Post Yield Analysis and Fracture in Notch Tension Pieces

J. G. Sumpter, Imperial College, London, D. J. Hayes, K. S. L., Amsterdam,
G. T. Jones C. A. Parsons, Newcastle,
and C. E. Turner Imperial College. London.

INTRODUCTION. This report uses a finite element approach based on incremental plasticity to analyse a series of single edge notch (SEN), double edge notched (DEN) and centre notch (CN) tension geometries. The majority of the computations are for non-workhardening and plane strain; the effect of plane stress and workhardening has been assessed in individual runs.

Typical mesh sizes were 450 nodes and 850 elements (triangular constant strain). In the region of the crack tip an element size of a/160 was used (a-crack length SEN, half crack length CN). In all cases, loading was applied as uniform displacements at the specimen ends. Special attention was given to the behaviour of two parameters which have been suggested as measures of the elastic-plastic crack tip stress-strain field; the J contour integral, and COD, the crack opening displacement.

J CONTOUR INTEGRAL. The derivation and formal definition of J as a line integral around the crack tip and as J = dP/da, the potential energy release rate, is given in (1). The integral may be evaluated using the field values of the finite element analysis. Its property as a crack tip characterising parameter is demonstrated theoretically only for non-linear elasticity or total theory plasticity. Justification is provided for its use in the case of incremental plasticity by the apparent maintenance of path independence in all the finite element solutions obtained by the authors. Similar results have been reported elsewhere (2), but other work (3) reports path dependence.

The energy interpretation of J, again requires the assumption of total theory plasticity. For linear elastic behaviour J = G, the elastic energy release rate. This interpretation has recently been used to determine J experimentally (4). This reference also indicates that fracture initiation in alloy steels is characterised by a critical value of J over a wide range of geometry and loading conditions.

CRACK OPENING DISPLACEMENT. The main limitation of the constant strain elements is that they cannot adequately represent near crack-tip conditions. At high loads where the deformed crack has a well-defined "nose" COD may be interpreted as the extrapolated tangent to the crack profile. Values defined in this way for bend specimens (5) have been shown to agree with approximate expressions known to represent experimental trends, but for tension, especially below general yield, the computed profile is more rounded and the COD tangent definition no longer seems relevant. An alternative suggestion (6) adopted here, is to take the displacement at the elastic-plastic interface on the crack profile. Other proposals are made (2).

RESULTS. The following main features emerge: (i) in SEN and CN plane strain, plastic zones extend at approximately 45° to the axis of the crack until they reach a free surface. The remainder of the plate does not yield, even after further deformation. The shortest crack length SEN a/W = .1, behaves rather differently. Here, yielding at the ends is extensive before plastic collapse from the notch occurs. The whole plate is consequently plastic with the exception of a small elastic enclave on the crack surface. Collapse loads are in general agreement with those predicted by slip line theory for CN, but higher for DEN and SEN. Tensile stress elevation at the crack tip is initially about $2\sigma_y$. There is some tendency for this to fall off near and after general yield, but complete loss of triaxality (i.e. σ_y = 1.155 CN plane strain) is not achieved. (ii) Fig.1 contrasts CN, SEN and DEN crack profiles. In the CN geometry after general yield a 1:1 relationship is established between COD and overall deflection. (iii) In all geometries, the standard deviation of J for 8 path lengths

was less than ± 5% of the mean for plane strain (± 2% for plane stress) up to and well beyond gross yield. Integration paths were rectangular in shape and varied in length between 0.5a and 10a. For shorter paths the results became inconsistent at higher loads. (iv) A linear relationship is established between J and overall deflection after general yield (Fig.2). The slope is $1.155\sigma_y$ for CN plane strain and SEN a/W = 5; about $1.0\sigma_y$ for CN plane stress and DEN 2a/W = .625. For SEN a/W = .1 the relationship is more complex (Fig.3). Workhardening causes a slight decrease in J for a given overall deflection. (v) A simple estimation of J (7) has been shown to give good agreement with the experimentally determined values in (4). Fig.2 shows, for CN 2a/W = .3125, the excellent correlation with the computed values. Similar agreement exists for other geometries. The estimation procedure is : below general yield $J = \frac{K_1^2}{E}$, where $K = F(a/W)\sigma\sqrt{\pi(a+r_y)}$; ($r_y$ and E^1 have the appropriate values for plane strain or plane stress); above general yield $J = \frac{dP}{da}$, where P = limit load Q x overall deflection Δ. Thus, for CN plane strain $J = \frac{-d}{d(2a)}\left[1.155\sigma_y(W-2a)\Delta\right]$ $= 1.155\sigma_y\Delta$; and for DEN plane strain $J = \frac{-d}{d(2a)}\left[1.155\left(1+\log\frac{(W-a)}{(W-2a)}\right)\sigma_y W-2a)\Delta\right]$

$$= 1.015\sigma_y\,\Delta\left(\text{for } 2a/W = .625\right)$$

(vi) The results establish a relationship beyond general yield of $J = 1.155\sigma_y\delta$ for CN and deep notch SEN tests. This suggests an analogy with the Dugdale model relationship $J = M\sigma_y\delta$ (1), where $M\sigma_y$ is the constrained yield stress. The relationship for DEN 2a/W = .625 for which M = 1.85 shows that this does not hold in general where collapse load is not just proportional to ligament width.

EXPERIMENTAL RESULTS. Tests are being conducted on various geometries of of notched plate. It is hoped to present experimental data, interpreted in the light of the present computations at the Conference.

CONCLUSIONS. The evidence here indicates that J is a suitable parameter for the characterisation of elastic plastic crack tip stress fields. Provided collapse loads are known, the confirmation given to approximate estimates of J from simple l.e.f.m. and collapse is very encouraging. Further experimental evidence is needed to substantiate the use of J as a fracture criterion.

The finite element analysis may not contain a clear definition of COD in tension geometries, but where COD is definable, as in the bend test, J and COD correlate well.

REFERENCES

1. Rice, J.R. Mathematical Analysis in the Mechanics of Fracture. Ed. Liebowitz, H. Vol.2, Academic Press, 1968
2. Boyle, E., Belfast University. Ph.D. Thesis, September 1972.
3. Levy et al. Int.Jour. of Frac.Mechs., 7 (2), 1971. 143.
4. Landes and Begley. J.Integral as a Failure Criterion. Fifth National Symposium on Fracture Mechanics, University of Illinois, September 1971.
5. Hayes, D.J., and Turner, C.E. An application of Finite Element Techniques to post yield Analysis of Proposed Standard three-point Bend Fracture Test Pieces. Submitted Int.J. Fract.Mechs.
6. Wells and Burdekin. Int.Jour.Fract.Mechs., 7, 1971, 233-41.
7. Bucci, Paris et al. J Integral Estimation Procedures. Westinghouse Scientific Paper, 71-1E7-FMPWR-P1, December 1971.

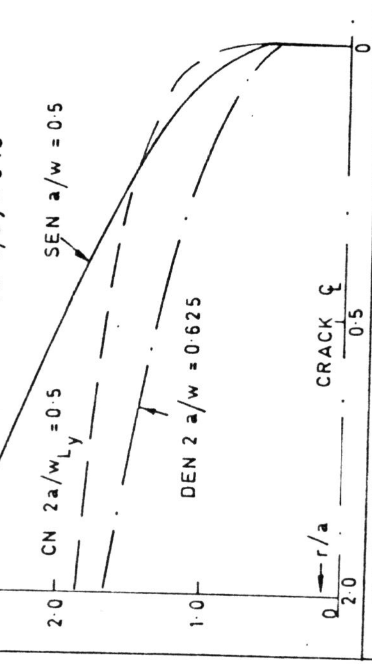

FIG.1: COMPARISON OF CRACK PROFILES IN TENSION AT $e/e_y = 0·75$

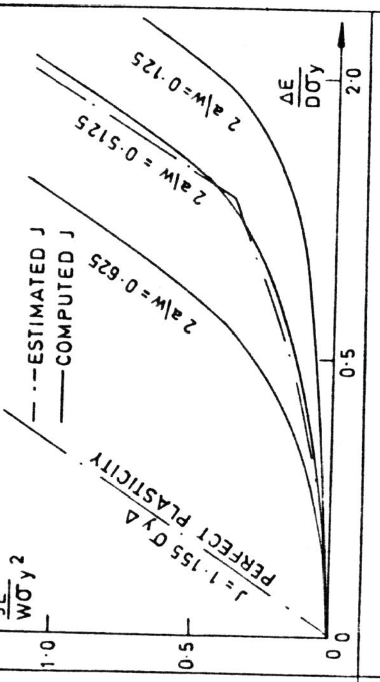

FIG.2: CN PLATES. PLANE STRAIN, J AGAINST OVERALL STRAIN, GAUGE LENGTH D= 2·5W

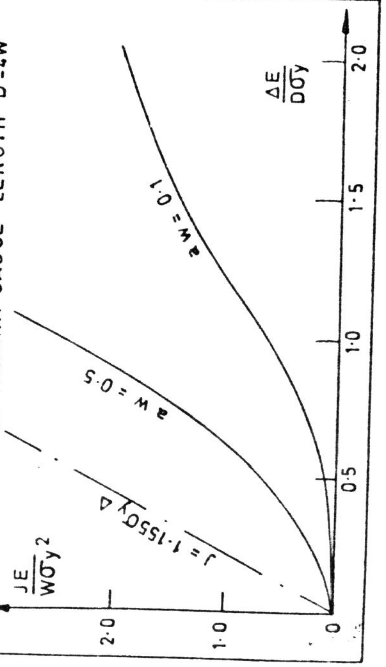

FIG.3: SEN TENSION PLANE STRAIN, J AGAINST OVERALL STRAIN. GAUGE LENGTH D=4W