# Introduction to the Diffusional Theory of Metal Fatigue

S. Pilecki
Warsaw, Poland

Several hypothesis have been advanced up till now aiming at the elucidation of the role of crystal lattice defects in the process of formation of fatigue microcracks. All of them, however, met with numerous reservations. The essential difficulty in studying the problem consist in the material impossibility of direct observations of the process. This prompted the present author to attempt to detect on theoretical premises the factors decisive for the mechanism of formation of fatigue microcracks. This paper forms a part of a more ample study.

The following main denotations are used:

$a_1$ - probability of meeting of cristal lattice defects;
b - coefficient of proliferation of dislocations;
c - coefficient of annihilation of dislocations;
D - coefficient of diffusion of dislocations;
E - energy;
F - obstacle or Frank-Read's source;
k - multiplication factor;
N - number of load cycles applied;
T - temperature;
t - time;
Z - outer surface;
U - dislocation;
$\psi$ - dislocation concentration - the ratio of the number of dislocations intersecting a surface unit to the number of nodes in cristal lattice in this unit;
m, im - index denoting "moving" or "immobile", respectively.

Let us write the formulae describing the typical processes and reactions occuring in pure metal between the dislocations under the action of external loadings. As a result of processes occuring between dislocations point defects may be formed. This, in turn, contributes to the interactions between the dislocations and the point defects and between the point defects mutually. Because of lack of place we do not write here coresponding formulae.

(1) Matrix - $(T, dT/dt) \rightarrow U_{im}$
(2) $U_m + 2F + E \rightarrow kU_m$
(3) $U_{im} + E \rightleftarrows U_m$
(4) $U_m + F \rightarrow U_{im}$
(5) $U_m + U_m \rightarrow 2U_{im}$
(6) $U_m + U_{im} \rightarrow 2U_{im}$
(7) $U_m + U_m \rightarrow U_{im}$
(8) $U_m + U_{im} \rightarrow U_{im}$
(9) $U_m + U_m + E \rightarrow 0$
(10) $U_m + U_{im} + E \rightarrow 0$
(11) $U_m + U_m + E \rightarrow U_{im}$
(12) $U_m + U_{im} + E \rightarrow U_{im}$
(13) $U_m + Z \rightarrow Z$.

The concentration of moving and immobile dislocations dependson their formation, vanishing, diffusion, blocking and putting in motion, as a result of the processes descibed by (1) - (13). Thus we obtain for the moving dislocations:

$$(14)\quad \frac{\partial \psi_m}{\partial N} = D_1 \frac{\partial^2 \psi_m}{\partial N^2} + a_2 kF\psi_m + a_3\psi_{im} - a_4 F\psi_m - 2(a_5+a_7+a_9+a_{11})\psi_m^2 - (a_6+a_8+a_{10}+a_{12})\psi_m\psi_{im}$$

and for immobile dislocations:

$$(15)\quad \frac{\partial \psi_{im}}{\partial N} = -a_3\psi_{im} + a_4 F\psi_m + (2a_5+a_7+a_{11})\psi_m^2 + (a_6-a_{10})\psi_m\psi_{im}.$$

The author obtained a similar system of equations describing the changes of concentration of vacancies, interstitial atoms and their colonies. In view of the complicated form of these equations and their non-linearity, it is impossible to obtain analitical solution of this system. However we obtain the approximate solution of the equation $\partial\psi/\partial N$, $(\psi = \psi_m + \psi_{im})$, in the polar coordinate system, assuming the distribution of $\psi$ in the specimen with circular cross-section to be independent of the angle $\varphi$, under initial condition $\psi(r,0) = \psi_0(r)$ and boundary condition $\psi(r_0, N) = 0$:

$$(16)\quad \frac{\psi(r,N)}{\psi_0} = \sum_{m=1}^{\infty} \frac{2J_0(\mu_m^{(0)}\rho)}{J_1(\mu_m^{(0)})} \exp\left[H\int_0^N \beta(N)\,dN\right],$$

where $\rho = r/r_0$; $\lambda_{m0} = (\mu_m^{(0)}/r_0)^2$; $\beta(N) = \psi_m/\psi$;

$$(17)\quad H = b_0 - c_2 - D_0\lambda_{m0};$$

and the values of $\mu_m^{(0)}$, $J_0(\mu_m^{(0)}\rho)$ and $J_1(\mu_m^{(0)})$ may be determined from the Tables of Bessel's functions. The series (16) is rapidly converging. Owing to this property, we may cofine the numerical computations to its first term only.





On the basis of this solution there are considered several problems, as follows.

Since $\psi$ depends - essentially - on the function $\beta(N)$ and the value of the integral $\int_0^N \beta(N)dN$, maximal concentration of dislocations in the course of metal fatigue we can evaluate directly from (16). The probability of proliferation of dislocation, that is to say, of the very existence of moving dislocations, depends, among others, on the stresses $\sigma_{act}$ acting thereup, and

$$(18)\quad \sigma_{act} = \sigma - m\psi^n,$$

where $\sigma$ - stress induced by external loading; m, n are constants. For $\psi$ sufficiently small, all of the dislocations may be moving dislocations (provided appropriate external stresses are applied), i.e. $\psi_m \cong \psi$ and $\beta(N) \cong 1$. With increasing $\psi$ stresses $\sigma_{act} \rightarrow 0$ and $\beta(N) \rightarrow 0$, too. In the initial period of the fatigue process the power exponent in (16) increases almost proportionally to N, consequently, $\psi(N)$ rises rapidly. In the subsequent stage the effect of N on $\psi(N)$ diminishes more and more, nevertheless, the value of $\psi(N)$ still continues to rise. At $N = N_{lim}$, the concentration $\psi(N)$ reaches its maximal value $\psi_{lim}$ which does not change any more (reaches the state of saturation).

The effect of the amplitude of varying stresses on the concentration of dislocations may be evaluated by the analysis of influence of coefficient H in (16). The intensity of proliferation of dislocations depends, among others, on the value of coeffi-cient $b_0 = a_2 kF$. Since $k = k(\sigma_{act})$,

$$(19)\quad b_0 = a_2 F(\sigma - m\psi^n).$$

With increasing of the amplitude of $\sigma$ the coefficients $b_0$ and H rise, and $\psi(N)$ increases more rapidly. With increasing N and $\psi(N)$, however, $b_0$ and H decrease; they influence on $\psi(N)$ similarly as $\beta(N)$, i.e. $\psi(\sigma, N) \rightarrow \psi_{lim}(\sigma)$.

The role of the surface layer in metal fatigue may be evaluated from the point of view of dislocation diffusion. The possible path of a given dislocation is restricted by the size of grains. Consequently, the region wherein the diffusion of dislocations towards the outer surface proceeds unhampered, is the surface layer. The dislocations situated more deeply, i.e. in the interior of the specimen /in a core/ have no possibilities to penetrate to outer surface. Since for the grains of the core there is $D_{oc}=0$ and , consequently, $H_c=b_o-c_2$ while for the grains of surface layer we have $D_{os}>0$ and $H_s=b_o-c_2-D_{os}\lambda_1$, thus with the same value of $b_o$ /depending on $\sigma_{act}$/ and $c_2$ /depending on $\psi$/ the value $H_c>H_s$. This is to say that the possibilities to attain the value $H_c>0$ are higher in the core than $H_s>0$ in the surface layer. Thus, if under certain conditions in the surface layer, too, there is $H_s>0$, then in view of $H_c>H_s$ the increase of $\psi(N)$ is more rapid in the core than in the surface layer. Moreover, in view of lack of reflux of the dislocations outwards, in the core $\sigma_{act}\rightarrow 0$ and $\psi\rightarrow\psi_{lim}$. This involves a decrease of the value of $b_o$ and an increase of $c_2$ which, in turn, leads to the decrease of $H_c$; at the same time the decreased mobility of dislocations leads to a decrease $\beta(N)\rightarrow 0$ and consequences of it discussed above. As regards the surface layer we are faced with quite a different situation: since $H_s<H_c$, the process of increase of $\psi(N)$ runs more gently than in the core and moreover - and this is more important - in view of the constant diffusion of dislocations towards the outer surface, the value of $\psi<\psi_{lim}$. Thus, in surface layer in view of high values of $b_o$ and $D_o$ the process of generation of new dislocation and of their flight to surface run intensely during the whole period of varying loadings. In other words, the surface layer is characterised by the existence of a stream of dislocations flowing through the whole thickness of the layer towards the outer surface. An important conclusion may be inferred from our considerations: the meetings of dislocations may occur mainly in the surface layer, while the generation of point defects only in this layer, especially at a certain definite depth beneath the surface. Thus, the generation of fatigue cracks cannot be ascribed only to effect of dislocations.

In the specimen three zones may be distinguished differing by the character of $\psi(r,N)$ changes. Two of them /outer and transient/ are situated in the surface layer, while the core is the third zone. The existence of these zones as well as the impossibility of predominance of the process of proliferation of dislocations over those of their vanishing in the vicinity of the outer surface leads to following statement: If the diameter of specimen is smaller than the critical one, the growth of $\psi(r,N)$ cannot occur. Owing to this fact, the strength of whiskers is extremely high. We have shown also that the maximal strength of whiskers within a certain range of their diameters should decrease with the increase of the diameter. Due to similar phenomena occurring in the surface layer of polycrystals the size of grains exerts an effect on the strength of polycrystals.





The differentiation of properties of particular zones of the surface layer leads to the formation of slip bands. In the transient zone there is H=0 but in the neighbouring microregions of the metal certain material properties /among them also of F/ deviate from the average value pertaining to the macroregion. Consequently, in microregion of the transient zone, where - due to the higher local F - the sign of H is positive, a rapid growth of $\psi(N)$ occurs with increasing N. This process is irreversible. Instead, in microregions, where - due to the lower local F - the sign of H is negative, with increasing N a rapid decrease of $\psi(N)$ occurs and this process is irreversible, too. The diffusional tendencies of dislocations contribute to the extension of these microregions towards the outer surface.

The conclusions resulting from those theortical considerations are fully substantiated by numerous facts known from experiments. The problem considered, and, in particular, the conclusions following from the aproximate solution of equations reffering to the dislocations paved the way for the formulation of the hypothesis of diffusional mechanism of numerous phenomena accompanying the fatigue of metals.

References: S. Pilecki - Bulletin de l'Académie Polonaise des Sciences, Sér. Sci. Techn., 17 /1969/, 489 [847]; 543 [949]; 18 /1970/, 57 [57]; 119 [195]; 171 [293]; 245 [467]; 317 [567]; 513 [887]; 521 [895]; 573 [971].