = A\exp\left[-\frac{U-\Phi(\sigma_{en},\sigma_{ed})}{RT}\right], \quad (16)$$

where

Φ = an increasing function with respect to σ_{en} and σ_{ed},

U = activation energy,

R = gas constant,

T = absolute temperature,

and $\quad A$ = constant.

Thus we can see that fracture strength will decrease by the thermal agitation. When load is not applied, then Φ = 0, and in such case

$$\mu = A \exp\left(-\frac{U}{RT}\right) . \qquad (17)$$

It can be seen from the argument mentioned above that U may be nearly equal to heat of sublimation.[18] On the other hand, from the standpoints of both safety engineering and new materials developement it is very important to clearify the term $\Phi(\sigma_{\ell n}, \sigma_{\ell d})$. It concerns the mechanism of fracture of solids with the size larger than atomic scale. Φ can be obtained by using the concept described in § 2. It is interesting that Φ can be in good approximation obtained as the equation of the following type:[19) 20)]

$$\Phi = V \ln \sigma_\ell \qquad (18)$$

for most important cases such as yield phenomena,[21] brittle fracture[22] and fatigue crack propagation.[20] V is some constant. It should be noted that in yield phenomena Φ has the same type as Eq. (18) whether the mechanism may be based on the Cottrell locking mechanism or on the dislocation dynamics mechanism.[23]

Thus for the case of the crack propagation based on the nucleation rate process, the nucleation rate μ of the crack given by Eq. (16) is expressed substituting

Eq. (18) into Eq. (16) as follows:

$$\mu = A \sigma_\ell^{\delta} \exp(-U/RT), \qquad (19)$$

where $\delta = V/RT$. In Eq. (19) σ_ℓ is expressed as follows:

$$\sigma_\ell \propto K \qquad \text{for elastic crack}, \quad (20)$$

$$\sigma_\ell \propto K^{\frac{2\lambda}{1+\lambda}} \qquad \text{for elastic-plastic crack}, \quad (21)$$

[24]

where

λ = strain hardening exponent, given by the relation $\sigma = \sigma^* \varepsilon^{\lambda}$.

$K = \sqrt{\pi c}\, \sigma$ = stress intensity factor.

Thus the theory may be regarded as a kinetic theory related with continuum mechanics approach in terms of stress intensity factor, K . It is to be noted that K enters in general into this kinetic theory in terms of two different natures as mentioned in § 2 . That is, one is the stress field as included in relaxation of strain energy in the neighbourhood of the crack of critical size, and the other is as the local concentrated tensile stress as included in activation volume. Eq.(19) is useful in analysing the kinetic aspect of fracture, such as the strain rate dependence of fracture stress, the problems of creep and fatigue as shown in later.

§ 4. Statistical approach

It can be understand from the argument described in § 3 that fracture phenomena have microscopically statistical nature since separation between neighbouring atoms may be helped by thermal agitation. The statistical aspect of this type is of microscopical nature and is included[25] in Eq. (16). On the other hand, the size of the stress raiser such as crack-like flaw has variability and rather random distribution, and thus fracture phenomena have a statistical aspect of another type, say, macroscopicaly statistical nature. It can be shown that when this variability is not so large, then this effect is included in A and the coefficients of σ_{en} and σ_{ed} in Eq. (16). Thus in the case of including microscopic and macroscopic statistical variability, μ of Eq. (16) can also be expressed as the same type as Eq. (19) as follows:[26]

$$\mu = L\sigma^{\eta} \qquad (22)$$

with the only difference that the factor of macroscopic variability is confined to L in Eq. (22). In this case the probability P of fracture (the probability of the fracture stress not exceeding applied stress σ) is given as[26]:

$$P = 1 - \exp\left(-\frac{L}{(\gamma+1)\dot{\sigma}}\sigma^{\eta+1}\right), \qquad (23)$$

where

L = constant proportional to the volume of the specimen,

$\dot{\sigma}$ = applied stress velocity,

η = constant dependent on temperature.

It is interesting to notice that Eq. (23) becomes Weibull's distribution function[27] when temperature and stress velocity are assumed definite values respectively.

§ 5. An interdisciplinary approach to fatigue fracture

I would like to emphasize the following three propositions connected with elucidating the essence of the phenomena of fatigue.

(1) Striations as shown in Fig. 2 are observed on fatigue fracture surface.[28] Most researchers believe each striation corresponds to each cycle, that is, fatigue crack propagates a distance of the width of one striation per every cycle. On the other hand, all over the range of crack propagation rate studied the experiments show that crack propagation rate does not equal to the striation spacing, that is, the former is larger than the latter in the range of higher value of dc/dN at the crack tip and vice versa in the range of lower value of dc/dN as shown in Fig. 2 for alluminum alloy. This characteristic is the same for steels. That is, the propagation rate dc/dN is experimentally related with stress intensity factor ΔK as follows:

$$\frac{dc}{dN} = C_1 \Delta K^{\delta}, \qquad (24)$$

where δ assumes from about 2.5 to some 6.7 dependent of materials:[29] on the other hand, the striation spacing s is experimentally related with the same stress intensity factor ΔK as follows:[30)]

$$S = S_1 \Delta K^2 \qquad (25)$$

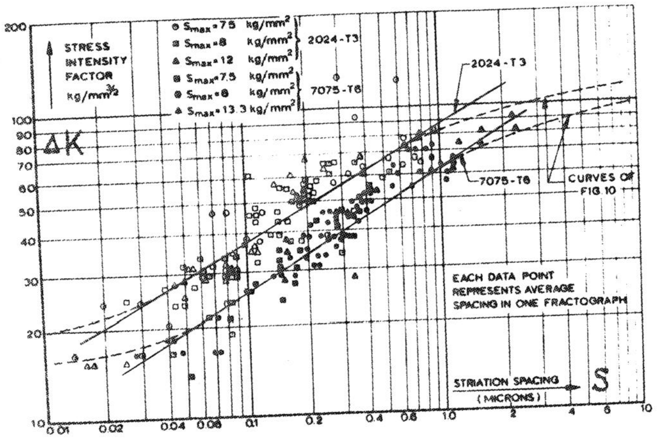

Fig.2 Relation between stress intensity factor and striation spacing [28)]

Therefore, line of log dc/dN versus logΔK will inevitably intersect with the line of log S versus logΔK except the case of δ = 2 all over the range of dc/dN. Thus we can see the similar situation in this case to Fig. for alluminum alloy.

As is the case in most problems on fracture or strength, one and the same equation will not cover the entire range of parameter covered. Thus in this case, it is more reasonable to divide three ranges of dc/dN, such as high, medium and low. For low range of dc/dN,

the characteristics in Fig. suggests that the extension of the crack will not occur every cycle, but probabilitic. For medium and high range of dc/dN, the probability becomes large and the crack will extend every cycle, that is, determistic, but the extension can become larger than s.

(2) Kinetic aspects of fatigue crack propagation.

The fatigue crack propagation rate experimantally obtained is expressed by equation of Arrhenius type as shown in Fig. 3.[31)] Thus taking into account of the characteristics mentioned above, the fatigue crack propagation may be regarded as stochastic process in terms of thermaly activation process physically.

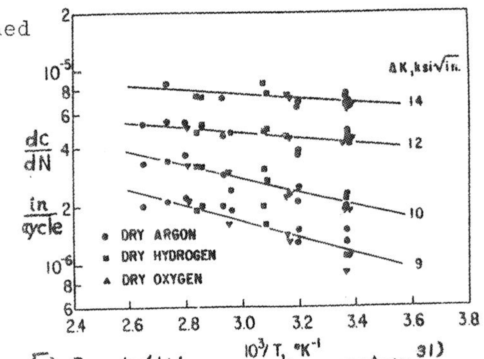

Fig.3 dc/dN versus temperature [31)]

(3) The effect of time and cycle.

Why does materials fracture under smaller stress when respected than when monotonic loading ? This suggests that materials will decrease its strength with time or cycle during loading. Then question will arise whether its life will be determined by the total time or by the total cycles applied. As we see in the following, life is time dependent. On the other hand, fatigue fracture is quite different from creep fracture under constant applied stress which is determined by time effect only. Thus fatigue life is dependent of both cyclic loading effect and time effect.

The following two lines of considerations may be responsible to answer these propositions, that is, the first one concern the low range and the second one concern the medium and high ranges of the crack propagation rate.

The first line of consideration is one that each discontinuous extension of the crack is considered to occur as a result of stochastic process. In this consideration the next stage of propagation is impossible unless applied load is repeated.[20] Thus let us assume each process of discontinuous crack propagation may consist of two successive stochastic processes. Let us denote the two probabilities by μ_1 , and μ_2 , respectively.* In the case when $\mu_1 \gg \mu_2$, the crack propagation rate dc/dN is given by:

$$\frac{dc}{dN} = \varepsilon \mu_2 , \qquad (26)$$

where ε = the increment of the crack length associated with each discontinuous propagation stage. μ_2 is the probability per cycle of the occurrence of the discontinuous propagation after the conditions ready for the

* This does not mean that the total life consists of these two successive stochastic processes, but it is composed of discontinuous propagation stages, each of them consists of these two successive stochastic processes. Furthermore, if the conditions ready for the next discontinuous propagation stage are established each time stress is reversed in the direction and is applied in the previous direction, then the first process is not probabilitic, but deterministic.

the propagation been attained. The probability μ_2 is one corresponding to the probability μ given by Eq.(19).

The second line of consideration is one that each discontinuous extension ε of the crack will occur exactly per cycle, that is :

$$\frac{dc}{dN} = \varepsilon \qquad (27)$$

On the other hand, the extension ε itself will be controlled by the probability of the stochastic process as based on kinetic theory, such as, on dislocation dynamics.[32)] ε in Eq.(27) in this case may be considered approximately proportional to dislocation velocity v , and v , in turn, may be expressed by the equation of the same type as the right hand side of Eq.(19). Thus ε in Eq.(27) will be expressed by the equation of the same type as μ given by (19).

More general consideration may be the one that using Eq.(26) in which ε is given by the second line of consideration, that is, dislocation dynamics.

Thus according to either of the two lines of considerations, the fatigue crack propagation rate, dc/dN is given by the equation of the following type :

$$\frac{dc}{dN} = \frac{A_0}{\omega} \exp\left[-\frac{U - V \ln \Delta K}{RT}\right], \qquad (28)$$

where

C = half length of the crack,

N = the number of repeated cycles of applied stress,

σ = stress amplitude,

$\Delta K = \sigma\sqrt{\pi c}$ = stress intensity factor,

ω = frequency or velocity of cycling,

U = activation energy,

and

A_o = constant.

Or Eq.(28) is rewritten as:

$$\frac{dc}{dN} = \frac{B}{\omega}\left(\sqrt{\pi c}\,\sigma\right)^{\delta} \exp\left[-\frac{U}{RT}\right], \qquad (29)$$

where

$$\delta = \frac{2\beta}{1+\beta}\cdot\frac{1}{mRT},$$

β = cyclic strain hardening exponent,

and

B and m = constants respectively dependent on materials.

The number of repeated cycles N_{P_2} for the length of the crack to reach critical one is given by:[33)]

$$N_{P_2} = \frac{1}{A_1\pi\sigma^2}\left[\frac{1}{K_{oc}^{\delta-2}} - \frac{1}{K_{fc}^{\delta-2}}\right], \qquad (30)^{**}$$

** If μ_1 is not negligible as compared with μ_2, then the life required for the propagation N_{Pt} is given as follows:

$$N_{Pt} = N_{P_1} + N_{P_2},$$

where

N_{P_1} = the total sum of the number of repeated cycles required for the attainment of the conditions ready for next each stage of propagation.

N_{P_1} may depend on cycling number only, but not so much on time, and, on the other hand, N_{P_2} depends on time only as can be seen from Eq.(28), where $N = \omega t$ and thus $(dc/dN) = (1/\omega)(dc/dt)$. Also see the Foot Note denoted by * .

where

$K_{oc} = \sqrt{\pi C_o}\,\sigma,$

$K_{fc} = \sqrt{\pi C_f}\,\sigma,$

Co = half length of the critical crack which will eventually lead to final fatigue fracture, say, the initial length of the main crack or of the crack-like notch,

and

C_f = half length of the critical length at which length the final catastrophic static fracture occur.

K_{fc} may be regarded as nearly equal to so-called fracture toughness, Kc.

The argument mentioned above concerns the specimen with an initial main crack or crack-like notch. In the case of smooth specimen, in low carbon steel[34)] the propagation process corresponds to the joining process of the microcracks initiated. The harder,[35)] however, the materials, the smaller the number of the microcracks joining, and, thus, in very hardened steel, such as heat treated bearing steel[36)] one fatigue crack initiated will propagate until final catastrophic fracture. From this reason Eq.(30) will hold also in smooth specimen, where Co may be at least smaller than grain size, say, may be the size of non-metallic inclusion.

In the case of $C_f \gg C_o$, which may be statisfied in smooth specimen, the second term in the parenthes of the right hand side of Eq.(30) may be neglected, and thus Eq.(30) may be written as:

$$N_{P_2} = \frac{1}{A_1 \pi \sigma^2 K_{oc}^{\delta-2}} \quad (31)$$

It can be seen from Eqs.(30) or (31) that fatigue fracture corresponding to a definite life N_{P_2} may not be determined by toughness in terms of stress intensity factor, but both by stress σ and stress intensity factor $\sqrt{\pi C_o}\,\sigma = K_{oc}$.

Also it can be seen from Table 1 that δ in Eq.(29) is not constant with respect to materials. It is interesting to note that the value of δ decreases with decrease of strain hardening exponent, λ . In deriving Eq.(29), cyclic stress strain exponent β is assumed instead of static one λ . If β shows similar trends to λ with respect to materials, then the influence of λ on the value of δ as shown in Table 1 can be expressed from Eq.(29).

Table 1

Materials	δ	Ultimate tensile strength, Kg/mm²	Elongation, %	Strain hardening exponent
Heat treated ball bearing steel[36]	4.7	185 (fracture stress)	5.8	λ_1
Mild steel[34]	4.0	38	42	λ_2
Tempered martensitic high strength steel (low carbon)[35]	2.5	84	40	λ_3

(It was experimentaly observed that $\lambda_1 > \lambda_2 > \lambda_3$)

If the relation of cyclic stress and strain $\sigma = \sigma_o \varepsilon_p^{\beta}$ is substituted into Eq.(31) for smooth specimen and is rearranged, we get:

$$\varepsilon_p \left(\frac{N}{\omega}\right)^{\xi} = R_o \quad (32)$$

where Ro is constant. This is Coffin-Manson[37] [38] law where $\xi = (1+\beta)\, mRT/2\beta^2$.

On the other hand, Eq.(28) or (29) is in well agreement with experimental data on the influence of stress intensity factor ΔK, temperature[31) 39)] as shown in Fig.3 and velocity of cycling on the fatigue crack propagation.

Furthermore, the concept mentioned above predicts that high fatigue resistant materials will be developed if the cyclic strain hardening exponent β is decreased and K_{fc} corresponding to the catastrophic fracture is increased.

§ 6 Conclusions

There remains much to be solved in developing the line of considerations mentioned above. Then it may be useful to see how the dissimilarities of microstructure in materials will enter the equation expressing the macroscopic behavior associated with strength and fracture. Thus it may also be known how to regulate what parameter for the purpose of developing high fracture resistant materials.

The author should appreciate useful discussions with Drs. M. Ichikawa and A. Kamei, and Mr. M. Yoshida.

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