Theoretical Calculation of the Griffith Criterion of Fracture in Anisotropic Media to Investigate the Correlation between Crystalline Directions and the Orientation of the Crack Surface

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When a material contains an inclusion where its elastic moduli are different from those of the remainder (matrix), an applied stress will be disturbed due to the presence of the inclusion. Such disturbed stress is the main subject to be investigated. When the elastic moduli of the inclusion vanish, the inclusion problem becomes a crack (or void) problem. Therefore, the general ellipsoidal inclusion problem in anisotropic media is considered first. Then, the inclusion will be specified to a void and finally to an elliptical crack which is degenerated from the ellipsoid.

According to Eshelby [1], the stress disturbance of a uniformly applied stress at infinity caused by an ellipsoidal inclusion is equivalent to that of eigen-strain problem when suitable eigen-strain is uniformly distributed in that ellipsoidal domain by assuming that the material is homogeneous everywhere. The ellipsoidal eigen-strain problems in the homogeneous material were solved by Eshelby [1] for isotropic and by Kinoshita and Mura [2] for anisotropic case. The internal stresses inside the ellipsoidal domain for both cases are found to be constant and linear functions of the eigen-strain. Therefore, a system of six linear simultaneous equations is formulated to determine the unknown six eigen-strain components by equating the six components of stress for the inclusion problem to the corresponding stress components in the eigen-strain problem.

By modifying the solution of eigen-strain problem and using the method stated before, Eshelby [1] solved the problems of plane and elliptical cracks for isotropic case. In the present calculation, the elliptical crack problem is extended to anisotropic material by using the solutions of Kinoshita and Mura [2].

Consequently, the interaction energy which is defined as the change of the total potential energy due to the presence of the crack will be calculated. By using this, the Griffith fracture theory is applied to determine the critical applied stress for a given size of penny-shaped crack. Since the crack surface is not necessarily on one of the crystalline planes of the material, a more general case which is the case under both tension and shear applied stresses is considered. The interaction energy and the critical stress depending on the correlation between the crystalline direction of the material and the orientation of the crack surface are evaluated.

In addition, the method can be equally applied to calculate the elastic strain energy of the system when a dilatational misfit strain is distributed in a thin elliptical inclusion (i.e., disc-shaped precipitate) and no external force is applied. The strain energy associated with the misfit becomes minimum when the plane of the inclusion coincides with one of the crystalline planes, where the matrix and the inclusion are assumed as two different cubic crystals having the same crystalline directions. The minimum value however, surprisingly, depends only on the elastic moduli of the inclusion for the infinitely thin elliptical inclusion. It is also a surprise to observe that when crystalline directions of the inclusion are parallel to the principal axes of the ellipsoid, the elastic strain energy is independent of the

orientation of the inclusion with respect to the crystalline directions of the matrix and the constant value is equal to the minimum value in the first case. If an inclusion is simply defined by giving a uniform dilatational misfit strain in an ellipsoidal domain in a homogeneous material (i.e., matrix and inclusion are same material), the condition for the minimum elastic strain energy is the same as that of the first case, namely, the minimum occurs when the plane of the inclusion is parallel to one of the crystalline directions.

REFERENCES

[1] Eshelby, J. D., Proc. Roy. Soc. A241, 376 (1957).

[2] Kinoshita, N. and Mura, T., phys. stat. sol. (a) 5, 759 (1971).