Influence of Non-Singular Stress Terms on Small Scale Yielding at Crack Tips in Elastic-Plastic Materials

S. G. Larsson, A. Janne Carlsson
Dept. Strength of Materials, Roy. Inst. Tech.,
Stockholm, Sweden.

Introduction

One of the basic assumptions behind the applications of linear elastic fracture mechanics to elastic-plastic materials is that plastic deformation at the crack tip is governed by the intensity of the stress singularity, i.e. the stress intensity factor K_I. For this to be true the plastic zone size has to be small compared to other geometric dimensions of the problem as e.g. crack length.

The extent, $2r_p$, of the plastic zone is approximately

$$2r_p = \frac{1}{3\pi}\left(\frac{K_I}{Y}\right)^2$$ for plane strain, where Y is yield strength.

A requirement of a maximum allowable K_I, e.g.

$K_I < \frac{1}{\sqrt{2.5}} Y\sqrt{a}$ as in the ASTM - Tentative Methods for Testing of Plane Strain Fracture Toughness, thus guarantees that plastic zone size is much smaller than crack length.

However, the application of LEFM to elastic-plastic materials requires that also the form of the plastic zone and the state of stress and strain at the crack tip is determined by K_I. If this is true the elastic-plastic crack problem can be solved using a boundary layer (b.l.) approach, [1].

In the present investigation the elastic-plastic problem has been solved for cracks in different types of specimens using the actual boundary conditions instead of the above approach. The solution is restricted to one linearly-elastic-ideally plastic material obeying von Mises flow rule.

It is then found that the b.l. results for the plastic state do not agree with the results obtained using the actual boundary conditions. In order to get agreement one has to add to the boundary stresses in the b.l. case the non-singular term of the x-direction stress, which is in a large region at the crack tip independent of x. The magnitude of this term is found from the FEM-solution for the actual geometry.

Finite element method and element model

A FEM-program for incremental treatment of elastic-plastic structures loaded in plane strain has been developed [2]. The program is based on an elastic-plastic constitutive matrix obtained through inversion of Prandtl-Reuss equations for a material obeying von Mises flow rule, [3]. Triangular, constant strain elements are used. For symmetry reasons only the marked parts of the specimens in fig. 1 has to be treated. In all cases the FEM-representation shown in fig. 2 was used for the region close to the crack tip. The region inside $r_o = 0.024\ a$ is composed of 48 elements. The region $0.024\ a < r < 0.8\ a$ contains 375 elements (degrees of freedom 476). About 100 elements are used to form the remaining parts of the specimens. Loading is incremental and at most one element is plasticized at each load increment. Further the load increments must not be larger than corresponding to $\Delta K_{I\ max} = 0.01\ Y\sqrt{a}$. This gives a better than 0.5% correspondence between values of effective stress and effective plastic strain determined this way and the stress-strain curve of the material. The computations were made on an IBM 360/75 and each load increment required a computing time of 6.3 sec. The number of load increments was about 200.

Numerical results

The specimens studied are characterized by the dimensionless parameters $\frac{W}{a} = 2.0$, $\nu = 0.3$, and $\frac{E}{Y} = 400.0$. In all cases yielding begins in the element just above the crack tip (b in fig. 2). This occurs at the values of K_I given in table 1 where K_I is determined according to [4] - [6]. Mean values of stress obtained from the analytic solution by integration over an area corrsponding to the elements in fig. 2 show that the mentioned element should yield first and at $K_I = 0.174\ Y\sqrt{a}$.

In fig. 3 the obtained plastic zones are compared for the different specimens and for load levels below the highest allowable for small scale yielding, according to ASTM, viz. $K_I = 0.6\ Y\sqrt{a}$. Results are also given for the b.l. solution. Length coordinates x and y are normalized with the characteristic length parameter $\left(\frac{K_I}{Y}\right)^2$.

The variation of displacement in the first nodal point behind the crack front, i.e. point k in fig. 2, is shown as a function of load in fig. 4. The coordinates of point k are $x = -0.006\ a$, $y = 0$. Fig. 5 shows the stress σ_y in the first nodal point in front of the crack ($x = 0.006\ a$, $y = 0$ or point l in fig. 2) as a function of K_I . Fig. 6 shows the effective plastic strain in the first plasticized element as a function of K_I .

The case is also studied when to the boundary conditions of the b.l. solution is added the non-singular term of the x-direction stress, $T_{xx} = -58.93 \cdot 10^{-2}\ K_I/\sqrt{a}$, of the center-cracked specimen. Corresponding results are given in fig. 4-6 as circles and in table 1. These results are to be compared with results for the center-cracked specimen. In fig. 7 also the plastic zone sizes at $K_I = 0.6\ Y\sqrt{a}$ have been compared for these two cases.

References

[1] J.R. Rice, Fracture, An Advanced Treatise, Vol. II, Academic Press, New York (1968).

[2] G. Härkegård, S.G. Larsson, Hållf.publ. 177, KTH, Stockholm (1972).

[3] Y. Yamada, N. Yoshimura, T. Sakurai, Int. J. Mech. Sci., 10, 343 (1968).

[4] Annual Book of ASTM Standards, Part 31, 911 (1970). American Society for Testing and Materials, Philadelphia.

[5] O.L. Bowie, J. Appl. Mech., 31, 208 (1964).

[6] M. Isida, Int. J. Fracture Mech., 7, 301 (1971).

Table 1

Problem	K_I	Ref.
Center cracked specimen	$0.151\ Y\sqrt{a}$	[6]
Double edge cracked specimen	$0.153\ Y\sqrt{a}$	[5]
Bend specimen	$0.166\ Y\sqrt{a}$	[4]
Compact tension specimen	$0.170\ Y\sqrt{a}$	[4]
Boundary layer approach, $T_{xx} = 0$	$0.161\ Y\sqrt{a}$	
-"- -"- -"- , $T_{xx} = - 58.93\cdot10^{-2} K_I/\sqrt{a}$	$0.151\ Y\sqrt{a}$	

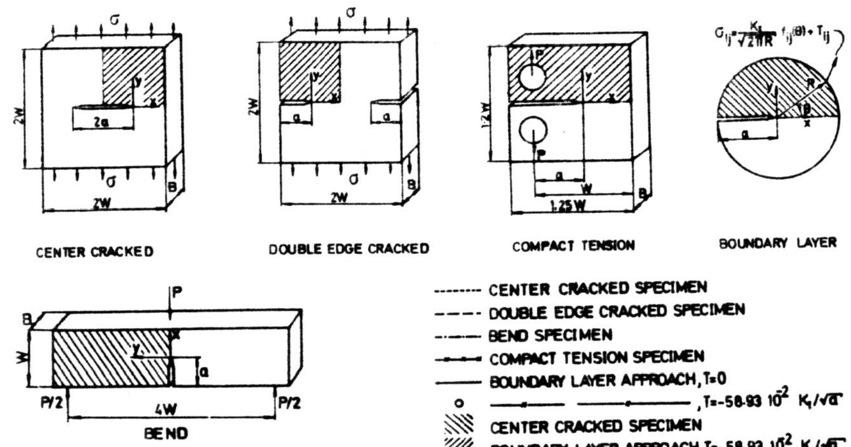

FIG 1.

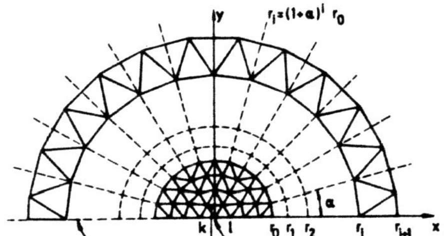

FIG 2.

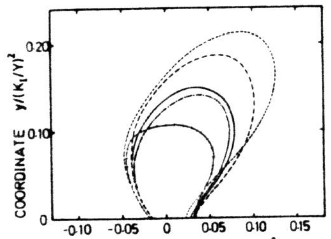

FIG 3.

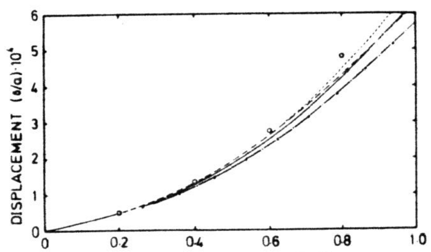

FIG 4. STRESS INTENSITY FACTOR $K_I/Y\sqrt{a}$

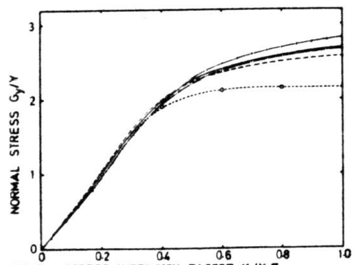

FIG 5. STRESS INTENSITY FACTOR $K_I/Y\sqrt{a}$

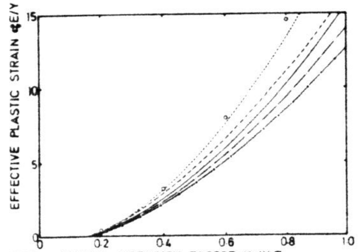

FIG 6. STRESS INTENSITY FACTOR $K_I/Y\sqrt{a}$

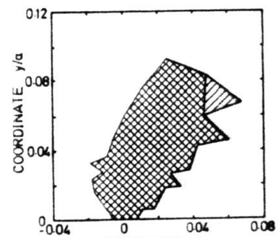

FIG 7. COORDINATE x/a