Kinetic Theory Approach to Fatigue Crack Propagation

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§1. Introduction

As for the model of kinetic theory of fatigue crack propagation it may be classfied into the following two types::

The first type: The model in which microcrack nucleates in the neighbourhood of the main crack through the mechanism of separation of atoms caused by stress concentration due to the notch effect[1)2)3)4)] (Fig. 1 a) or due to the slip (Fig. 1 b) or the mechanism of the condensation of vacancies[5)] (Fig. 1 c), and the main crack join this, the process being repeated.

The second type: The model in which the main crack itself extends without nucleating a microcrack by the mechanism of separation of atoms[6)] (Fig. 1 d) or the mechanism of slip (Fig. 1 e).

With respect to the model of the first type, some theoretical approaches have been attempted in the previous papers.[1)-5)] In the present article, from the viewpoint of the second type an attempt has been made based on dislocation dynamics.

§2. Theoretical considerations

In the present article it is assumed that an irreversible slip occurs at the tip of crack every half cycle under tension, but not during half cycle under compression. As far as this respect is concerned, the model is not inconsistent with those of McEviley,[7] Tomkins[8] and Laird.[9] It may be reasonable to assume the amount of the slip as $\alpha n b$, where n is the number of dislocations emitted from the crack tip, b is Burgers vector, and α is geometrical factor. For instance, in the case of

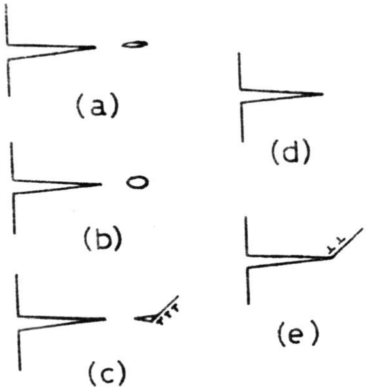

Fig.1 Typical examples of models in kinetic theory of fatigue crack propagation.

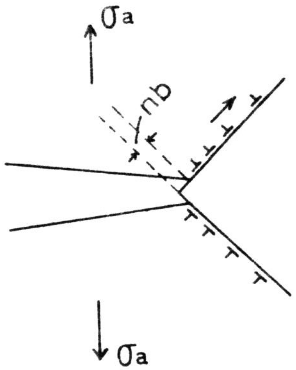

Fig.2 The model for the present analysis.

tension-compression loading as shown in Fig.2, α is $1/\sqrt{2}$. Then fatigue crack propagation rate, dC/dN is written as:

$$\frac{dC}{dN} = \alpha n b , \qquad (1)$$

where C = half length of crack, and N is repeated cycles. On the other hand, n may be approximately estimated

by the following relation:[10]

$$n = \frac{L \pi \Delta\tau}{G b} , \qquad (2)$$

where L = the distance traveled by the lead dislocation. G = modulus of regidity. $\Delta\tau$ = the difference between the effective shear stress τ_{eff} on the dislocation and the applied shear stress τ_a, that is, $\tau_{eff} - \tau_a$. It is also shown that $\Delta\tau = \frac{3}{4(1-\nu)}(\tau_a - \tau_s)$ and thus[10] $\tau_{eff} \simeq 2\tau_a$, thus $\Delta\tau \simeq \tau_a$.

Eq.(2) was derived assuming a semilogarithmic stress-velocity relation for dislocation dynamics, and, however, it was concluded[10] that their arguments are equally valid for the power relation. Thus in the present article we assumed the following power relation:

$$V = V_o \left(\frac{\tau_{eff}}{\tau_o}\right)^m \simeq V_a \left(\frac{\tau_a}{\tau_o}\right)^m , \qquad (3)$$

which is the more conventional expression. V = dislocation velocity. m and τ_o are constants, respectively. $\tau_{eff} \simeq 2\tau_a$ as described above.[10]

In the case of alternating stress, L may be given as:

$$L = \int_0^{\frac{1}{2\omega}} V_1 \, dt , \qquad (4)$$

where V_1 = the velocity of the lead dislocation. ω = stress repeating frequency.

In such a region, as in this case, in the neighbourhood of the crack the averaged local stress $\overline{\tau_l}$ concentrated in the concerned region should be used in Eqs.(2) and (3) instead of the uniformly applied stress τ_a, that is, the nominal stress. Therefore in the present case $\overline{\tau_l}$ should be taken as $\Delta\tau$. At any time the averaged stress $\overline{\tau_l}$ over the distance ε_1 ($>L$) from the crack tip is written as:[2)3)]

$$\tau_l = \frac{(1+\beta)\sigma_y}{2}\left(\frac{\sin 2\pi\omega t\ \Delta K}{\sigma_y\sqrt{(1+\beta)\pi\varepsilon_1}}\right)^{\frac{2\beta}{1+\beta}}, \qquad (5)$$

where $\Delta K = \sqrt{\pi C}\,\sigma_a$ (stress intensity factor), σ_a = applied stress amplitude in terms of tensile stress, σ_y = yield stress and β = cyclic strain hardening exponent.[3)] With respect to time, taking the averaged value of $\sin 2\pi\omega t$ over half cycle, that is, $2/\pi$, then the time averaged value of $\overline{\tau_l}$ is written as:

$$\overline{\tau_l} = \frac{P}{2}\Delta K^{\frac{2\beta}{1+\beta}}, \qquad (6)$$

where $P=(1+\beta)\sigma_y\left(2/\pi\sigma_y\sqrt{(1+\beta)\pi\varepsilon_1}\right)^{\frac{2\beta}{1+\beta}}$. On the other hand, substituting Eq.(6) into Eq.(3), the averaged value $\overline{V_1}$ of the velocity of the lead dislocation during the half cycle is written as

$$\overline{V_1} = V_a\left(\frac{P}{\tau_0}\right)^m \Delta K^{\frac{2\beta}{1+\beta}m}. \qquad (7)$$

Thus L is obtained approximately as:

$$L = \int_0^{\frac{1}{2\omega}} \overline{V_1}\,dt = \frac{V_a}{2\omega}\left(\frac{P}{\tau_0}\right)^m \Delta K^{\frac{2\beta}{1+\beta}m}. \qquad (8)$$

Substituting Eqs.(5) and (8) into Eq.(2), and then putting Eq.(2) into Eq.(1), we obtain:

$$\frac{dC}{dN} = \frac{\pi}{2\sqrt{2}G}\frac{V_a P}{\omega}\left(\frac{P}{\tau_0}\right)^m \Delta K^{\frac{2\beta}{1+\beta}(1+m)}, \qquad (9)$$

which corresponds to the well-known power relation[11)] with respect to ΔK. In many materials $m \simeq 10 \sim 35$ at room temperature.[12)] Taking[8)] $\beta \simeq 0.1$, then, the power coefficient $\frac{2\beta}{1+\beta}(1+m)$ in Eq.(9) ranges from 1.8 to 6.3, which is in good agreement with the trend of the data[11)] on dC/dN. In the case of m having of the type[13)] $m = \alpha_0/RT$ (α_0 = constant),

Eq.(9) may be expressed as the equation of the following type:

$$\frac{dC}{dN} = \frac{A}{\omega}\exp\left[-\frac{U - \frac{2\beta}{1+\beta}(\alpha_0+RT)\ln\Delta K}{RT}\right], \qquad (10)$$

where $U = \alpha_0 \ln(\tau_0/p)$. Furthermore when V_a in Eq.(3) is expressed as[14)] $V^* \exp(-\Delta H/RT)$, then U in Eq.(10) is written as $\Delta H + \alpha_0 \ln(\tau_0/p)$. The equation of the type of Eq.(10) is in well agreement with experimental data on the influence of stress intensity factor,[15)] temperature,[15)] and velocity of cycling on the fatigue crack propagation.

§3. Discussion and conclusions

The temperature dependence of yield stress σ_y included in P should be studied. On the other hand, if elastic stress distribution is taken instead of elastic-plastic, then similar equation as Eq.(10) is still obtained, where σ_y is not included.

Which of the first type and the second type mentioned in §1 is critical is future problem needed to be studied, although it appears that the first one concerns the lower range and the second one the higher range of dC/dN.

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