Significance of Stress and Strain Fields around Inclusions with Respect to Fatigue Failure.

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Introduction.

Non-metallic inclusions are known to play an important role in the fatigue of metals by serving as potential fatigue-crack nuclei. In the fracture-mechanics approach to fatigue an inclusion is treated as a sharp defect, notwithstanding its finite root radius. A less conservative procedure, allowing for a period of crack initiation, will be outlined in the following.

Stress-intensity factors.

In view of the inherent weakness of the phase boundary between inclusion and matrix, it will be assumed that the inclusion acts as a cavity during the tensile part of the loading cycle. The fracture-mechanics part of the analysis thus requires knowledge of the stress-intensity factor, K, for a crack originating from a notch. A first approximation of K is given by

$$K^2 = (\zeta\,\alpha\sigma_o)^2\,\pi l \qquad l \le l^*$$
$$K^2 = (\gamma\,\sigma_o)^2\,\pi(a+l) \qquad l \ge l^* \tag{1}$$

where

$$l^* = \frac{a}{(\zeta\alpha/\gamma)^2-1} \tag{2}$$

and

a = notch depth
l = crack length
σ_o = nominal applied stress
α = stress-concentration factor of the notch
γ = crack-geometry factor
ζ = surface-crack correction factor (= 1.127)

Fig. 1 illustrates the application of Eq.(1) to a circular hole with symmetric side-cracks ($\alpha = 3$, $\gamma = 1$). The more accurate Bowie solution [1], tabulated by Paris and Sih [2], has been included for comparison. From Fig.1 it is inferred, that for $l > l^*$ the influence of notch shape can be neglected, and the effect of the notch merely becomes that of increasing the effective crack length. In the above case $l^* = 0.096\,a$, and in general, except for very mild notches, $l^* \ll a$.

Crack initiation.

A prerequisite for crack nucleation is localized, cyclic plastic flow, which may occur, under nominally elastic conditions, at a free surface or at an internal discontinuity [3]. In the latter case, crack extension by fatigue may be divided into two separate stages:

(i) an initiation stage, N_i, comprising nucleation of a crack and its extension through a region of general, cyclic plastic flow ($l \le l^*$) ;

(ii) a propagation stage, N_p, to which the concepts of linearly elastic fracture mechanics may be applied ($l \ge l^*$).

In the following work it is tentatively assumed, that the initiation stage is governed by a Coffin-type relationship

$$N_i = \left(D/\Delta\,\varepsilon^{p}_{max}\right)^{m} \qquad (3)$$

If Eq.(3) is regarded as a damage criterion [4], m and D are material parameters. If, contrary to this interpretation, Eq. (3) results from the integration of a crack-growth relation [5], D will also depend on the initial and the final crack length. The choice of the former and the duration of the nucleation stage then remain undetermined.

Consider in particular a spherical cavity in a linearly elastic-perfectly plastic matrix subject to a remotely applied uniaxial alternating stress. If the stress range exceeds the shake-down range, a region of repeated plastic flow will spread from the equator of the sphere. Using a finite-element computer program for the solution of this axisymmetric, elastic-plastic boundary-value problem, plastic strains at point A of Fig.2 have been calculated. The hysteresis loop in Fig.2 shows the axial plastic strain as a function of applied stress, and Fig.3 illustrates the accumulation of equivalent plastic

strain. σ_Y and $\varepsilon_Y = \sigma_Y/E$ denote yield stress and yield strain respectively. Poisson's ratio is 0.3 in the present example.

The size of the initiation region is $l^* = 0.083\ a$, which is obtained from Eq.(2) with $\alpha = 2.05$ and $\gamma = 2/\pi$. Assuming $\varepsilon_Y = 0.0025$, $m = 2$ and $D = 1$, and substituting the axial plastic strain range at A into Eq.(3)yields a duration of the initiation stage versus applied stress range as depicted in Fig. 4.

Crack propagation.

The existence of a threshold-stress-intensity range, ΔK_{th}, below which crack growth does not occur, has been established by several authors [6-8]. Furthermore crack-propagation laws including this feature have been suggested [8,9]. In particular, using a COD-approach, Donahue et al. [9] arrived at

$$\frac{dl}{dN} = \frac{8}{\pi}\left[\left(\frac{\Delta K}{E}\right)^2 - \left(\frac{\Delta K_{th}}{E}\right)^2\right] \qquad (4)$$

Integration of Eq.(4) from the initial crack length l^*, given by Eq.(2), to the critical crack length, corresponding to the fracture toughness K_c, results in

$$N_p = \frac{N_o}{s^2}\ \ln\frac{(\varkappa/\lambda)^2 - 1}{(s/s_{th})^2 - 1} \qquad (5)$$

where

s	=	$\Delta\sigma_o/\sigma_Y$	$\varkappa$	=	$K_c/\Delta K_{th}$
s_{th}	=	$\sqrt{w/a}/\beta$	λ	=	$K_{max}/\Delta K$
w	=	$(\Delta K_{th}/\sigma_Y)^2/\pi$	N_o	=	$1/8\ \gamma^2\,\varepsilon_Y^2$
$1/\beta^2$	=	$1/\gamma^2 - 1/(\zeta\alpha)^2$			

Application of Eq.(5) to crack growth from a spherical cavity under pulsating tension ($\lambda = 1$) yields the propagation curve of Fig.4, if the following numerical values are adopted:

a	=	100 μm	K_c	=	50 $MNm^{-3/2}$
σ_Y	=	500 MNm^{-2}	ΔK_{th}	=	5 $MNm^{-3/2}$
ε_Y	=	0.0025			

The radius of a cavity in the shape of a sphere or a circular cylinder corresponding to $\Delta\sigma_{th} = \sigma_Y$ is given in Table 1 for values of σ_Y and ΔK_{th} typical of steel.

Table 1. Cavity radii (um) corresponding to [illegible] = σ_Y.

σ_Y (MNm^{-2})	500	1000	2000	500	1000	2000
ΔK_{th} ($MNm^{-3/2}$)	5	5	5	10	[illegible]	10
Sphere	73	18	5	290	3	18
Cylinder	29	7	2	120	[illegible]	7

The validity of continuum models, when the inclusion size becomes of the order of a few microns, as well as the applicability of linearly elastic fracture mechanics to stress ranges in excess of the yield stress, are matters open to discussion. Fatigue tests on steel containing natural inclusions are presently undertaken with a view to investigating some of the assumptions and predictions of the foregoing analysis.

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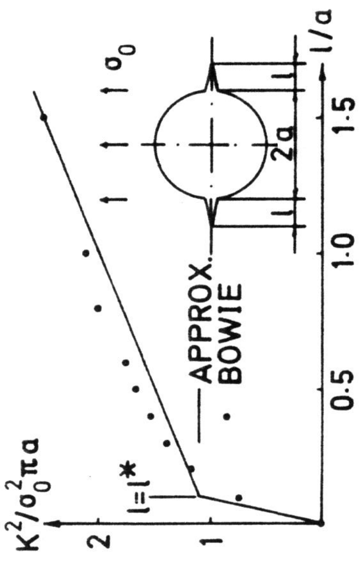

Fig.1. Comparison between K-solutions.

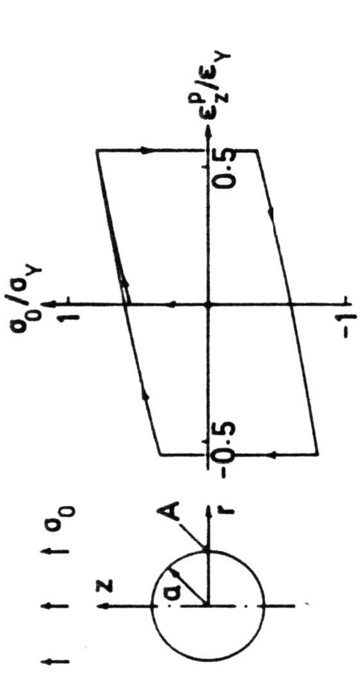

Fig.2. Axial plastic strain at A.

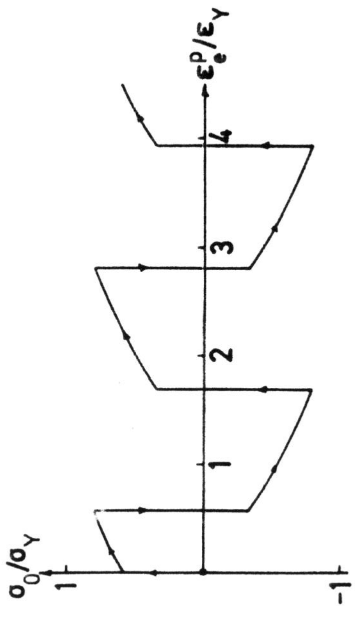

Fig.3. Equivalent plastic strain at A.

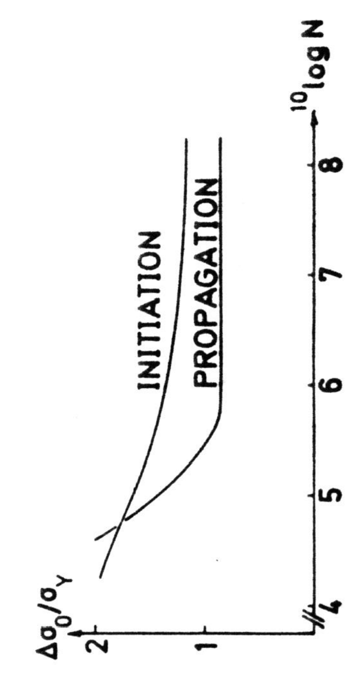

Fig.4. S-N-curves for initiation and propagation.